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ELECTRICAL MEASUREMENTS IN PRACTICE

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BY

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PREFACE

The subject of electrical measurements has received much attention in the literature of electrical engineering. In general, however, the treatment has been from a theoretical or academic point of view. In this volume the author has endeavored to present the subject in a simple, practical manner and from the standpoint of engineers who are actively engaged in making measurements, tests and investigations in the electrical industry.

All classes of measurements that the laboratory and testing engineer is ordinarily called upon to make have been covered. The method of treatment and part of the material is the same as that in the author's contribution on this subject to the Fourth Edition of the Standard Handbook for Electrical Engineers.

Instruments must naturally form a prominent part of any discussion of electrical measurements, but detailed descriptive matter pertaining to commercial instruments has been limited to those instruments in most general use and without which the book would be obviously incomplete. Maximum demand meters have, however, been described rather extensively because they are a comparatively recent development. A short chapter has also been devoted to curve-drawing instruments because of their important part in commercial measurements and in laboratory investigations.

The author wishes to acknowledge his obligation to the Electrical Testing Laboratories for much data and information made available to him; to Dr. C. H. Sharp and Mr. Gordon Thompson for reading portions of the proof and making valued criticisms; to those manufacturers who furnished electrotypes; and, finally, to express the most grateful appreciation to his friend Mr. Little M. Dudley for able and enthusiastic assistance in the preparation of the manuscript.

The author will appreciate having his attention brought to any errors of omission and commission that the reader may note.

F. M. F.

New York City,
June, 1917.

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ELECTRICAL MEASUREMENTS IN PRACTICE

CHAPTER I

INTRODUCTORY

1. Measurements.—The measurement of any given quantity is the comparison of that quantity with another quantity of the same kind which has been chosen as a unit. In discussions of the theory of measurements, measurements are usually classified as direct or indirect. Direct measurements are those in which the numeric, or number expressing the magnitude of the quantity being measured, is determined by direct comparison with a standard, as, for example, the determination of a length with a yardstick. In indirect measurements the numeric is obtained directly from a formula showing the relation between the quantity being measured and one or more other quantities which have been directly measured. For example, when current is measured in terms of fall of potential and resistance, it is calculated from the relation $I = E/R$.

2. Units.—As stated above, a unit is that quantity with which another quantity of the same kind is compared when making a measurement. It may be a purely arbitrary quantity with no rational significance, such as the foot and the pound. On the other hand, a unit may have a very definite meaning. For example, a centimeter is one-hundredth part of a meter which in turn is supposed to be the ten-millionth part of the northern quadrant of the earth or the distance between the pole and the equator at the meridian of Paris. Similarly, the gram is the mass of 1 c.c. of water at 4°C., the temperature at which the density of water is greatest.

3. Fundamental or Absolute Electrical Units.—The fundamental electrical units are based on the c.g.s. system of absolute units. This system is founded on the centimeter, gram and second—units which have a perfectly definite significance. All physical measurements are established in this system on the theory that all physical phenomena are the result of matter and motion, that is, space (centimeters), mass (grams), and time (seconds).

There are two c.g.s. systems of electrical units, the electromagnetic and the electrostatic. The electromagnetic system is derived from the unit magnetic pole defined as a pole of such strength that it repels a similar pole at a distance of 1 cm. with a force of 1 dyne. The electrostatic system is but little used and all electrical measurements are based on the electromagnetic system.

The various units in these two systems have never been officially named, but some writers prefix the name of the practical unit with "ab" or "abs," and "stat" when referring to the corresponding c.g.s. unit. Thus "absohm" refers to the absolute unit of resistance in the electromagnetic system and "statohm" refers to the corresponding unit in the electrostatic system.

4. Practical Units and Standards.—A standard is the concrete representation of a unit. The fundamental c.g.s. units are difficult to represent and cannot, therefore, be used directly in ordinary electrical measurements. It was recognized early in the development of the art that convenient standards were necessary. Consequently a system of "practical" units was established which was derived from the c.g.s. electromagnetic units and which could be represented by definite, concrete and reproducible standards.

The following are the definitions of the three principal practical units and the corresponding standards, which were adopted by the International Electrical Congress held in Chicago in 1893 and made official in the United States by Act of Congress, July 12, 1894:

The international ohm, "which is based upon the ohm equal to 10^9 units of resistance of the c.g.s. system of electromagnetic units, is represented by the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 grams in mass, of a constant cross-sectional area and of the length of 106.3 cm."

The international ampere "is one-tenth of the unit of current of the c.g.s. system of electromagnetic units, and is represented sufficiently well for practical use by the unvarying current, which when passed through a solution of nitrate of silver in water and in accordance with the accompanying specifications, deposits silver at the rate of 0.001118 of a gram per second."

The international volt "is the electromotive force that, steadily applied to a conductor whose resistance is one international ohm, will produce a current of one international ampere, and which is represented sufficiently well for practical use by 1,000/1,434 of the electromotive force between the poles of the voltaic cell, known as Clark's cell, at a temperature of 15°C."

It will be noted that these definitions are not absolutely consistent. The unit of current is specifically fixed by its value in absolute or c.g.s. electromagnetic units, while the unit of resistance is specifically fixed by the resistance of a certain column of mercury. The International Electrical Conference held in London in 1908 eliminated this inconsistency. It also adopted separate names and definitions for the practical units defined in terms of absolute units and for those defined in terms of concrete standards.[1] The three principal units so defined are as follows:[2]

The *ohm,* the unit of electric resistance. It has the value of 1,000,000,000 (10^9) in terms of the centimeter and the second.

The *ampere,* the unit of electric current. It has the value of one-tenth (0.1) in terms of the centimeter, gram and second.

The *volt,* the unit of electromotive force. It has the value of 100,000,000 (10^8) in terms of the centimeter, gram and second.

The *international ohm* is the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 grams in mass, of a constant cross-sectional area and of a length of 106.300 cm.

The *international ampere* is the unvarying electric current which, when passed through a solution of nitrate of silver in water, deposits silver at the rate of 0.00111800 of a gram per second.

The *international volt* is the electrical pressure which, when steadily applied to a conductor the resistance of which is one international ohm, will produce a current of one international ampere.

In all ordinary measurements, although based on the "international" values, it is customary to omit the prefix. Hence unless otherwise stated, the values as fixed by the above definitions for "international" units are always implied.

5. Relation between Absolute and Practical Units.—The following table[3] shows the relation between the practical units and the absolute units in both the electromagnetic system and the electrostatic system.

[1] "The Principles Involved in the Selection and Definition of the Fundamental Electrical Units to be Proposed for International Adoption," F. A. Wolff, *Bulletin,* Bureau of Standards, vol. 5, p. 243 (1908–09). Also *Proceedings,* American Physical Society, meeting April 25, 1908.

[2] These definitions have not as yet been adopted officially by Congressional action, although subscribed to by the United States delegation to the London Conference.

[3] From "Standard Handbook for Electrical Engineers," 4th edition, 1915, p. 15.

Quantity	Symbol	Practical unit	Absolute C.G.S. Unit.	
			Electromagnetic	Electrostatic
Electromotive force.	E	volt = 10^8 abvolts	abvolt = 10^{-8} volt	statvolt = 300 volts
Resistance........	R	ohm = 10^9 absohms	absohm = 10^{-9} ohm	statohm = 9×10^{11} ohms
Current...........	I	ampere = 10^{-1} absampere	absampere = 10 amperes	statampere = 3.333×10^{-10} ampere
Quantity..........	Q	coulomb = 10^{-1} abcoulomb	abcoulomb = 10 coulombs	statcoulomb = 3.333×10^{-10} coulomb
Capacitance.......	C	farad = 10^{-9} abfarad	abfarad = 10^9 farads	statfarad = 1.111×10^{-12} farad
Inductance........	L	henry = 10^9 abhenrys	abhenry = 10^{-9} henry	stathenry = 9×10^{11} henrys
Energy...........	W	joule = 10^7 abjoules (ergs)	abjoule (erg) = 10^{-7} joule	statjoule (erg) = 10^{-7} joule
Power............	P	watt = 10^7 abwatts $\frac{\text{(ergs)}}{\text{(sec.)}}$	abwatt $\frac{\text{(ergs)}}{\text{(sec.)}}$ = 10^{-7} watt	statwatt $\frac{\text{(ergs)}}{\text{(sec.)}}$ = 10^{-7} watt

6. Primary and Secondary Standards.—The distinction between primary and secondary standards is largely a matter of viewpoint. In general, however, primary standards may be considered as those which represent directly by definition the practical unit involved, such as the mercury ohm standard, the silver voltameter standard and the saturated cadmium cell standard.

Secondary standards are the more practicable working standards which are standardized by comparison with primary standards and used as the basis of all ordinary measurements. They include, for example, the manganin standard resistors and the Weston-type standard cell. Primary standards, are, in general, maintained only by the government custodians of the standards in the various countries;[1] whereas secondary standards or working standards, based on these primary standards, serve as the fundamental basis of practical measurements in engineering and commercial fields.

7. Precision.—The absolute true value of a quantity cannot be determined by measurement, due to unavoidable errors in methods, limitations of the human senses, and so forth. The probable true value can, however, be stated, as a value which lies between two assigned limits. The precision of the measurement is the numerical expression indicating these limits. In other words, it is the degree of reliability of the result obtained and indicates the probable limit of the difference between the value obtained and the absolute true value. It is thus apparent that the result of a measurement is, strictly speaking, incomplete unless it includes a numerical statement of the precision. For example, if the measured value of an electromotive force is 110.23 volts, and it is believed that that value is within 0.03 volt of the absolute value, the precision of the measurement is ± 0.03 volt and the result of the measurement would be stated as 110.23 ± 0.03 volts.

The precision of a measurement obviously depends upon the unknown errors which enter the measurement. These are usually classified as systematic and accidental. Systematic errors are those which affect similar measurements in the same way, such as, for example, the degree of reliability or the unknown error

[1] England, at National Physical Laboratory, Teddington; France, at Laboratoire Central d'Electricité, Paris; Germany, at the Physikalisch-Technische Reichsanstalt, Berlin; United States, at Bureau of Standards, Washington.

of the standard. Accidental errors are those over which the observer has no control and which are as likely to be positive as negative, and therefore, tend to become eliminated from the average of a number of observations. The value to be assigned to the precision is based on a determination of the accidental errors and an estimate of the systematic errors after a careful examination of the various factors in the measurement.

In precise measurements, various mathematical methods may be employed to determine the precision of the measurement, after systematic errors have been eliminated. One method is that which gives the "probable error," a strictly mathematical figure by means of which the relative precision, compared to other measurements, can be judged.

In direct measurements, the most probable result—that is, the value which is probably nearest to the true value—may for all practical purposes be taken as the arithmetical mean of the various observations. The precision of the measurement will then be the sum of the estimated systematic errors plus the mean of the differences between the mean of the observations and each individual observation.

In indirect measurements, that is, measurements made by computation from one or more direct measurements, the precision may be calculated from the precision of each of the direct measurements. Thus let

$$A = f(m \times n \times o \times \ldots).$$

Then a small variation, dm, in m, will produce a certain variation in A which will be indicated by Xm. Similarly, a small variation, dn, in n, will produce a variation in A of X_n, etc. The total or combined variation in A due to each of the small variations in the several variables will be

$$X = \sqrt{X_m^2 + X_n^2 + X_o^2 \ldots}.$$

The values of X_m, X_n, etc., are obtained by differentiating the function with respect to each variable.

As an illustration let us assume that the power consumption of an electrically heated device has been measured and it is desired to determine the precision of the measurement. The resistance and current were measured and the power calculated from the relation:

$$W = I^2R.$$
$$I = 10.4 \text{ amp., reliable to } 0.1 \text{ amp.}$$
$$R = 11.85 \text{ ohms, reliable to } 0.05 \text{ ohm.}$$
$$W = 1{,}281.7 \text{ watts.}$$

Differentiating, first with respect to I,

$$\frac{dW}{dI} = d(I^2)R = 2IRdI = 2 \times 10.4 \times 11.85 \times 0.1 = 24 = X_m$$

in general case.

Then differentiating with respect to R,

$$\frac{dW}{dR} = I^2dR = 10.4^2 \times 0.05 = 5.0 = X_n \text{ in general case.}$$

The precision or reliability of W is, therefore,

$$X = \sqrt{24^2 + 5^2} = 25 \text{ watts.}$$

It is to be noted that it is useless to use more than two significant figures in computations of this character. While the numbers 10.4 and 11.85 in the above example are shown in full in the differentiating operation to facilitate identification, the actual multiplication was performed using 10 and 12 respectively.

8. Precision of Commercial Measurements.—The precision obtainable in an electrical measurement depends upon the various factors which enter into the determination; among these are the correctness of the principle employed and the method used, accuracy of the standards, number and magnitude of possible errors, correctness of calculations and so forth. In many precision measurements, a precision of 1 part in 100,000 in certain classes of measurements is regularly attained. In commercial measurements, the cost of such a high degree of precision is not justified. The limits, however, are being gradually raised as the art develops and greater refinements are introduced. The following table indicates the precision which may be reasonably expected in various classes of commercial measurements made by average observers, under ordinary conditions and with standard commercial instruments.

Average Precision to be Expected in Various Classes of Commercial Measurements

Class of Measurements and Method. (Deflection of two-third scale assumed in indicating instruments)	Probable precision, per cent.
Current	
Potentiometer, high-grade types	0.03
Portable ammeter, continuous-current	0.4

Portable ammeter, alternating-current	0.5–1.0[1]
Switchboard ammeter, continuous-current	1.5
Switchboard ammeter, alternating-current	1.0–2.5[1]
Curve-drawing instrument	1.5–3.0
Potential	
Potentiometer, high-grade type	0.02
Portable voltmeter, continuous-current	0.25
Portable voltmeter, alternating-current	0.5–1.5[1]
Switchboard voltmeter, continuous-current	1.0
Switchboard voltmeter, alternating-current	1.0–2.5[1]
Curve-drawing instrument	1.5–3.0
High potential (as employed for testing insulators, etc.)	5.0
Power	
Portable wattmeter	1.0
Laboratory wattmeter (non-portable or semiportable)	0.25
Switchboard wattmeter	2.0
Curve-drawing instrument	2.0–4.0
Energy (watt-hour meters)	
Continuous-current	2.5
Alternating-current, single-phase, no transformers	2.0
Alternating-current, single-phase, with transformers	2.5
Alternating-current, polyphase, with transformers	3.0
Frequency	
Portable instrument	0.5
Switchboard instrument	1.5
Power-factor	
With portable instruments (ammeter, voltmeter, wattmeter)	2.0
Switchboard instrument (above 90 per cent.)	1.0
Switchboard instrument (below 90 per cent.)	2.0–4.0
Resistance	
Medium: 1.0 to 10,000 ohms with Wheatstone bridge, high-grade	0.1
Less than 1 ohm, over 10,000 ohms with Wheatstone bridge, high-grade	0.2–1.0
Portable bridges	0.5–2.0
Low: Fall of potential method	0.5
Thomson double bridge	0.05
High: (Insulation measurements)	5.0
Conductivity	0.25
Magnetic measurements	2.5

[1] Depends upon capacity of instrument and whether used with a transformer.

9. General Precautions in Electrical Measurements.—The following are some of the sources of error which are most frequently found in electrical measurements and which, therefore, should be borne in mind.

(*a*) The probable accuracy of the standards, instruments and methods should be known but care should be taken to distinguish between accuracy and sensitivity. For example, an indicating instrument may be so sensitive that a change of 0.1 per cent. in the quantity being measured can be detected, but, due to defects or errors in the design, it is not reliable to less than 1 per cent. Obviously the latter limits the accuracy of the instrument. Again the certified value on a certificate accompanying a standard resistor may be given to 0.05 per cent., or 5 parts in 10,000, when the limit of reliability is only 0.2 per cent. due to thermo e.m.fs., a temperature coefficient or change in potential drop with current because of improper location of the potential taps.

(*b*) As a general proposition, in other than rough determinations, one measurement should not be relied upon Several readings should be taken, and the conditions should be altered, wherever possible, in order to avoid systematic errors. Thus, in measuring a resistance by the fall of potential method, the result of one reading of amperes and volts is not as reliable as the mean of several, each taken with a slightly different current, noting, however, that the precision increases only as the square root of the number of readings. Or, if a resistance is measured with a bridge, the average of several measurements with different settings is more accurate than a single measurement.

(*c*) Indicating instruments should be of such a range that the quantity being measured will produce a reasonably large deflection on the scale. The percentage observational error decreases in direct proportion as the magnitude of the deflection increases.

(*d*) The possible presence of external or stray magnetic fields, both direct and alternating, should always be borne in mind. Such fields may be produced by current in neighboring conductors, or by various classes of electrical machinery and apparatus, structural iron and steel in buildings, and so forth. These fields introduce errors by combining with the fields of portable indicating instruments, galvanometers and other instruments utilizing a magnetic field, and also in the case of alternating fields, by inducing small e.m.fs. in the loops formed in bridge po-

tentiometers, etc. Commercial indicating instruments equipped with magnetic shields are available, but it is not always safe to depend absolutely upon them and the instrument should be placed as far as possible from any suspected source of the disturbance.

The presence of an external field may be detected by noting the change in the indication (assuming that the quantity being measured remains constant) when the instrument is turned through an angle of 180°. In the case of continuous current the average indication will be the correct value. Stray fields are discussed further in connection with the measurement of the various electrical quantities.

(*e*) In measurements involving high resistances and galvanometers, such as bridges and potentiometers, possible "leakage" or shunt circuits should be eliminated. This is done by providing a "guard" circuit, the purpose of which is to keep all points to which the current might flow improperly, at the same potential as the highest in the apparatus or to shunt the leakage current around the measuring instrument. See potentiometers and insulation resistance measurements.

(*f*) Temperature changes in various parts of bridge, potentiometer and similar circuits should be avoided because of thermo e.m.fs. produced at the junction of dissimilar metals. Such effects are often produced if the observer's hand comes into contact with the metal parts of the galvanometer key, switches, and so forth.

(*g*) Instruments with glass and hard-rubber covers should not be rubbed, especially with a dry cloth. The induced electrostatic charge on the moving element is often sufficient to change the deflection materially.

(*h*) At potentials of 500 volts and above, the electrostatic attraction between moving and fixed parts may become serious. This is prevented by keeping the two parts at the same electrostatic potential. When grounding is permissible, this can be done by connecting the circuit to earth at the point where the instrument is connected, care being taken that the moving-coil end of the instrument is on the ground side. In very high-potential work this electrostatic attraction becomes very troublesome, so that the instruments must be connected in circuit at a grounded part of the line, or else thoroughly insulated from ground and the moving element connected to the case or to an electrostatic shield around the instrument. The latter may be conveniently made from ordinary iron-wire netting.

CHAPTER II

GALVANOMETERS

10. General.—A galvanometer is an instrument for measuring electric current by utilizing the magnetic field produced by the current. Strictly speaking, this would include such instruments as ammeters and voltmeters, but the term is ordinarily understood to apply to instruments used to measure very small currents.

In the early days of the art galvanometers were used for the direct measurement of all electrical quantities, large as well as small. While they are still used for direct measurement of very small electrical quantities, their very wide application is as detectors or indicators in connection with other apparatus for the measurement of current, electromotive force, quantity of electricity, electrostatic capacity and so forth. Galvanometers and other types of detectors are used so extensively in all classes of electrical measurements, both continuous-current and alternating-current, that their discussion in a separate chapter is considered justifiable.

In general, galvanometers are classified as moving-magnet or moving-coil for both continuous current and alternating current. In a galvanometer of the moving-magnet class a small, temporary or permanent magnet is suspended at the center of a coil of wire through which flows the current to be measured, producing a deflection. In moving-coil galvanometers, a coil of relatively fine wire is suspended in a permanent or electro-magnetic field; the current to be measured flows through this coil, producing a deflection. Important examples of moving-magnet instruments are the tangent galvanometer and the Kelvin galvanometer; of moving-coil instruments the D'Arsonval and electrodynamometer type galvanometers.

The principal types of galvanometers are briefly described in the final paragraphs of this chapter arranged in accordance with the above classification.

11. Mounting.—High-sensitivity reflecting instruments are more or less affected by mechanical vibrations and should be so

mounted that this effect is avoided or minimized. In a heavy brick or concrete building, the walls are usually free from vibration and a stone shelf attached to the wall makes an excellent support. Masonry piers carried to solid earth and kept free from contact with the building are excellent but costly. Satisfactory results will usually be obtained with a pier about 3 ft. high resting on about 3 in. of felt.

Where the vibration is very serious, its effect may be eliminated by mounting the instrument on a platform hung from springs and carrying an adjustable weight, *W*, as shown in Fig.

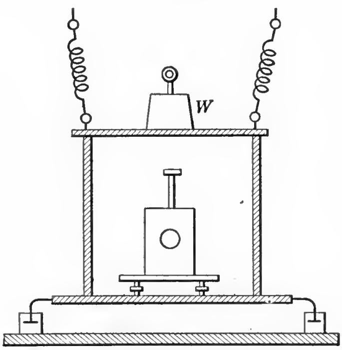


Fig. 1.

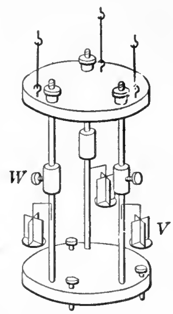


Fig. 2.

1. The theory of this method of mounting is that the natural period of the whole mass is made so large that the vibrations are damped out; slow vibrations or motions are prevented by oil dashpots. The Julius suspension, Fig. 2, utilizes the same principle, the weights, *W*, being used to adjust the center of gravity. This apparatus is suspended from small steel wires of considerable length. The scales of reflecting instruments should be on a separate support unless the instrument itself is on a very solid support.

Characteristics of Galvanometers

12. Sensitivity and Constants.—The sensitivity of a galvanometer is expressed in one of several ways, the numerical expression of which is called the galvanometer constant.

The constant of a galvanometer is variously defined as follows:

(*a*) The *ampere constant* is the current in microamperes (millionths of an ampere) which will produce 1 mm. deflection on a scale 1 meter distant. Or it may be expressed as the deflec-

tion which will be produced by 1 microampere. This constant is obviously the fundamental one, because the deflection is always produced by the current which passes through the instrument, rather than the quantity of electricity or the potential. The other constants are, therefore, simply derived from the ampere constant.

(*b*) The *megohm constant* is the number of megohms in series with the galvanometer through which 1 volt will produce 1 mm. deflection on a scale 1 meter distant. In routine tests of insulation resistances a "working constant" is often used. It is the number of scale divisions deflection which the working voltage will produce through one megohm.

(*c*) The *volt constant* is the potential in microvolts (millionths of a volt), across the galvanometer terminals which will produce a deflection of 1 mm. on a scale 1 meter distant. Or it may be expressed as the deflection in millimeters which will be produced by 1 microvolt.

(*d*) The *coulomb constant* refers to ballistic galvanometers and is the quantity in microcoulombs (millionths of a coulomb) which will produce 1 mm. deflection on a scale 1 meter distant.

High-current sensitivity is desirable for high-resistance measurements, such as insulation resistance. High-voltage sensitivity is desirable for measurements involving small potentials, such as low-resistance bridges, potentiometers, and so forth. High-coulomb sensitivity is desirable in magnetic tests.

13. "Figure of merit" is a term formerly used to measure the sensitivity of galvanometers and is the current which will produce 1 scale division deflection. Obviously it is meaningless for comparative purposes unless the value of a scale division and the distance to the scale are given.

14. "Factor of merit"[1] is a term sometimes used which is intended to be a complete relative measure of the sensitivity. That is, in addition to the current sensitivity as defined above (ampere constant) it includes the period and the resistance of the galvanometer. It is calculated as follows:

$$K_m = \frac{100 \times K_i}{t^2 \, (r)^{3/5}}$$

where K_m = factor of merit, K_i = ampere constant expressed as millimeters deflection produced by 1 microampere at 1 meter,

[1] Scientific Instrument Company, Ltd., List No. 126, p. 8 (Dec., 1913).

t = galvanometer period (undamped) in seconds and r = galvanometer resistance in ohms. In other words, this quantity assumes that the sensitivity of a galvanometer varies directly with the ampere constant, inversely as the square of the undamped period and inversely as the $\frac{2}{5}$ power of the resistance.

The sensitivity of galvanometers where a microscope is used, as in the Einthoven galvanometer, obviously cannot be expressed in the above constants. The same is true of alternating-current galvanometers where the deflection is proportional to the square of the current. It is, therefore, usual to indicate the sensitivity of such galvanometers by stating the minimum current which can be detected. This may be fairly compared with one-tenth of the ampere constant (microamperes per millimeter deflection) of the other types of galvanometers on the assumption that 0.1 mm. deflection can be detected.

15. Damping.—This term is applied to those forces which collectively bring the moving system to rest after it has been set in motion. In order to shorten the period and facilitate readings, damping sometimes is intentionally produced mechanically with air vanes, or more generally, by electrical means. The latter means include: (*a*) use of a metal frame on which the moving coil is wound and in which eddy currents are produced; (*b*) use of a closed loop of copper wire attached to the coil and in which eddy currents are produced; (*c*) use of the generator action of the swinging coil when its circuit is closed. The latter is the more convenient method for general purposes because the damping can be readily changed by means of resistances in series or in parallel with the galvanometer.

16. Critical damping[1] is that condition of damping where the movable system comes to zero position, or equilibrium, but does not pass through it, in the shortest time after deflection. A galvanometer is *aperiodic* when it is critically damped. It is *dead beat* when the movable system deflects to its final position or reading without oscillation. The *critical resistance* is that resistance in series with the coil, including the suspension and connecting leads, which will produce critical damping.

The damping of a galvanometer is often measured by the *logarithmic decrement* which is the ratio of the logarithm of the amplitude of the swing on one side of the zero position to the

[1] "General Design of Critically Damped Galvanometers," Frank Wenner, *Bulletin*, Bureau of Standards, vol. 13, No. 2, p. 211 (1916–17).

logarithm of the amplitude of the next succeeding swing on the other side of the zero position.

17. Period.—The period of a galvanometer is the time in seconds required for one complete oscillation, or the time between two successive passages through the zero or mid-position in the same direction.

Galvanometer Shunts

17a. General.—A galvanometer shunt is a combination of resistances so arranged and connected to a continuous-current galvanometer that the constant of the galvanometer can be quickly changed from one value to another. In null methods, the first adjustments are usually such as to cause currents to flow in the galvanometer circuit which are far beyond the capacity of the galvanometer, hence a shunt is employed which is gradually removed as a condition of exact balance is approached. In deflection methods, the range of the instrument is greatly extended by using a shunt so that it is applicable to measurements of current beyond the capacity of the instrument.

18. Plain Shunts.—An ordinary resistance box may be employed as a shunt in many classes of measurements, being con-

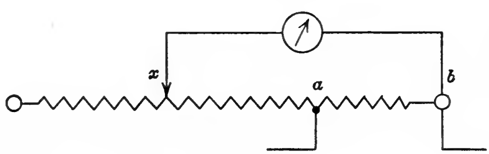


Fig. 3.

nected as shown in Fig. 3. The reading of the galvanometer is multiplied by the constant

$$k = \frac{R_g + R_s}{R_s}$$

where R_g = resistance of galvanometer plus the resistance aX, and R_s = resistance ab. Where a shunt of this type is regularly employed, the resistances are so adjusted that certain convenient values of k are given directly without calculation. These values are usually multiples of 10 and

$$R_s = \frac{R_g}{9}, \frac{R_g}{99}, \frac{R_g}{999}, \text{ and } \frac{R_g}{9{,}999}$$

thus making k = 10, 100, 1,000 and 10,000 respectively.

It is to be noted that the ratio always involves the galvanometer resistance so that a given shunt can be used only with the galvanometer for which it is adjusted if deflection readings are to be taken. Furthermore, the value of k, as calculated above, applies only when the current in the main circuit is the same whether the galvanometer is shunted or not shunted. This will

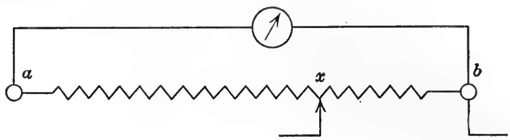


FIG. 4.

obviously not be the case in a low-resistance circuit. In null measurements, this precaution is unnecessary because the exact value of the shunt ratio is unimportant.

19. Universal Shunts.—In an Ayrton or universal shunt, the resistance is so arranged that the shunt can, theoretically, be used with a galvanometer of any resistance. When, in Fig. 4, the movable contact is at a,

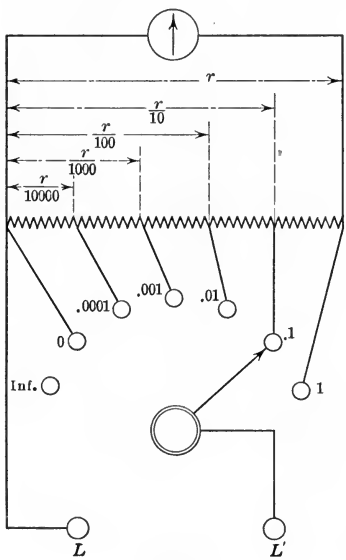


FIG. 5.

$$I'_g = I \frac{r_{ab}}{r_{ab} + r_g}$$

where I'_g = current through galvanometer, I = current from battery (current to be measured), r_{ab} = total resistance of shunt between a and b and r_g = resistance of galvanometer. When the contact is moved to X

$$I''_g = I'_g \frac{r_{bx}}{r_{ab}}$$

assuming that the battery or main current is not altered by the change.

In this equation, the resistance of the galvanometer does not appear. Hence, if the galvanometer constant is obtained with the entire shunt in circuit (x at a), the shunt ratio at any other portion of x is r_{bx}/r_{ab}. The universal shunts supplied by the

Leeds and Northrup Co. are arranged as shown diagrammatically in Fig. 5. The ratio is changed by moving a brush contact arm over the various contact studs as shown, the ratio at each position being indicated.

While, theoretically, such a shunt can be used with any galvanometer, it is customary to select a shunt which has about 10 times the resistance of the galvanometer. Although such a resistance reduces the sensitivity about 10 per cent., the effect in the combined resistance with different shunt ratios is small except for relatively low-resistance circuits.

Selection of a Galvanometer

The various characteristics of a galvanometer are so interrelated that in general an advantage in any one direction is gained at the expense of some other desirable feature. In certain special cases, some one characteristic may be the important consideration but for ordinary electrical measurements, the galvanometer best suited to the particular condition or class of work contemplated is a compromise between several features, the more important of which are discussed in the following paragraphs.

20. Ruggedness.—It is extremely desirable that a galvanometer which is to be used under ordinary laboratory conditions be sufficiently robust to withstand moderate mechanical disturbances and occasional violent swings without getting out of adjustment. Where the air gap is made small and the clearance between the coil and the pole faces reduced to a minimum in order to gain sensitivity, much time and patience is required to get the coil to swing freely. Furthermore, a particle of lint, dust or iron may cause endless trouble.

The construction should be such that the suspension can be easily changed or replaced.

21. Stability of Zero.—The suspension should be a material which will not take a set after being deflected and fail to return to a constant zero. If the suspension is made too light in order to increase the sensitivity, its torque becomes so weak that the zero position becomes unstable. The most commonly used suspensions are phosphor bronze in the form of flat strips. Steel is also being used to some extent.

22. Resistance.—The resistance of a galvanometer should be suitable for the work on which it is to be used. In general, high-resistance galvanometers (over 1,000 ohms) should be used for

the measurement of small currents or high resistances such as the resistance of an insulating material. A low-resistance galvanometer (less than 100 ohms) should be selected for measurements involving small e.m.fs., such as measurements with a low-resistance potentiometer and thermocouples. For general purposes, such as Wheatstone-bridge measurements, a medium-resistance instrument (100 to 1,000 ohms) is best suited.

23. Period.—Rapid movement of the moving system is usually highly desirable not only on account of economy of the observer's time, but because of changes in the conditions which affect the measurement. It is always desirable to check the measurement or balance at least once and where the conditions are apt to change, this must be done quickly. Therefore, the period should be as short as possible without sacrificing the sensitivity which varies as the square of the period. In general, a period of the order of 4 or 5 sec. is suitable for zero method measurements, although a period of the order of 15 to 20 sec. is necessary in ballistic measurements.

24. Damping.—The damping should in general be somewhat less than the critical value. If a galvanometer is overdamped, the zero position becomes somewhat uncertain because of the slowness in approaching the final position. Therefore, to insure a prompt and definite return to zero, the movable system should pass through the zero position once or possibly twice before coming to rest.

25. Magnetic Impurities.—Care should be taken that the copper, coil support and other parts of the movable system in the magnetic field are as free as possible from magnetic impurities.

26. Sensitivity.—The sensitivity of a galvanometer is, in the great majority of measurements, of decidedly secondary importance because usually the other factors entering into the final result of the measurement determine the precision obtained. In other words, a relatively low sensitivity represents, in most cases, a precision much higher than can be assigned to the other elements in the measurement. Therefore, some sensitivity can usually be sacrificed to gain the operating advantages incident to ample clearance in the air gap, short period and sufficient torsional moment in the suspension. With these features properly provided for, the sensitivity of moving-coil instruments is dependent upon the strength of the field, the number of turns,

the area of the coil and the moment of inertia. The latter is made small by making the coil narrow.

Continuous-current Galvanometers

The best-known *moving-magnet* galvanometers are the tangent, sine, Kelvin and Broca instruments. The tangent and sine galvanometers were formerly used extensively in electrical measurements but are now obsolete. They are described here as a matter of historical interest and because the principles employed are still used in modern instruments.

27. Tangent Galvanometer.—The magnetic needle is suspended or pivoted above the center of a circular scale over which moves the pointer attached to the magnet. The coil carrying the current is in a plane perpendicular to that of the scale so that the direction of the field which it produces is in the plane of the magnet. Since the earth's magnetic field is the directing force, its horizontal component must be known and the instrument always placed in the same position relative to it. This position is usually where the magnet is in the plane of the coil when no current is flowing. The tangent of the angle of deflection is proportional to the current in the coil, hence the name of the instrument. The current in amperes is approximately

$$I = \frac{5rH \tan \alpha}{\pi n} \qquad \text{(amperes)}$$

where r = radius of coil in centimeters; H = horizontal component of earth's field in gausses; n = number of turns in coil; and α = angle of deflection.

28. Sine Galvanometer.—This instrument is similar to the tangent galvanometer, the essential difference being that the coil is moved as the needle deflects so that they are finally in the same plane. In this instrument, the above formula becomes:

$$I = \frac{5rH \sin \alpha}{\pi n}. \qquad \text{(amperes)}$$

The principal causes for these two types of galvanometers becoming obsolete are: dependence upon the earth's field which is not only continually changing but is affected by nearby magnetic bodies and current-carrying conductors; wide variation in sensi-

tiveness at different parts of the scale; and the long period which made the instruments too slow for commercial measurements.

29. Kelvin Galvanometer (astatic type).—In this type (Fig. 6), the earth's field is very largely eliminated by having two magnetic needles, or sets of needles, of slightly different strengths at opposite ends of a light quartz rod and oppositely directed. Each is at the center of a pair of coils through which flows the current to be measured, these pairs being oppositely wound so that the moment exerted on the two is in the same direction. Thus the moving system is almost, but not quite, astatic. The controlling force is the earth's field modified as desired by means of a movable permanent magnet. In the particular instrument illustrated there are two such magnets mounted above the case. The deflections are measured by means of a minute mirror attached to the quartz rod and a telescope (or lamp) and a scale. This galvanometer must be calibrated against a standard. Due to the weak torque produced by the earth's field because of the astatic arrangement and the small mass of the moving system, this is one of the most sensitive types of galvanometer,—instruments with a current sensitivity of the order of 1×10^{-10} amp. having been constructed.[1] In the modern 2,000-ohm instrument made by the Cambridge Scientific Instrument Co. and shown in Fig. 6, a sensitivity of about 1×10^{-9} amp. is obtained with a period of about 15 sec.

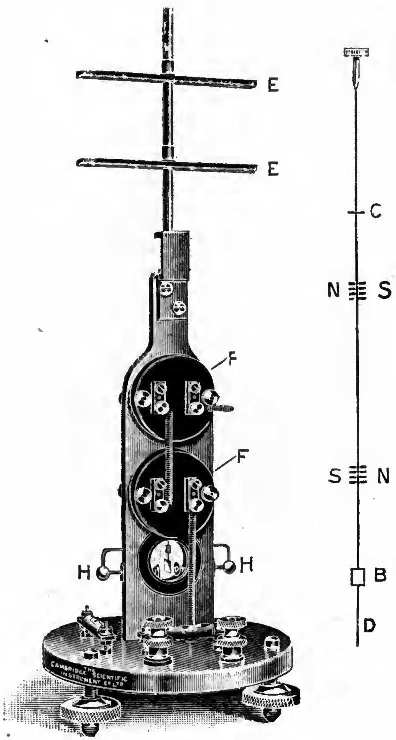


Fig. 6.

While the Kelvin galvanometer is still found in commercial laboratories to a limited extent, the magnetic and vibration

[1] "The Galvanometer," E. L. Nichols, p. 80.

disturbances usually present in such laboratories make this type of instrument unsuitable for ordinary work. It is therefore used principally where the greatest sensitivity is required as in certain kinds of research work.

30. Broca Galvanometer.—This modern galvanometer (Fig. 7) employs the principle of the Kelvin instrument. The distinguishing feature is the moving element. The two magnet systems are formed by two short, steel wires hung vertically and very close to each other, each wire being so magnetized that the ends have the same polarity and a consequent pole is formed at

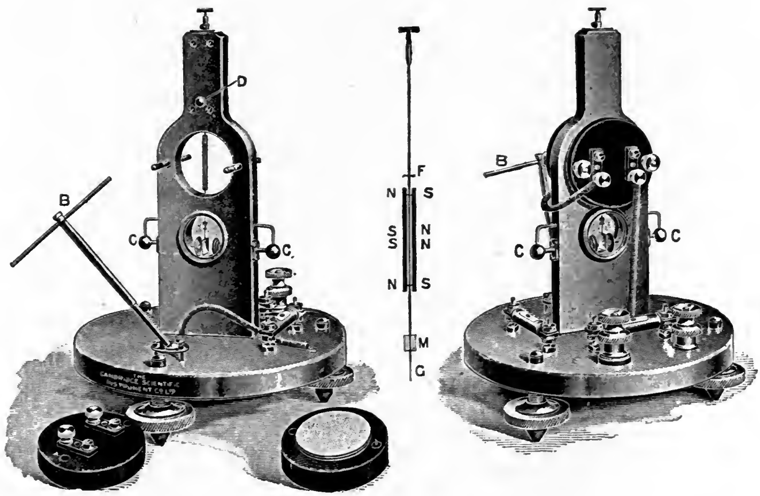


FIG. 7.

the center (see center of Fig. 7). Thus each magnet system has a small moment, the moment of inertia of the whole moving element is kept very small and a very astatic arrangement is obtained. Variable air damping is provided and the current coils are readily changed, as shown, so that a range of sensitivities can be had with one instrument. Control is obtained with a permanent magnet underneath the instrument controlled by the lever *B*. A small mirror is attached to the movable system and deflections are read with a telescope or lamp and scale. The following table gives some data on Broca galvanometers published by the manufacturer, the Scientific Instrument Co.

Sensitivities of Typical Broca Galvanometers

Resistance of galvanometer, ohms	Period, seconds	Sensitivity[1]	
		Ampere constant	Coulomb constant
8.8	10.0	2.9×10^{-3}	4.6×10^{-3}
8.8	17.3	9.3×10^{-4}	2.6×10^{-3}
110.0	10.0	1.0×10^{-3}	1.6×10^{-3}
110.0	17.3	3.3×10^{-4}	9.1×10^{-4}
860.0	10.0	4.5×10^{-4}	7.2×10^{-4}
860.0	17.3	1.5×10^{-4}	4.1×10^{-4}

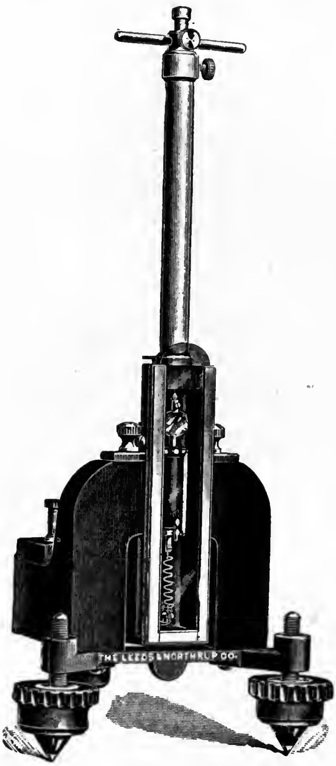


Fig. 8.

31. D'Arsonval Galvanometers.—There is practically only one type of moving-coil galvanometer employed for continuous-current measurements and that is the D'Arsonval type. Furthermore there are not only many more D'Arsonval type galvanometers in use than any other type, but the D'Arsonval principle is the basis of most continuous-current measuring instruments.

This type of galvanometer consists essentially of a coil of fine wire suspended between the poles of a permanent horseshoe magnet. The coil is usually suspended by a phosphor-bronze or steel wire, or flat strip, which not only conducts the current to the coil but provides the controlling force. Current is conducted from the coil by a helix of fine wire at the bottom. The strength of the field in the region occupied by the coil is increased by mounting a soft iron core in the central space enclosed by the coil, the pole pieces

[1] Ampere constant = current in microamperes which will produce 1 mm. deflection at 1 meter. Coulomb constant = quantity of electricity in microcoulombs which will produce 1 mm. deflection at 1 meter.

being shaped to give a uniform field throughout the space in which the coil moves. When the galvanometer is to be used only as a detector, as in bridge and potentiometer measurements, the pole pieces are sometimes pointed in order to concentrate the field and increase its intensity. While the D'Arsonval galvanometer cannot be readily used to make absolute measurements, but requires calibration, its many advantages have made it the most generally used type of galvanometer in the electrical laboratory. It is practically independent of the earth's field or other external fields; its period can be made short, which, with its dead-beat qualities, makes it a much faster instrument than other types; it is comparatively rugged, simple to use and one instrument can be employed for a wide range of work, by changing the suspension strip or the entire movable system. In the Leeds and Northrup instrument, shown in Fig. 8, the movable system is mounted in a tube which slides between the pole faces. This is readily replaced by another tube in which a coil of the desired resistance is already mounted.

The following typical data are from publications of the Leeds and Northrup Co.

SENSITIVITIES OF D'ARSONVAL GALVANOMETERS

Resistance of galvanometer, ohms	Period, seconds	Ampere constant[1]
50	7.0	1.3×10^{-2}
50	12.0	5.0×10^{-3}
1,500	7.0	1.6×10^{-3}
1,500	12.0	8.0×10^{-4}
12	7.5	2.5×10^{-3}
550	17.5	1.0×10^{-5}

D'Arsonval galvanometers may be classified according to the resistance. High-resistance galvanometers have resistances of about 1,000 ohms or more and are designed for use in high-resistance circuits where high-current sensitivity is required, as in measurements of insulation resistance. Low-resistance galvanometers have resistances of the order of 100 ohms or less, and are intended for low-resistance circuits where very small e.m.fs.

[1] Ampere constant = current in microamperes which will produce 1 mm. deflection at 1 meter. Coulomb constant = quantity of electricity in microcoulombs which will produce 1 mm. deflection at 1 meter.

are to be detected, as in conductivity measurements, low-resistance bridges, low-resistance potentiometers, and so forth. Those with a resistance between 100 ohms and 1,000 ohms are best adapted to general purposes, such as Wheatstone bridges and high-resistance potentiometers.

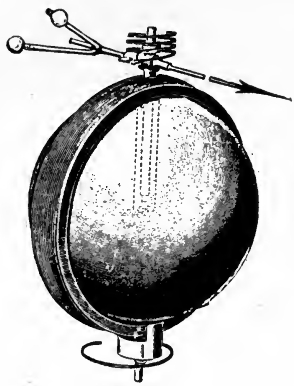


FIG. 9.

32. Portable D'Arsonval galvanometers are essentially millivoltmeters with short scales and high sensitivity, designed as detectors for portable "zero method" instruments, such as Wheatstone bridges and potentiometers. The "Unipivot" portable galvanometer, made by Paul, has a particularly high sensitivity because of the unique construction. The coil is circular in shape, while the whole moving system is supported by a single pivot which is placed at the center of the coil and the center of gravity of the system, thus greatly reducing friction (see Fig. 9). It is suitable for a variety of purposes because it has a

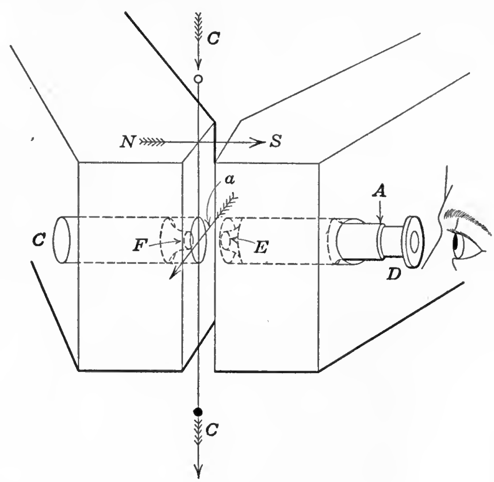


FIG. 10.

hundred-division scale and can be used as a deflection instrument as well as a zero instrument. The sensitivity is of the order of 1 microampere (0.000001 amp.) per division for a 50-ohm instrument.

33. Einthoven String Galvanometer.—This galvanometer is essentially a form of D'Arsonval galvanometer, the coil being reduced to a half turn or single wire in a very narrow gap between the poles of a powerful electromagnet. The essential features are shown diagrammatically in Fig. 10. When current passes through the "string," *CC*, it is deflected at right angles, to the field *NS* as indicated by the arrow, *a*. This movement is measured by means of a microscope fitted with a micrometer eyepiece as indicated in the figure or by projecting an enlarged image of the string on a graduated screen by means of a projection lantern. The instrument is used as an oscillograph (Chapter XIV) by fitting it with suitable photographic attachments.

The air gap being very small and the poles small compared with the size of the electromagnets, an intense field is produced so that the sensitivity is very high. The magnetic circuit is worked above the saturation point; consequently variations in the magnetizing current have no effect. Various materials are used for the string so that the resistance may vary from a few ohms to 10,000 ohms. The period depends upon the tension and can be adjusted through a very wide range. The weight is several thousand times less than that of the moving system of ordinary moving-coil galvanometers and the inertia is extremely small. Thus the period can be reduced to less than 0.005 sec. and by adjustment of the tension, the period can be increased to 8 or 10 sec.

Typical data applying to this type of galvanometer as published by the manufacturer, the Cambridge Scientific Instrument Co., are given in the following table.

CHARACTERISTICS OF RECENT EINTHOVEN STRING GALVANOMETERS

String		Resistance of galvanometer, ohms	Period, seconds	Current sensitivity (*a*)
Material	Diameter, millimeter			
Silvered glass.....	0.0025	12,030.0	(*b*)	1.5×10^{-5}
Silvered glass.....	0.0035	3,060.0	(*b*)	2.1×10^{-5}
Silvered glass.....	0.0030	8,720.0	0.004	0.14
Aluminium wire ..	0.0350	6.9	0.0390–0.0012	7.6×10^{-2}–76
Silver wire.......	0.0170	8.2	0.0820–0.0026	2.3×10^{-2}–21

(*a*) Current in microamperes which will produce 1 mm. deflection with a magnification in the microscope of 600.

(*b*) Instrument quite dead beat, about 10 sec. required to reach full deflection.

34. Ballistic Galvanometers.—Either the moving-magnet or moving-coil type of galvanometer may be made a ballistic galvanometer. In order that a ballistic galvanometer may be perfectly ballistic the quantity of electricity to be measured must be completely discharged through it before the movable system has moved appreciably. The period, or time of vibration, must therefore be long compared with the time of discharge. This is accomplished by increasing the inertia of the movable system. In the magnet type, a relatively large magnet is used. In the coil type of galvanometer, a detachable weight (often in the form of a wire loop similar in shape to the coil) is added or the coil is made very wide compared with its length.

The magnitude of the first deflection of a ballistic galvanometer is a measure of the quantity discharged into the instrument. In an instrument in which there is little damping such as the moving-magnet type, the quantity may be calculated directly from the constants of the instrument. Thus

$$Q = \frac{2Ht \sin (\tfrac{1}{2})\alpha}{\pi G} \qquad \text{(coulombs)}$$

or, for small angles (5° or less)

$$Q = \frac{Ht \sin \alpha}{\pi G} \qquad \text{(coulombs)}$$

where Q = quantity of electricity in coulombs, H = field strength in gausses, G = constant computed from the coils, t = period in seconds, α = angle of deflection. In practice, however, ballistic galvanometers are usually standardized and the formula becomes simply $Q = kd$ where d = deflection and k = galvanometer constant or quantity per unit deflection.

The constant, k, is determined with a standard condenser or mutual inductance. The deflection obtained upon suddenly discharging a charged condenser through the galvanometer is d, and $kd = Q = CE$; and hence

$$k = \frac{CE}{d}$$

where Q = quantity of electricity in coulombs, E = potential to which the condenser had been charged in volts, and C = capacity of condenser in farads. When a mutual inductance is used, the galvanometer is connected in series with the secondary and

$$kd = Q = \frac{MI}{R}$$

when the primary circuit is opened or closed. Then

$$k = \frac{MI}{dR}$$

where Q = quantity of electricity in coulombs, M = coefficient of mutual inductance in henrys, I = steady value of the current in the primary circuit of the mutual inductance in amperes; and R = resistance of secondary circuit (including the mutual inductance) in ohms. When the galvanometer is used in magnetic measurements it may be calibrated with such a mutual inductance directly in lines of magnetic flux per centimeter deflection.

Alternating-current Galvanometers

The increase in the use of alternating current has been accompanied by the development of instruments and methods for alternating-current measurements so that instruments for detecting and measuring very small alternating currents are now available. Consequently many of the fundamental methods used in continuous-current work, such as bridge and potentiometer methods, are used in alternating-current measurements with equal facility though with (as yet) less precision.

The principal types of moving-coil galvanometers for alternating current are the electrodynamometer, thermal and vibration. There is also a moving-magnet type of vibration galvanometer.

35. Electrodynamometer Galvanometers.—The principle employed is the electrodynamic action between two conductors carrying current. A coil of fine wire is suspended between two fixed coils. When all three coils are connected in series and a current flows through them, the moving coil will move through an angle which will be proportional to the square of the current. Thus the sensitivity decreases rapidly with the current so that in null-method measurements, the sensitivity is a minimum where the greatest sensitivity is desired. By separately exciting the fixed coils this condition is improved because the deflection then varies directly with the current in the moving coil.

Reflecting instruments are usually made practically astatic by using two sets of fixed and movable coils, one set above the other with the two movable coils mounted on the same support. The general construction and appearance is similar to direct-current galvanometers. A Leeds and Northrup galvanometer

is shown in Fig. 11. A sensitivity is readily attained with this type of instrument of 1 microampere per centimeter deflection at 1 meter distance with 100 ohms in the moving coil and with the fixed coils separately excited.

The Sumpner electrodynamometer made by R. W. Paul is

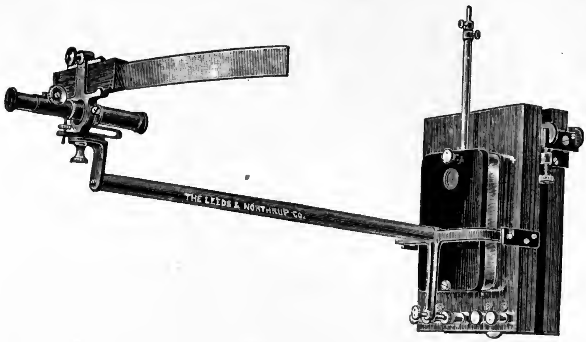


FIG. 11.

distinguished by the fact that the field is produced by an electromagnet constructed of very thin iron laminations having a high resistance and low hysteresis losses. The magnetic circuit is such that the magnetic field is accurately proportional to and in quadrature with the applied potential. By separately exciting the field coils, potentials of the order of 1 microvolt can be detected.

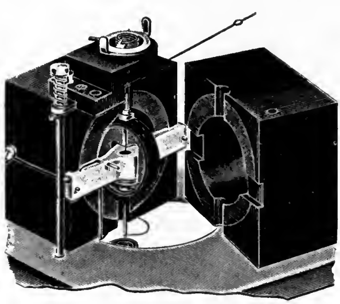


FIG. 12.

A portable electrodynamometer-type galvanometer is made by R. W. Paul in which the single point of a support is employed ("unipivot" construction, see corresponding D'Arsonval galvanometer). The working parts are shown in Fig. 12. The most sensitive form of this instrument deflects full scale with 0.01 amp. with fixed and movable coils in series.

36. Thermal Galvanometers.—The principal example of this type of galvanometer is the Duddell thermogalvanometer shown in Fig. 13. The principle is very simple and is shown in the diagram at the right of the illustration. A fine wire movable

coil is suspended between the poles of a permanent magnet as in the D'Arsonval galvanometer but the coil circuit includes a bismuth-antimony thermocouple which is placed very close to an electric heater through which passes the current to be measured. The instrument is practically free from inductance and capacitance, and is, therefore, independent of frequency. It is particularly applicable to the measurement of high-frequency

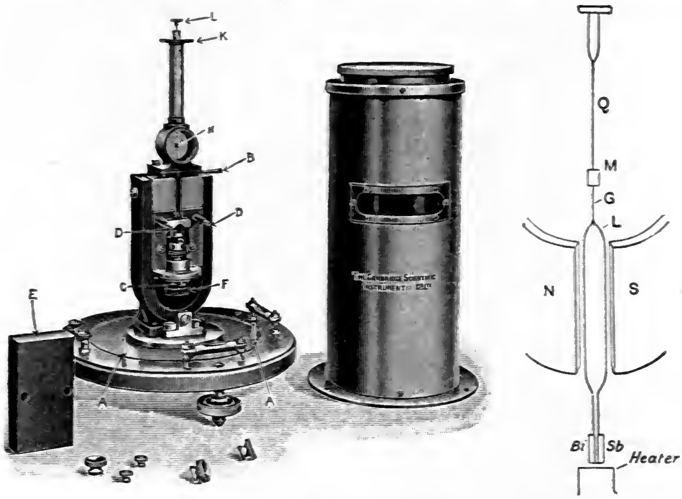


FIG. 13.

currents—as high as 100,000 cycles per second. A wide range in sensitivity is obtained by changing the heater element and the distance between the heater and the thermocouple. An important feature is that the instrument can be calibrated on continuous current. The following data are published by the manufacturer, the Cambridge Scientific Instrument Co.

SENSITIVITY OF DUDDELL THERMOGALVANOMETERS

Approximate resistance of heater, ohms	Current to produce 250 mm. deflection, microamperes[1]	Current to produce 10 mm. deflection, microamperes[1]
1,000	110	22
100	350	70
10	1,100	220
4	1,750	350
1	3,500	700

[1] At 1 meter distance. Deflections are practically proportional to the square of the mean effective values of current.

37. Vibration Galvanometers.[1]—In a vibration galvanometer, the natural period of vibration of the movable system is such that it is set into vibration by a current having the same period as the movable system. The period of the movable system having been adjusted to the period of the current which it is desired to measure, the strength of the latter is indicated by the amplitude of the vibration produced.

These instruments are useful principally as detectors in null-method measurements, such as inductance and capacitance determinations with a Wheatstone bridge. While vibration galvanometers are being used[2] to some extent where conditions are favorable, their application in commercial laboratories is very limited, despite the fact that they are simple and comparatively robust. This is due to the disturbing effect of mechanical vibrations and to the necessity of having the frequency of the current being measured very steady if maximum sensitivity is to be had.

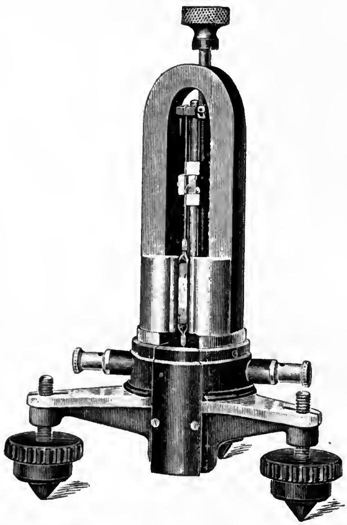


FIG. 14.

The several forms of vibration galvanometers may be classified as moving-coil or moving-magnet types. Typical galvanometers of each form are described briefly in the following paragraphs.

Moving-coil Type.—The Leeds and Northrup vibration galvanometer, shown in Fig. 14, is an example of this type. The moving system consists of a very narrow coil of rectangular section suspended between the poles of a permanent magnet as in the D'Arsonval galvanometer. The period is roughly adjusted by altering the effective length of the suspension by means of a

[1] "Characteristics and Applications of Vibration Galvanometers," F. WENNER, *Transactions*, A. I. E. E., vol. 31 p. 1243 (1912).

[2] "Measurement of Inductance by Anderson's Method Using Alternating Current and a Vibration Galvanometer," E. B. ROSA and F. W. GROVER, *Bulletin*, Bureau of Standards, vol. 1, p. 291 (1905).

movable bridge or support. Fine adjustment is obtained by altering the tension. The instrument is usually used with a lamp and scale, the width of the band on the scale which is produced by the image of the lamp filament reflected from the vibrating mirror being a measure of the current. It is, of course, first adjusted to give maximum width of band with a steady current. Fig. 15 is a resonance curve of the galvanometer illustrated in Fig. 14, which shows the sharpness of tuning that can be obtained. It is to be noted that current sensitivity increases rapidly with the sharpness of tuning. This is accompanied, however, by a decrease in the resonance range, so that the instrument becomes more responsive to variations in the frequency

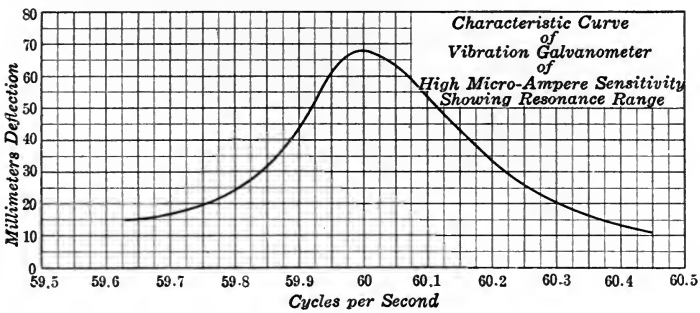


FIG. 15.

of the current being measured. Hence, in a practicable instrument, the design must be a compromise of current sensitivity and range of resonance. A 700-ohm galvanometer of the kind illustrated has a 60-cycle current sensitivity of 2.5×10^{-2} microampere per millimeter at 1 meter. A 0.16-ohm instrument has a 60-cycle voltage sensitivity of 2 microvolts per millimeter at 1 meter.

The Campbell vibration galvanometer made by R. W. Paul is similar to the one just described. Provision is made for readily changing the coils. One form having a bifilar suspension has a range of frequency of about 50 to 1,000 cycles per second and with a coil such that the total effective resistance is 35 ohms, the sensitivity on 1,000 cycles is 5 microamperes per millimeter at 1 meter. A 500-ohm instrument has a sensitivity of 1.5×10^{-2} microampere per millimeter at 1 meter and 50 cycles. Another form having a single suspension and 50 ohms resistance

has a sensitivity of 4×10^{-2} microampere at 1 meter and 50 cycles.

The Einthoven string galvanometer, described in par. 33, may be used as a vibration galvanometer.

Moving-magnet Type.—The Drysdale-Tinsley vibration galvanometer (Fig. 16) is an example of the moving-magnet type. A very small piece of soft iron is suspended by a silk fiber between the poles of a permanent horseshoe magnet. Adjacent is

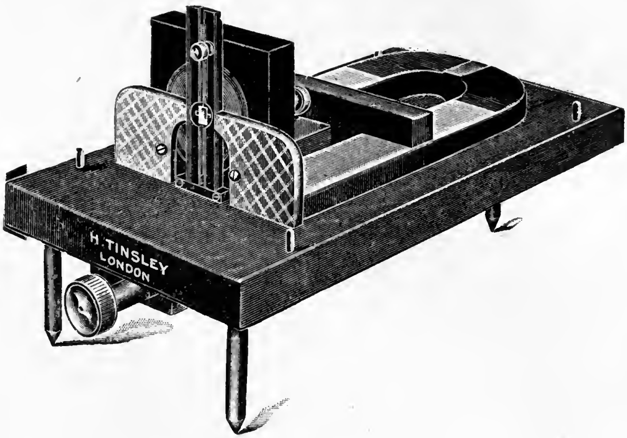


FIG. 16.

a coil carrying the alternating current to be measured which produces a field perpendicular to that of the permanent magnet. The temporarily magnetized needle vibrates in synchronism with the period of the alternating current and with an amplitude proportional to the strength of the current. The indication is a band of light of varying width, as in the movable-coil type. The effective period of the moving system is altered, or adjusted to resonance, by varying its "magnetic inertia," that is, the strength of the soft-iron magnet. This is accomplished by shunting more or less of the permanent field by means of a movable keeper across the limbs of the magnet. The sensitivity of a 40-ohm instrument on 50 cycles is about 0.25 microampere per millimeter at 1 meter.

38. Rectifiers.—In alternating-current measurements made by null methods, a D'Arsonval galvanometer in conjunction with some form of rectifying device can be employed as the detector. There are advantages and disadvantages in this method compared

to the use of standard alternating-current galvanometers. While it permits the use of a form of galvanometer which any laboratory must already have, another equally expensive piece of apparatus is required. The greater sensitivity (in general) of the D'Arsonval instrument is usually of little value because other conditions generally limit the precision to a value much below that corresponding to a sensitivity of only an ordinary order. On the other hand, the question of phase relation between the movable-coil current and the fixed-coil current in dynamometer galvanometers (they are usually in different circuits) is eliminated. It is to be noted that this type of detector responds to the average value instead of the mean effective value of the current.

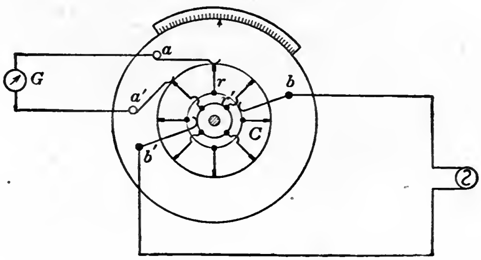


FIG. 17.

Synchronous Commutator.—The scheme of the synchronous commutator is shown in Fig. 17. A commutator, C, and two slip rings, r and r', are mounted on the shaft of a small synchronous motor. The bars in the commutator are narrow, and equal in number to the poles of the motor and equally spaced. Two brushes, a, a', connected to the galvanometer are so spaced as to bridge two bars simultaneously. These brushes are mounted on a disc which can be rotated about the shaft. The bars are connected alternately to the two rings which in turn are connected through the brushes, b, b', to the alternating current being measured. It is apparent that the connections to the galvanometer are reversed every half cycle, so that the indication is a steady one, the value of which may be made anything from zero to a maximum by shifting the angular position of the brushes. Thus the most sensitive position can be readily found, irrespective of the phase relation between the current in the circuit being measured and that in the motor armature. The variation in contact resistance at high speed and the possibility

of thermo e.m.fs. usually cause trouble where the resistances or potentials are very low, as in low-resistance bridge measurements.

Synchronous Reversing Key.[1]—This device is designed to overcome the objections to the commutator scheme mentioned in the preceding paragraph by eliminating sliding contacts. The principle of the device is shown in Fig. 18. A cam, *C*, mounted on the shaft of a small synchronous motor is so designed that it moves the lever, *l*, up and down in such a manner that the pair of contacts, *a* and *a′*, at the end of the lever make alternate contact with the pairs of stationary contacts, *c* and *c′*, once per cycle. The number of projections on the cam correspond to the number of pairs of poles on the motor. The contacts are arranged as

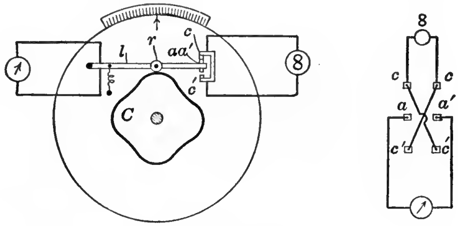


Fig. 18.

shown diagrammatically at the right of Fig. 18 from which it will be seen that the connections to the galvanometer are reversed every half cycle, so that a steady deflection is obtained in the continuous-current galvanometer. The contacts are all supported on a solid disc which can be rotated around the shaft and hence reversal can be made at any point on the wave as in the case of the synchronous commutator. The contacts are all made of platinum, the cam is hardened steel, and the lever, *l*, is kept in contact with the cam at all times by means of a spring. The roller, *r*, is necessary to insure contact at the low portion of the cam.

Galvanometer Deflections

39. Reflecting-galvanometer deflections are read with a telescope and scale or with a lamp and scale. In the former, the scale is reflected from the plane mirror (attached to the moving

[1] "Recent Progress in Exact Alternating-Current Measurements," C. H. Sharp and W. W. Crawford, *Transactions*, A. I. E. E., vol. 29, p. 1517 (1910).

system) to the telescope through which movements are observed. With a lamp and scale, an image of a narrow beam of light (issuing from a narrow slit in a vessel enclosing a lamp, or from a portion of an incandescent lamp filament) is thrown on to the scale by the mirror. In order to get a sharp image, either the mirror is made concave (with a 1-meter focus if the scale is the usual distance of 1 m. away), or a lens is used. Fig. 19 shows the general arrangement—lamp and scale at the left, telescope and scale at the right.

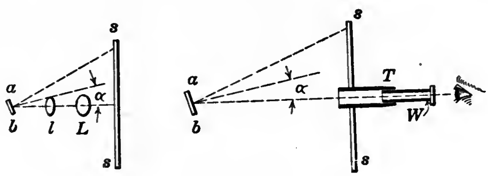


FIG. 19.

40. Galvanometers as Detectors.—The majority of galvanometers are used as detectors only, that is, in zero-method methods where the kind of scale or proportionality of deflections does not enter into the determinations. In such cases a very short, straight scale is sufficient and space may be economized by placing the galvanometer on the wall above the table, with the scale directly underneath.

41. Galvanometer Scales.—When readings are to be taken, care should be exercised that they are proportional to the angular deflections. On a straight scale the deflection in millimeters is

$$d = A \tan 2\alpha$$

where A = distance from mirror to scale in millimeters and α = angle through which the moving coil turns. If the angle is small, d is proportional to α. The error becomes about 0.5 per cent. at 5°. For larger angles, d may be corrected according to the following table before substituting in the formulas or the scale may be calibrated directly. Obviously a curved scale will eliminate this error, and by properly adjusting the curvature the readings may be made proportional to the angle or to any desired function of the angle.

Scale Errors of Reflecting Galvanometers (A. E. Kennelly)

(Corrections in millimeters to be subtracted from deflections in millimeters)

Deflection, millimeters	Distance from mirror to scale, millimeters					
	1,000	1,100	1,200	1,300	1,400	1,500
50	0.05	0.05	0.00	0.00	0.00	0.00
60	0.05	0.05	0.05	0.05	0.05	0.00
70	0.10	0.05	0.05	0.05	0.05	0.05
80	0.15	0.10	0.10	0.10	0.05	0.05
90	0.20	0.15	0.15	0.10	0.10	0.10
100	0.25	0.20	0.20	0.15	0.15	0.10
120	0.45	0.35	0.30	0.25	0.25	0.20
140	0.70	0.55	0.50	0.40	0.35	0.30
160	1.00	0.85	0.70	0.60	0.50	0.45
180	1.40	1.20	1.00	0.85	0.70	0.65
200	1.95	1.65	1.40	1.15	1.00	0.90
220	2.60	2.15	1.80	1.55	1.30	1.20
240	3.30	2.80	2.35	2.00	1.75	1.50
260	4.25	3.50	3.00	2.55	2.20	1.90
280	5.30	4.40	3.70	3.15	2.75	2.40
300	6.45	5.35	4.50	3.90	3.35	2.95
320	7.80	6.50	5.50	4.70	4.05	3.55
340	9.30	7.75	6.55	5.60	4.85	4.25
360	10.95	9.15	7.75	6.65	5.75	5.00
380	12.80	10.70	9.05	7.80	6.70	5.90
400	14.85	12.40	10.50	9.05	7.85	6.85

CHAPTER III

CONTINUOUS E.M.F. MEASUREMENTS

42. Unit of E.m.f.—The fundamental unit of e.m.f. is based on the centimeter and the second. It has no official name but is frequently called the "abvolt" in the electromagnetic system or "statvolt" in the electrostatic system. This unit is not readily represented in concrete form and, therefore, all ordinary electrical measurements employ the practical unit, the international volt, which is the legal unit of e.m.f. in the United States at the present time.

The international[1] volt is that e.m.f. which, when steadily applied to a conductor whose resistance is 1 international ohm, will produce a current of 1 international ampere. The Act of Congress of 1894 establishing this unit also stated that the international volt is "represented sufficiently well for practical use by 1,000/1,434ths of the e.m.f. between the poles or electrodes of the voltaic cell known as Clark's cell at a temperature of 15°C. and prepared in the manner prescribed in the accompanying specifications."

Standards

43. Clark Cell.—The Clark cell was the first form of voltaic cell to meet the requirements of a reliable standard of e.m.f., namely, reproductibility and reasonable permanence. The elements are: positive electrode, metallic mercury; negative electrode, amalgamated zinc; electrolyte, saturated solution of zinc sulphate and mercurous sulphate (Fig. 20). Its e.m.f. was originally taken as 1.434 international volts at 15°C., which value was made legal in 1894 as stated above. Later and more careful determinations have shown, however, that 1.4328 is a more correct figure.

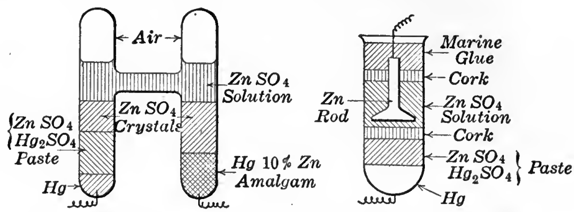
The e.m.f. of the Clark cell changes with temperature in accordance with the formula:

$$E = 1.4328\,[1 - 0.00077\,(t - 15)]$$

[1] The prefix "international" is ordinarily omitted.

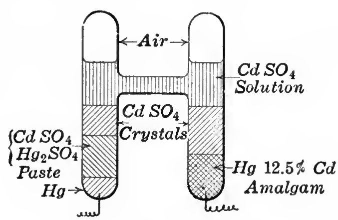
where E = international volts at any temperature, t, and t = deg. C.

With the development of the art, more precise measurements became necessary and certain characteristics of the Clark cell made it unsatisfactory as a standard for highly precise work. The principal defects are the relatively large temperature coefficient which necessitates the determination of the *true* temperature of the cell; and the time lag between the temperature and the e.m.f. Thus great care is necessary to insure that the e.m.f. has reached the value corresponding to the temperature of the cell. Because of these objectionable features, the Clark cell, while still the official standard, has been superseded in practice by the Weston cell which is practically free from these objections.



Clark Cell "H" Form Clark Cell-Carhart Form

FIG. 20.



Weston Normal Cell

FIG. 21.

The elements of the Weston cell are essentially: positive electrode, metallic mercury; negative electrode, amalgamated cadmium; electrolyte, solution of cadmium sulphate and mercurous sulphate. There are two forms—in one the solution is saturated, and in the other it is unsaturated above 4°C.

44. Weston Cell, Normal.—The form in which the electrolyte is saturated is called the Weston normal cell (Fig. 21), which is the form that has been adopted as the official standard cell of

the Bureau of Standards[1] because it is more permanent and can be reproduced with greater accuracy than when the electrolyte is unsaturated. When carefully made according to the official specifications, cells will agree with each other within a few parts in 100,000. The e.m.f. changes slightly with temperature according to the following formula:

$$E = 1.0183\,[1 - 0.000041\,(t - 20)]$$

where E = international volts at any temperature, t, and t = deg. C.

45. Weston Cell, Unsaturated.—In this form the electrolyte is saturated at 4°C. and therefore it is unsaturated at all ordinary temperatures. The unsaturated cell is not so uniform in e.m.f. as the saturated form but the temperature coefficient is only 0.00001 per degree C. which is ordinarily negligible. Therefore, while the unsaturated cell has to be standardized, it is more convenient for general use as a secondary standard.

The cells furnished by the Weston Electrical Instrument Co. are of the unsaturated type. The manufacturers recommend that their cells be not subjected to temperatures below 4°C. or above 40°C. and that no current greater than 0.0001 amp. be passed through them.

Methods of Comparison with Standard Cell

The principle of the various methods which may be employed in comparing an e.m.f. or a difference of potential with the e.m.f. of a standard cell is indicated in the following paragraphs.

46. Substitution Method.—The current flowing through a high resistance (not less than 15,000 ohms) is measured with a high-sensitivity galvanometer, first with the standard cell in the circuit and then with the unknown e.m.f. substituted for the standard cell. The resistance being the same in the two cases,[2] the deflections are proportional to the e.m.f. and

$$E = \frac{ed'}{d}$$

where E = unknown e.m.f., e = standard-cell e.m.f., d = deflection with standard cell and d' = deflection with unknown

[1] See Circular No. 29, Bureau of Standards. The value of the e.m.f. of the Weston normal cell was established by the Bureau on Jan. 1, 1911 as 1.01830 international volts at 20°C.

[2] This method is obviously not applicable if the resistance of either source of e.m.f. is high.

e.m.f. If the deflection with the unknown e.m.f. is too large, the galvanometer may be shunted (see galvanometer shunts, pars. 18 and 19).

47. Equal-deflection Method.—This is a modification of the above method in which the deflections are kept the same in the two cases by changing the resistance. Then

$$E = \frac{er'}{r}$$

where r = total resistance of the circuit (including the galvanometer and source) with the standard cell in the circuit and r' = total resistance with the unknown e.m.f. in circuit. This method is better than the first one because it is a constant-deflection method, thus eliminating the question of proportionality of deflections. Furthermore, the result depends on the known values of two resistances, rather than observed deflections and is, therefore, more accurate.

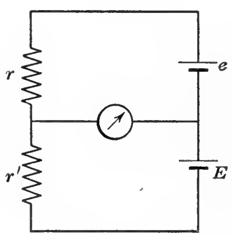


FIG. 22.

48. Wheatstone's Modification of the Equal-deflection Method.—In this method the galvanometer resistance does not have to be known. The deflection, d, is noted when the unknown e.m.f., E, and a known high resistance are in circuit. Additional resistance, r', is added and the deflection, d', again noted. Similarly with the standard cell of potential difference, e, the resistance is adjusted until the same deflection, d, is obtained and then an amount of resistance, r, is added until the deflection d' is again obtained. The unknown e.m.f. is

$$E = \frac{er'}{r}$$

the symbols having the same significance as before.

49. Lumsden Method.—A Wheatstone bridge is formed with the two e.m.fs. comprising two arms and one subdivided high resistance forming the other two arms, as shown in Fig. 22. Obviously when r and r' are adjusted so that the galvanometer shows no deflection,

$$E = \frac{er'}{r}.$$

This method has an advantage over the preceding ones in the respect that it is a zero-deflection method.

50. Condenser-discharge Method.—In this method a condenser is charged, first from the unknown e.m.f. and then from the standard cell, discharge in each instance being through a ballistic galvanometer. The deflections will be proportional to the e.m.fs. hence

$$E = \frac{ed'}{d}$$

as in the substitution method. Obviously, if the unknown e.m.f. is much smaller or much larger than the standard cell, the deflection can be made about equal to that of the standard cell by using a larger or a smaller condenser. In that case the ratio of the capacitances should be known, and then

$$E = \frac{ed'C'}{dC}$$

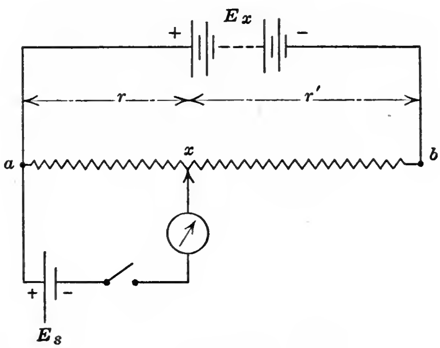


FIG. 23.

where C = capacitance of condenser used with the unknown e.m.f. and C' = capacitance of condenser used with the standard cell. This method has the advantage that practically *no current is required,* which is advantageous in making measurements of voltaic cells of very small capacity or rapid polarization.

51. Opposition or Potentiometer Method.—The principle of this method is that of opposing the e.m.f. of the standard cell against an equal difference of potential which bears a known ratio to the unknown e.m.f. This method is the most accurate and by far the most generally used, because it is both a zero-deflection and a zero-current method and the result depends only on the ratio of two resistances, quantities which can be very accurately determined.

Fig. 23 shows diagrammatically the application of the principle. E_x represents the unknown e.m.f., E_s the standard cell and ab a variable resistance connected to E_x. Since the fall of potential along ab is uniform, it is obvious that between one end, a, and some one point, x, the difference of potential is equal to that of E_s. This point is located by connecting E_s as shown and in series with a galvanometer and a key. Either the contact is moved along the resistance, or r and r' are adjusted until no deflection of the galvanometer is observed when the key is closed. Then

$$E_x = \frac{E_s\,(r + r')}{r}$$

Potentiometers

51a. General.—Potentiometers are instruments specially constructed and arranged for the rapid and convenient comparison

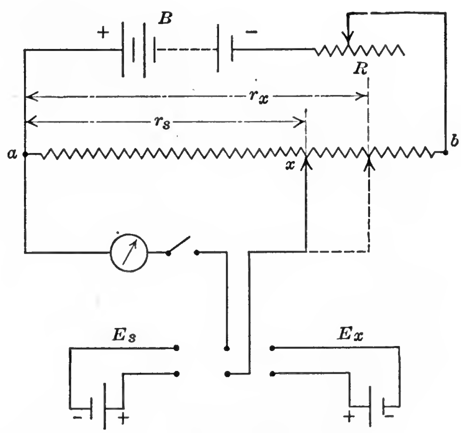


Fig. 24.

of e.m.fs. or potentials with a standard cell by means of the principle explained in the last paragraphs. They form a very important class of measuring apparatus because practically all e.m.f. and current standardization measurements, even those of the most precise character, are now made with potentiometers.

Fig. 24 shows diagrammatically the fundamental scheme of a potentiometer. A resistance, ab, is connected in series with a rheostat, R, and across a battery, B. The standard cell, E_s

and the unknown e.m.f. or potential, E_x, are connected to a suitable switching arrangement such that each can be connected through a key and galvanometer to the resistance *ab* as indicated. With E_s connected to *ab* and the key closed, the contact x is shifted until a balance is obtained. Noting the value of r_s, the switch is thrown over to E_x and the contact x again shifted until another balance is obtained, the new value of the resistance, r_x, being noted. The value of the unknown potential is evidently,

$$E_x = \frac{E_s r_x}{r_s}.$$

Now, if the rheostat, R, is so adjusted when balancing against the standard cell that balance is obtained with, for example,

$$\frac{E_s}{r_s} = \frac{1}{1{,}000}$$

then

$$E_x = \frac{1}{1{,}000} r_x.$$

Thus by making the drop across a certain definite resistance equal to 1 volt, the various subdivisions of the total resistance *ab*, may be calibrated and marked in decimal parts of a volt, that is, 1.0, 0.1, 0.01, 0.001 volt, etc., so that the unknown potential can be read directly. This is the scheme employed in all potentiometers.

It is to be noted that when balancing for E_x it is immaterial whether the contact x is moved along *ab* or the resistance between a and x is changed, provided that in the latter method a similar and opposite change is made between x and a, so that the total resistance *ab*, remains constant (see Wolff potentiometer, par. 55).

52. Classes.—Practical potentiometers are made by various instrument makers in a variety of forms which differ in method of operation and details of construction. The usual classification is, however, based on the value of the "potentiometer resistance," *ab* (Fig. 24), that is, whether it is "high" or "low." A potentiometer with about 100 ohms or less total resistance is classed as low-resistance and when the resistance is several hundred or several thousand ohms the potentiometer is classed as high-resistance. The former type is employed in the measurement of very small potentials in low-resistance circuits such as the e.m.i. in a thermocouple. Descriptions of typical instruments of each class are given in the following paragraphs.

53. Leeds and Northrup Potentiometer.—This is a low-resistance type potentiometer. The arrangement of the circuits is shown in Fig. 25 and the instrument itself in Fig. 26.

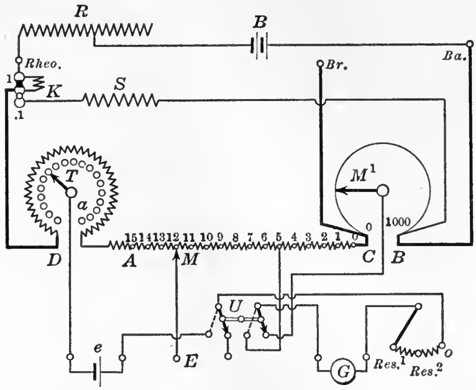


FIG. 25.

The essential part of the instrument consists of fifteen 5-ohm coils, AC, connected in series with the extended wire, ~~CE~~, the
CB

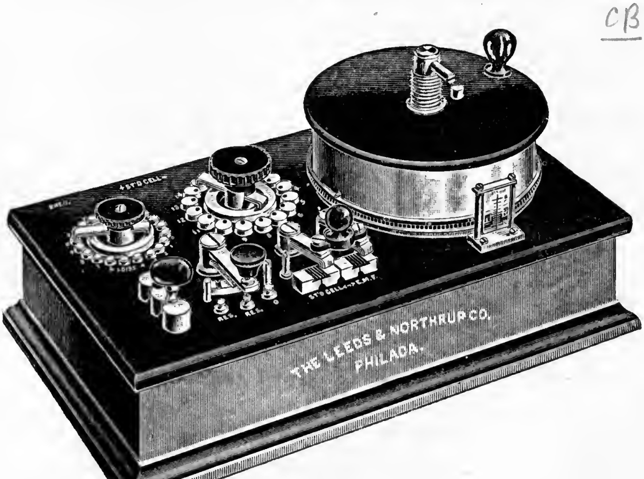


FIG. 26.

resistance of which from 0 to 1,100 scale divisions is 5.5 ohms. Thus when the current from the battery, B, is adjusted by the rheostat, R, to 0.02 amp., the fall of potential across each 5-ohm

coil in AC is 0.1 volt and across CB, 0.11 volt. Since the latter is divided into 1,100 parts, the e.m.fs. may be measured to 0.0001 volt. At point 5 in AC, a wire is permanently attached which connects to one point of the double switch, U. When this switch is thrown to the left, the standard cell is connected through the galvanometer to point 5 and the sliding contact T which moves over the dial at the left consisting of 19 resistance coils. Between a and A is a resistance which is adjusted to such a value that with 0.02 amp. flowing, the potential drop between 5 and a is 1.0175 with 0.0001 volt additional drop across each dial coil, giving a maximum of 1.0194. This range will include the usual differences between different Weston standard cells. To adjust the current to 0.02 amp. throw the switch U to the position indicated by the dotted lines, set the contact T to correspond to the e.m.f. of the standard cell and regulate R until the galvanometer shows no deflection. The unknown e.m.f. is then measured by throwing the switch U to the position indicated by the full lines and adjusting the contacts M and M' until no deflection is noted. After a balance has been obtained, the current may be checked by simply shifting U to the first position and pressing the contact key. The extended wire, CB, is mounted spirally on a disc of marble and the contact is carried on the inside of the revolving drum at the right of Fig. 26 so that the wire itself is protected from dust.

The two plug holes in the upper left-hand corner of the diagram are for the purpose of changing the range of the potentiometer to one-tenth of its normal value. When the plug is in the lower hole the resistance, S, is in parallel with the resistance between D and B, making the resistance between these points one-tenth of its previous value. At the same time, the resistance, K, is automatically inserted in the circuit. Its value is such that the total potentiometer resistance is the same as before and the current remains undisturbed. Thus, after adjusting the current in the regular way as described above and inserting the plug in the lower hole, each section between A and B represents one-tenth of the previous value. This arrangement permits low e.m.fs. to be measured to one additional decimal place if the galvanometer is sufficiently sensitive.

54. Paul Potentiometer.—This instrument, made by R. W. Paul, London and New York, is another example of low-resistance type. The connection diagram is shown in Fig. 27 and the in-

strument proper in Fig. 28. It is similar to the Leeds and Northrup potentiometer in that the lower part of the reading is obtained on an extended slide wire at one end of the potentiometer circuit and a switch, for quickly changing from the standard cell connected to N to the unknown potential, is provided as indicated

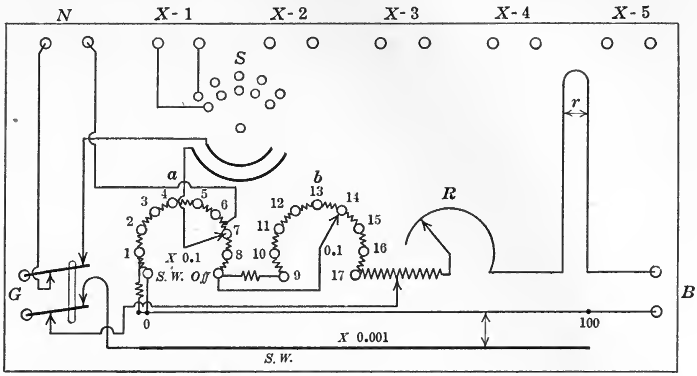


Fig. 27.

at the left of the diagram. There are 17 resistance coils of 1 ohm each so that the resistance is 10 ohms per volt. The slide wire is laid straight as shown and is divided into divisions about 0.4 cm. long, corresponding to 0.001 volt, which are subdivided into five parts. Thus the range is from 0.00002 volt to 1.8 volts.

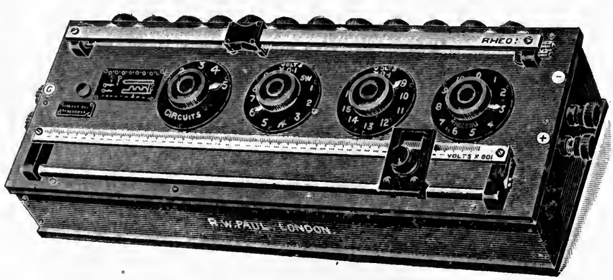


Fig. 28.

The switch S is provided to connect quickly the various circuits, $x - 1$, $x - 2$, etc., to the potentiometer circuit in turn. All contacts are enclosed.

The Paul potentiometer is also made with ranges as low as 0.000004 volt to 0.036 volt for thermo-electric measurements.

55. Wolff Potentiometer.—This instrument, made by Otto Wolff, Berlin (J. G. Biddle, Philadelphia, American agent), is a prominent example of the high-resistance class of potentiometers. It also differs from those described in the preceding paragraphs

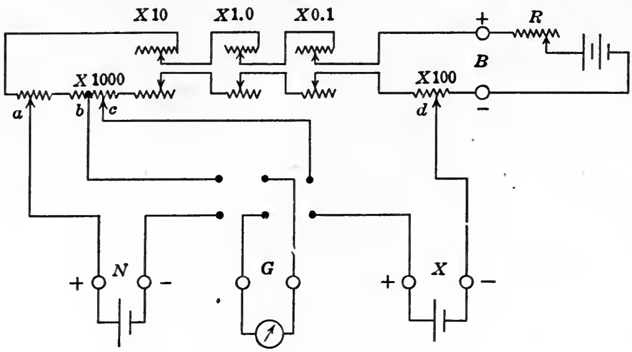


FIG. 29.

in that balance for E_x is obtained by altering the resistance between the E_x contacts.

Fig. 29 is a simplified diagram of the circuits. The current in the potentiometer circuit between $B+$ and $B-$ is adjusted in

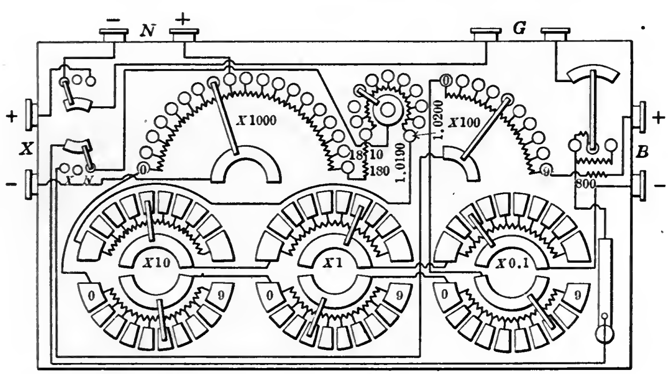


FIG. 30.

the usual manner by the rheostat, R. The standard cell, N, is connected to the circuit at the proper points ab, by closing the switch, S, to the left. With the switch closed to the right, the final balance for X is obtained by adjustment of the three rheostats marked "x10," "x1.0" and "x0.1," which are so arranged

that as the resistance between c and d is increased or decreased, an equal and opposite change is made automatically in that part of the circuit beyond cd so that the total resistance remains unchanged.

Fig. 30 is a diagram of the circuits as actually arranged, the notation being the same as in Fig. 29. The total resistance is 20,000 ohms or 10,000 ohms per volt. There is no slide wire, the entire resistance being comprised in the five dials having steps of 1,000, 100, 10, 1 and 0.1 ohms respectively, corresponding to 0.1, 0.01, 0.001, 0.0001 and 0.00001 volt respectively.

A separate dial is provided for the standard-cell adjustment, together with a separate resistance which can be altered to accommodate different cells without affecting the measuring circuits. A "throw-over" switch is provided for quickly connecting the standard cell and verifying the adjustment without disturbing any settings. The total range of the instrument is 1.9 volts.

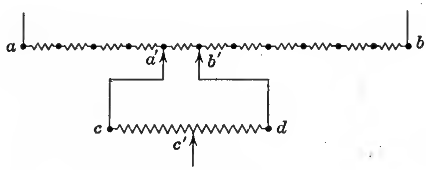


Fig. 31.

56. Leeds Potentiometer.—This instrument made by Leeds and Northrup is of the high-resistance type, the resistance being 10,000 ohms per volt. It employs the standard slide-wire principle but the subdivisions are obtained by the use of the Thomson-Varley slide instead of an extended wire. The principle of this scheme is indicated in Fig. 31. The double contact, $a'b'$, always spans one coil in the main circuit ab and by tapping from cd, the potential, $a'b'$, can be readily subdivided.

57. Deflection Potentiometers.—In this type of potentiometer, an approximate balance is obtained in the usual manner. The unbalanced potential is shown by the deflection of the galvanometer which has been previously calibrated. This deflection converted to volts added to the potential shown by the setting of dials of the potentiometer proper gives the value of the unknown potential. The Brooks deflection potentiometer[1] made by Leeds

[1] *Bulletin*, Bureau of Standards, vol. 2, p. 225 (1906); vol. 4, p. 275 (1907–08); vol. 8, p. 395 (1912).

and Northrup Co. is of this type. It contains a portable galvanometer, regulating rheostats, keys and so forth, so that the instrument is self-contained and portable. A low-resistance deflection potentiometer is also made by R. W. Paul.

The application of the deflection potentiometer is found where the potential is not sufficiently steady to permit the use of the standard-type instrument and where a higher precision than that obtainable with secondary standards (voltmeters) is desired. The average of a slightly fluctuating potential can be readily estimated on the galvanometer. Also, where a great many determinations are to be made and rapidity is important, the deflection instrument is convenient.

58. Potentiometer Volt Boxes.—Potentiometers are usually constructed for a maximum potential of only 1.5 volts. When a higher potential is to be measured a form of resistance box, called a volt box is used in conjunction with the potentiometer.

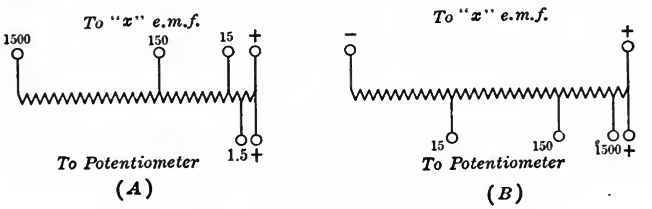


Fig. 32.

It consists of a number of resistance coils connected in series. with taps brought out to binding posts. Fig. 32 shows the diagrammatic arrangement of two forms. In (*A*), the potentiometer is connected to posts marked "+" and "1.5." When the potential to be measured is between 1.5 and 15 volts, it is connected to the "+" and "15" posts, between which the resistance is just 10 times that between "+" and "1.5." Hence only one-tenth of the potential is applied to the potentiometer, the readings of which must, therefore, be multiplied by 10. Similarly, when the potential is between 15 and 150 volts, it is connected to the "+" and "150" posts and the ratio is 100. In (*B*), the unknown e.m.f. is always connected to the posts, "+" and "−," and the potentiometer connections are shifted to obtain the desired ratio. Theoretically, the former method is the better because the resistance in parallel with the potentiometer is constant and the sensitivity is, therefore, constant. In the latter case, the sensitivity varies, being a minimum with the lowest ratio, or

when the maximum amount of resistance is in parallel with the potentiometer. On the other hand, the unknown e.m.f. is always connected to a high resistance, and hence there is little danger of burning out the volt box; this occasionally happens with the first form (A) due to accidental connection with the wrong terminals.

59. Use of Potentiometers.—The following notes apply to the care and use of potentiometers in ordinary measurements:

(*a*) The *accessories* should be suitable for the particular type of instrument, that is, high- or low-resistance, and for the class of measurement to be made. The galvanometer should be sufficiently sensitive to give a perceptible deflection when there is an unbalance equal to the smallest scale division in the potentiometer circuit. A low-resistance galvanometer, of the order of 100 ohms, should be used with a low-resistance potentiometer; and a high-resistance galvanometer of 500 to 1,000 or more ohms, with a high-resistance potentiometer. Similarly, the resistance of the volt box for low-resistance potentiometers should be as low as the permissible power loss in the resistance coils will permit. This is usually about 5,000 ohms for 150 volts. For high-resistance instruments, the resistance is 1,000 ohms or more per volt.

(*b*) The *secondary or auxiliary source* of e.m.f. from which the potentiometer current is obtained must usually be from 2 to 3 volts. A storage battery of one or two cells is necessary with low-resistance types where the current is relatively large. Storage batteries, however, require attention if reasonable service and life are to be obtained. Furthermore, the e.m.f. is unsteady immediately after charging and at first falls off rapidly. Dry cells are more convenient, require no attention and are entirely satisfactory for high-resistance potentiometers; they are also satisfactory for instruments of moderately low resistance if not in constant use. In the latter case, the large-size cells or two sets of cells in parallel should be used. In ordinary use dry cells will be serviceable for 8 to 12 months.

(*c*) The *first trials for balances* should always be made with a resistance in series with the galvanometer. This is usually provided in the instrument, with facilities for readily cutting it out of circuit when an accurate balance is being obtained. The resistance protects the galvanometer and also the standard cell from the effects of excessive currents.

(*d*) *When not in use* the instrument should be kept covered in order to keep the contacts free from dust and to prevent unnecessary oxidation of the hard-rubber top which decreases the surface insulation.

(*e*) Trouble is sometimes experienced, especially in damp weather, due to current "*leaking to ground*" from the potentiometer circuits in a manner which produces a false deflection. This can be obviated by providing a "guard circuit." In one scheme of this kind all of the apparatus is placed on small, hard-rubber pillars each of which in turn rests on a small metal plate. These plates are connected together and to the positive "x" binding post by fine bare wire. Thus all points to which current might "leak" over the surface are kept at the highest external potential to which the potentiometer is connected. When the surfaces become noticeably moist, conditions can be improved by carefully wiping with a cloth moistened with grain alcohol.

(*f*) In very *precise measurements* the potentiometer and volt box are placed in a shallow tank of oil so that the coils are completely immersed. The oil is kept thoroughly stirred and its temperature is held constant by a thermostat operating on an electric heater. When the natural radiation is not sufficient to cool the oil down to the temperature where the thermostat operates, a coil of pipe is provided through which cold water is circulated. For all ordinary measurements, however, that is, a precision of 0.02 or 0.03 per cent., oil immersion is quite unnecessary if the apparatus has been designed for air cooling which is the usual case.

60. Calibration.—In calibrating a potentiometer, the only requirement is that the ratio of the resistance of each step to the resistance betwen the standard-cell terminals shall be the same as the ratio of the corresponding potentials. In other words, it is essential that the various coils be equal but not necessarily of any particular value. For example, if the standard-cell e.m.f. is 1.0183 volts and the resistance between its terminals is 101.83 ohms, the resistance of the various steps should be adjusted to 10 ohms per 0.1 volt. If the standard-cell resistance is 102.848 ohms the potentiometer is still accurate if the resistance throughout the circuit is adjusted to 10.1 ohms per 0.1 volt. A potentiometer is checked, therefore, by measuring the resistance of the various steps and comparing the total at each setting with the resistance between the standard-cell terminals. The deviation

of this ratio from that of the corresponding potentials is the error that would be introduced in a measurement. Potentiometers are usually provided with facilities for conveniently measuring the resistance of each coil.

VOLTMETERS[1]

61. General.—Voltmeters are direct-reading indicating instruments for the measurement of e.m.fs. and potential differences and are used very extensively for all ordinary engineering measurements.

A voltmeter for direct current is essentially a low-sensitivity galvanometer provided with a scale over which moves a pointer attached to the moving system. The scale is calibrated in volts by comparison with an indicating instrument of higher sensitivity or by means of a potentiometer. Practically all continuous-current voltmeters employ the principle of D'Arsonval galvanometers, as shown in Fig. 33. They consist essentially of a light rectangular coil of fine copper wire wound upon an aluminium frame, pivoted in jeweled bearings and capable of rotating in the annular space between the soft-iron core and the pole pieces of the permanent magnet. The aluminium frame, being a closed secondary circuit, acts as a brake or damper when the coil is deflected; the instrument is thus made "*dead beat.*" The pole pieces are so shaped that the magnetic field strength is uniform throughout the space in which the coil moves. The field strength varies widely in different makes of instrument, ranging from 200 to 3,000 lines per square centimeter in the steel. A light tubular pointer attached to the coil moves over a calibrated scale. The current is introduced into the coil by two spiral springs which also provide the controlling force. Since the field strength and the gradient of the controlling forces are uniform, the deflection is strictly proportional to the current passing through the coil,

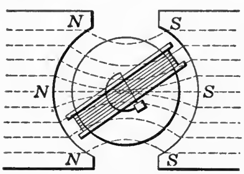


FIG. 33.

[1] *Circular* No. 20 (2d edition) of the Bureau of Standards, entitled "Electrical Measuring Instruments," contains an extensive discussion of the principles of construction, operation and use of the various types of voltmeters in general use.

and the scale divisions are uniform; in such a case the instrument is said to have an "equal part" scale. A large amount of resistance is connected in series with the moving coil in order to make the current small. Thus the same instrument can be made suitable for a wide range of voltages by changing the amount of series resistance. This resistance is made of wire having a low temperature coefficient in order to neutralize as much as possible the effect of the large coefficient of the copper in the coil.

61. Characteristic Data.—The resistance of portable voltmeters of the D'Arsonval type varies from 50 to 150 ohms per volt. The current sensitivity varies from about 7 to 20 milliamperes at full scale deflection. The resistance of the moving coil is of the order of 75 ohms. The torque varies from 2 to 6 gram-millimeters at full scale deflection with a ratio of torque to weight (grams) of about 1 to 5.

63. Laboratory-standard Type.—So-called laboratory-standard voltmeters are similar to portable instruments except that they are larger, have a longer pointer, a longer and more open scale and are made with greater care. They are only semiportable and are intended primarily for standardizing purposes. Measurements can be made with them more accurately because the error in reading is lower; for example, a potential which may be read to 0.1 volt with a portable voltmeter may be read to 0.05 volt with a laboratory standard.

64. Effect of Magnetic Fields.—External or stray fields may affect the indications of D'Arsonval instruments unless the instrument is shielded with an iron case. Even when shielded, it is advisable to keep the instrument as far as possible from large currents or other possible sources of magnetic fields, such as adjacent electrical instruments. With portable instruments, a simple test for stray fields is to note whether a given deflection remains the same when the instrument is turned through a horizontal angle of 180°; a stray field is indicated by a change in the deflection, and its effect may be eliminated either by taking the average of the indications in the two positions or by placing the instrument in the position where it indicates the mean of the two extremes. Of course, this can be done only where the deflection is steady.

The general effect of stray fields on the standard types of portable and switchboard instruments is shown in the accompanying table. These errors are usually only temporary and

disappear when the stray field is eliminated. However, if the field is very strong as under short-circuit conditions in a neighboring conductor, demagnetization of the instrument magnets may result in a permanent error. Shields are likely to be of little value under such conditions because the iron becomes saturated.

EFFECT OF STRAY MAGNETIC FIELDS ON D'ARSONVAL TYPE VOLTMETERS

Stray field,[1] lines per square centimeter	Error at two-thirds full scale deflection, per cent.	
	Shielded	Unshielded
5	0.5 –1.0	2.0
10	0.75–1.75	3.5–5.5
15	1.0 –3.0	6.0–7.5
20	1.25–3.25	7.5–10.0

65. Effect of Temperature Changes.—Increase in temperature normally causes the phosphor-bronze control springs to weaken slightly and, therefore, tends to make the instrument read higher. The magnets tend to weaken slightly and cause the instrument to read low. Thus the two effects are in opposite directions. By far the greatest temperature effect is in the moving coil which, being copper, suffers a resistance change of about 0.4 per cent. per degree C. However, the large amount of series resistance ordinarily employed is made of material having a negligible temperature coefficient so that the total net change in resistance is negligible. The final result is, therefore, that the temperature coefficient of D'Arsonval-type instruments is extremely small, about of the order of 0.01 or 0.02 per cent. per degree C.

66. Effect of Electrostatic Fields.—An external electrostatic field cannot ordinarily produce an error because of the shielding effect of the metal case. But in circuits of relatively high voltage, 500 volts or over, continuous-current instruments are subject to errors due to electrostatic attraction between the moving system and the cover where the latter is insulated from ground as would be the condition when the base is of wood or the instrument is on a non-conducting table or mounted on a switchboard. In such a case, a charge is induced on the cover and attraction of the moving system results. The test is to touch the cover

[1] The field produced at a distance of 30 cm. (12 in.) from a conductor carrying 3,000 amp. is about 20 lines per square centimeter.

with the hand; any movement of the pointer indicates electrostatic attraction has existed.

The remedy is to connect the cover to the moving-coil terminal of the instrument and thus bring the moving system and the surrounding metal cover to the same potential.[1] Where the circuit is grounded, or can be grounded, the moving-coil terminal of the voltmeter and the case should be connected to the grounded side of the circuit.

Rubbing the glass cover with dry cloth will also produce electrification, with similar results. The charge can be removed by rubbing the glass with the moist hand or by breathing on it.

67. Precautions, Portable Voltmeters.—Portable instruments should be approximately level when in use, although if in reasonably good condition as to mechanical balance of the moving element, a difference in height of 0.5 in. between any two edges will produce no readable error. Preferably they should not be left in circuit except when taking readings, in order to avoid possible "setting" of the springs and errors due to heating. Portable instruments should ordinarily be spaced about 12 in. center to center in order to eliminate mutual errors due to stray magnetic fields.

68. Switchboard Voltmeters.[2]—Continuous-current voltmeters for use on switchboards are usually of the *D'Arsonval type.* The construction is the same as that of portable instruments except that they are more substantial and rugged, especially as regards the moving system, in order to withstand the harder conditions of continuous service and excessive fluctuations. They are mounted in iron cases to protect them as much as possible from the normal stray fields due to busbars.

The discussion of the characteristics of portable and laboratory instruments in the preceding paragraphs applies in general to switchboard instruments.

69. Astatic Instruments.—The astatic class of switchboard continuous-current instruments (voltmeters and millivoltmeters)

[1] Care should be taken that the magnet system is connected to the cover. If they are insulated from each other, grounding the cover will obviously only partially remove the difficulty.

[2] An extensive discussion of switchboard voltmeters with design, performance characteristics, etc., will be found in the following article: "A Comparison of American Direct-current Switchboard Voltmeters and Ammeters," T. T. Fitch and C. J. Huber, *Bulletin,* Bureau of Standards, vol. 7, p. 407 (1911).

made by the General Electric Co. is designed particularly for use on switchboards carrying very large currents. In this instrument there are two fields produced by two electromagnets. The moving element consists of two flat coils connected in series. The control is provided by a piece of soft iron in the moving system which moves between the magnet poles and, as a temporary magnet, exerts a torque opposing that produced by the coils. The fields and the two moving coils are astatically arranged so that the instrument is not affected by external magnetic fields.

The single air-gap instruments of the Westinghouse Electric and Manufacturing Co. can be so arranged that an external field

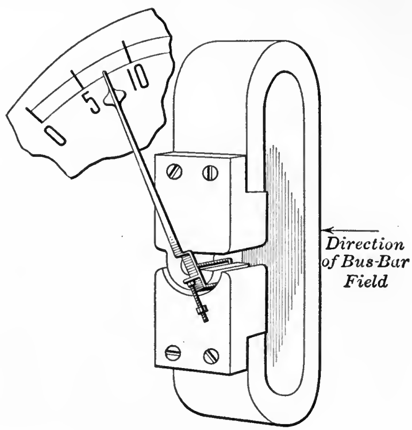


FIG. 34.

will have a negligible effect.[1] In this instrument the coil is pivoted at one edge and the other edge swings through the single air gap. In addition to other advantages claimed for this construction, the magnets can be arranged so that an external field is perpendicular to the air-gap field and, therefore, harmless. Fig. 34 shows the arrangement of the instrument and how the field from a vertical bus (the most troublesome source of stray fields) can have no effect.

70. Ground Detectors.—Ground detectors on ungrounded continuous-current systems are usually simply special forms of voltmeters. In one form, there are two coils, differentially wound on the moving system. One end of each coil is con-

[1] "Switchboard Instruments," PAUL MACGAHAN, *Transactions*, N. E. L. A., Technical Sessions, 36th Convention, p. 599 (June, 1913).

nected to ground and the two free ends are connected respectively to the two sides of the system. When there is no ground or fault on the system, the pointer stands at the center of the scale, in normal equilibrium. When a ground or fault occurs, current flows through the coil connected to the ungrounded side, producing a deflection. In another form of detector a standard voltmeter, without the series resistance is connected to the center of a resistance shunted across the line, with the remaining terminal connected to ground.

71. Measurement of Extreme Potentials.—The methods and instruments described in the preceding paragraphs can be used to measure a very wide range of voltages. Where the potential is not only small but the available power is also very small, as in thermocouples and miniature galvanic cells, the potentiometer method is the most convenient. A simple slide-wire instrument can be easily arranged but care should be taken that the galvanometer employed is sufficiently sensitive to respond with a current which will not alter the potential to be measured.

High potentials, where the power available is insufficient to operate an electromagnetic instrument in conjunction with series resistance, may be measured with electrostatic voltmeters (see par. 98) or with electrometers.

72. Electrometers.—The principle of the electrometer is as follows: A piece of thin aluminium is suspended by a metallic suspension over four quadrants of sheet metal which are insulated from each other and from the frame or support. Opposite quadrants are connected to each other and the two sets are connected respectively to the two sides of the circuit to be measured. If a charge from a condenser is placed on the moving vane, one end will be repelled and the opposite end attracted, producing a deflection which will be a measure of the potential applied to the stationary quadrants. This instrument is extremely sensitive, and while it is one of the earliest types of electrical measuring instruments it is still used extensively in research work where the available power is small because no current is required beyond that necessary to charge the quadrants.

CHAPTER IV

CONTINUOUS-CURRENT MEASUREMENTS

73. Absolute Unit and Standard.—The fundamental or absolute unit of current is based on the centimeter, gram and second. In the electromagnetic system of fundamental units, it is defined in terms of the dimensions of a conductor carrying the current and the strength of the field produced by that current. Measurements of current directly in terms of the c.g.s. unit are, therefore, made with instruments so carefully constructed that the current can be calculated from the various dimensions of the instrument.

Absolute measurements of current have usually been made with two types of standards. In one type of standard, the deflection of a magnetic needle at the center of a coil is measured and the current is calculated from the dimensions of the coil, the strength of the earth's field and the torsion of the suspension. This standard is a precision form of tangent galvanometer (par. 27). This method involves, of course, any error in the determination of the earth's field. In the other standard, the needle is replaced with a suspended coil. When the length and the radius of both movable and fixed coils are in the ratio of $\sqrt{3}:1$, when the centers coincide and when the dimensions of the fixed coil are large compared with those of the movable coil, the torque exerted by the moving system is expressed by

$$T = \frac{4\,\pi^2 N n r^2 I}{\sqrt{D^2 + L^2}} \qquad \text{(dyne-cm.)}$$

where N = number of turns in fixed coil, n = number of turns in movable coil, D = diameter of fixed coil in centimeters, L = length of fixed coil in centimeters, r = radius of movable coil in centimeters, and I = current (coils in series) in c.g.s. units. Hence by measuring the torque (weighing it), the current can be determined directly in c.g.s. units. This type of standard is in reality a precision form of Siemens electrodynamometer.[1]

[1] "The Gray Absolute Electrodynamometer," E. B. Rosa, *Bulletin*, Bureau of Standards, vol. 2, p. 71 (1906).

Obviously, the greatest care is necessary, not only in the construction of these absolute standards but in their use.[1]

74. Practical Unit and Standard.—It would be impracticable to make ordinary measurements in terms of the fundamental unit by the methods indicated above. The Act of Congress of 1894 which legalized certain practical units of electrical measure defined the practical unit of current, or the international ampere, as one-tenth of the fundamental c.g.s. unit. This Act also defined the practical standard of current as the rate of deposition of silver at the cathode of a silver voltameter (par. 74) constructed and operated under certain prescribed conditions, the international ampere being the current which will deposit 0.001118 gram of silver per second in a standard voltameter.

Precision Measurements

75. General.—Precision measurements of current are made with a voltameter or a potentiometer. Electrodynamometer instruments of the Siemens and Kelvin balance types were formerly used extensively as secondary standards but the potentiometer has not only superseded such instruments but also voltameters except in rare cases. The high state of development of potentiometer methods, the high precision obtainable and the reliability and convenience of the potentiometer have resulted in its being used almost exclusively even for measurements of the highest precision.

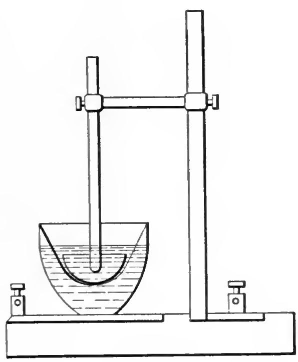


Fig. 35.

75a. Voltameters.—When a continuous current is passed through an electrolyte, the latter is decomposed at a rate which is proportional to the current strength, and an apparatus for measuring current by such means is called a voltameter.

In the silver voltameter, the cathode or negative electrode is platinum, the anode or positive electrode is pure silver, and the

[1] "A Determination of the International Ampere in Absolute Measure," E. B. Rosa, N. E. Darsey and J. M. Miller, *Bulletin*, Bureau of Standards, vol. 8, p. 269 (1912).

electrolyte is silver nitrate. Fig. 35 shows the general form. The anode is usually a silver rod projecting into a platinum bowl which rests in turn on a copper plate. In cheaper forms, plates of silver and platinum are supported in a glass jar.

The solution is usually about 15 per cent. (by weight) silver nitrate and the effective area of the anode is about 50 sq. cm. per ampere. In order to prevent disintegrated silver on the anode from dropping on to the cathode, the anode is surrounded with pure filter paper or a porous cup made of unglazed porcelain; in other cases, merely a shallow glass dish is placed beneath the anode. Recent investigations at the Bureau of Standards[1] have shown, however, that the chemical activity of filter paper results in an excessive deposit of silver and that the porous cup is much more suitable. Before using, the cathode should be carefully cleaned, a non-metallic scraper of horn or bone being employed to remove the silver. It next should be thoroughly washed with dilute nitric acid, distilled water and alcohol in succession, then dried in an oven at about 150°C., cooled in a desiccator and weighed. The current should flow during a period of 30 min. to 1 hr. from an unvarying source of continuous e.m.f., the time being carefully noted in seconds and fractions thereof. After the run, the cathode must be thoroughly washed with distilled water, until the rinse water shows no precipitation upon adding hydrochloric acid. The average current in amperes is computed from the formula

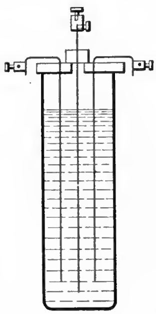


Fig. 36.

$$I = \frac{M}{0.001118t} \quad \text{(amp.)}$$

where M = weight of silver deposited in grams, t = total time in seconds, and 0.001118 = electrochemical equivalent of silver.

[1] "The Silver Voltameter, Part II. The Chemistry of the Filter Paper Voltameter and the Explanation of Striations," E. B. Rosa, G. W. Vinal and A. S. McDaniel, *Bulletin*, Bureau of Standards, vol. 9, p. 211 (1913). For experiments with the silver voltameter see Parts I and III of the same article, pp. 151 and 493 respectively.

"Studies on the Silver Voltameter," C. H. Hulett and G. W. Vinal, *Bulletin*, Bureau of Standards, vol. 11, p. 553 (1914–1915).

"Summary of Experiments on the Silver Voltameter at the Bureau of Standards and Proposed Specifications," E. B. Rosa and G. W. Vinal, *Bulletin*, Bureau of Standards, vol. 13, p. 479 (1916–1917).

The copper voltameter is used for the measurement of very large currents. The anode is electrolytically pure copper, the cathode is copper or platinum, and the electrolyte is a solution of pure copper sulphate in the proportion of 10 grams of crystals to 40 c. c. distilled water. Two anodes may be used in order to utilize both sides of the cathode (Fig. 36) and the current capacity may be further increased by connecting additional plates in parallel. The current density should not exceed 1 amp. per square centimeter of cathode area. If the cathode is copper, the exposed portion above the electrolyte should be coated with shellac or sealing wax to prevent oxidation. The cell should be kept cool to obtain the most accurate results. The rate of deposition varies slightly with the current density and the temperature as shown in the following table. The cathode must be carefully and thoroughly washed, dried and weighed as in the case of the silver voltameter.

ELECTROCHEMICAL EQUIVALENT OF COPPER AT VARIOUS CURRENT DENSITIES AND TEMPERATURES[1]

Square centimeters of cathode per ampere	Electrochemical equivalent, mg. per amp.-sec.		
	At 12°C.	At 23°C.	At 28°C.
50	0.3288	0.3286	0.3286
100	0.3288	0.3283	0.3281
150	0.3287	0.3280	0.3278
200	0.3285	0.3277	0.3274
250	0.3283	0.3275	0.3268
300	0.3282	0.3272	0.3265

[1] From "Standard Handbook for Electrical Engineers," 1st edition (1907).

In the gas or water voltameter the electrodes are two platinum plates and the electrolyte is a 10 per cent. solution of sulphuric acid. In the electrolysis of this solution, hydrogen gas is formed at the cathode and oxygen gas at the anode. The total gas formed collects above the liquid, and is measured volumetrically in a closed graduated tube forming the upper part of the containing vessel.

76. Measurements with Potentiometers.—A current is measured with a potentiometer by measuring the fall of potential across a resistor of known resistance and through which the cur-

rent to be measured is passed. The current is calculated by Ohm's law. Potentiometers are discussed in Chapter III and standard resistors in Chapter VII.

As previously stated, practically all precision measurements are now made in terms of the volt and the ohm with a potentiometer; the legal standard, the silver voltameter, being used only for reference measurements and primary standardization. The potentiometer method is much more rapid and convenient than the voltameter. The precision is at least equal to the voltameter method and usually higher because the measurement depends only on a resistance and a difference of potential, both of which quantities can be measured with a very high precision.

Ammeters[1]

77. General.—Indicating continuous-current instruments which indicate current directly by the deflection of a pointer over a marked scale are known as ammeters. Modern continuous-current ammeters are almost universally millivoltmeters of the D'Arsonval type connected to the terminals of a resistance (shunt) which in turn is connected in series with the main circuit whose current is to be measured. The deflections of the instrument will be proportional to the fall of potential across the shunt and therefore to the current flowing through it. A millivoltmeter may be made an ammeter of any capacity by simply changing the resistance of the shunt.

The principle of the D'Arsonval-type instrument is described in Chapter III on e.m.f. measurements, par. 60*a*. In fact, a large part of Chapter III applies equally well to ammeters. The only essential difference between a millivoltmeter and a voltmeter is in the resistance of the movable coil. This resistance in a millivoltmeter is much lower than in a voltmeter (order of 0.5 and 5 ohms respectively) in order to make the millivolt constant—and therefore the sensitivity as an ammeter when connected to the shunt—higher. This is accomplished by winding the movable coil with coarser wire or with several small wires in parallel. Also, the total ampere-turns are usually less than in voltmeters. This results in a lower torque requiring weaker springs. Also,

[1] See *Circular* No. 20 (2d edition) Bureau of Standards, for extensive discussion of principles of construction, operation, and use of the various types of ammeters in general use.

in order to reduce the resistance of the circuit, a higher conductivity alloy than phosphor bronze is sometimes used.

78. Portable Instruments.—Portable ammeters of about 25 amp. range and less are usually "self-contained," that is, the shunt is within the instrument case. Above about 25 amp., the shunt is separate from the instrument proper but connected to it by special detachable leads. The potential drop in the shunt at full scale deflection is usually about 50 to 200 millivolts. The lower-grade instruments are usually designed for full scale deflection at the lower potential while high-grade instruments require 100 to 200 millivolts at full scale. In the high-grade instruments, the shunt is usually separate from the instrument for all ranges of current so that 1 millivoltmeter may be used with a series of shunts. It is obvious that wherever the shunt is separate from the instrument, the leads form a part of the instrument circuit and should never be altered without recalibrating the instrument or taking care that the resistance of the leads is kept the same. Care should be taken that the terminals of leads, instrument and shunt are clean and that the connections are tight, in order not to introduce extra resistance. Semi-portable "laboratory-standard" millivoltmeters similar to laboratory-standard voltmeters (par. 62) are, in conjunction with high-grade shunts, extensively used as secondary standards of current.

79. Switchboard Instruments.[1]—Instruments for use on switchboards are similar to the portable instruments except that they usually indicate full scale with 50 to 75 millivolts in order to keep the energy loss in the shunt low. The discussion of switchboard voltmeters (par. 67) applies also to ammeters.

80. Cable-type Ammeter.—The principle of an instrument for measuring the current in a circuit without opening it is shown diagrammatically in Fig. 37.[2] It consists essentially of a split iron ring, r, arranged so that it can be slipped over a bus or cable, c, carrying the current to be measured. This ring is wound with a fine wire winding, w, which is connected in series with a rheostat, R, a low-voltage battery, B, and an ammeter, A. A

[1] For construction data, constants and performance characteristics of switchboard voltmeters see "A Comparison of American Direct-current Switchboard Voltmeters and Ammeters," T. T. Fitch and C. J. Huber, *Bulletin*, Bureau of Standards, vol. 7, p. 407 (1911).

[2] "Measuring the Current in Direct-current Circuits," Otto A. Knopp, *Electrical World*, Oct. 2, 1915.

small magnet needle, M, is mounted in the gap between the poles of the split ring. When a magnetic flux is set up in the ring by the current in C, an opposing current in W is adjusted until complete neutralization of the flux due to C is obtained as indicated by the position of M. The conductor C being simply one turn, the current in it is then equal to the product of the turns in W and the current in amperes.

81. Ammeter Shunts.—Ammeter shunts are so constructed as to have a resistance which will be constant, as nearly as possible, under all conditions. The resistance metal has a low temperature coefficient, and the temperature is kept low either by connecting several strips in parallel and making the current density low, or by making the current density high and using short lengths of the resistance metal with heavy copper terminals designed to dissipate the heat by conduction and radiation. The former method is most generally used, the strips being silver-soldered into relatively heavy copper or brass terminals which are connected into the circuit to be measured.

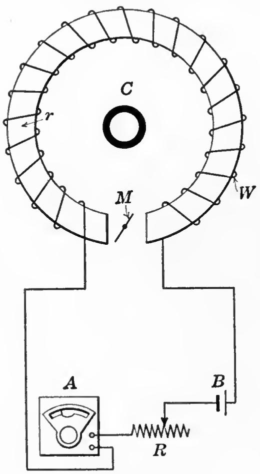


FIG. 37.

Furthermore, the resistance metal should also have a low thermo e.m.f. against copper for two reasons: First, if one end of the shunt becomes hotter than the other end due to poor contact at one current connection or to a poor connection in the cable or bus structure near to that end of the shunt, a thermal e.m.f. will be produced in addition to the potential drop across the shunt. Second, when current passes through the junction of two dissimilar metals which have a thermo e.m.f. with respect to each other a Peltier effect results, that is, heat is absorbed or produced. Hence in a shunt, one junction tends to become cool and the other to become hot and the resultant difference in temperature will produce an e.m.f. if the resistance metal has a thermo e.m.f. against copper. Where a thermo e.m.f. exists, it may be compensated for by introducing into the millivoltmeter circuit two similar pairs of thermo junctions, one junction of each pair being connected to each potential tap respectively with the other junction sufficiently far away

to be at room temperature. The scheme is shown diagrammatically in Fig. 38 where S and S' are the compensating "shunts." It will be seen that the various e.m.fs. will approximately neutralize each other.

82. Temperature Compensation.—The temperature-resistance coefficient of copper is 0.4 per cent. per degree C., consequently a temperature error of this magnitude would exist if the movable coil of the millivoltmeter was connected directly to the shunt. The most common method of eliminating this error is to connect sufficient resistance having a low temperature coefficient such as manganin, in series with the coil (Fig. 39) to reduce the total coefficient to a negligible value. Usually, however, to get a sufficiently low coefficient by this method, the shunt drop must be 150 to 200 millivolts.

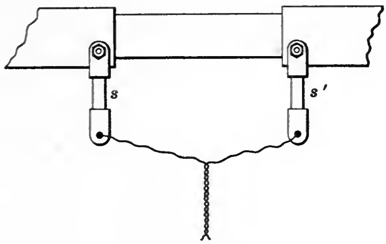


FIG. 38.

In another method, compensation may be affected with a lower potential and is, therefore, applicable to 50-millivolt instruments. The scheme is shown in Fig. 40. A small amount of low-co-

FIG. 39.

efficient resistance wire, R_m, is connected in series with the movable coil and the whole shunted by a copper wire R_c. Another small amount of low-coefficient resistance wire, R'_m is connected in series with this multiple circuit. By adjusting these various

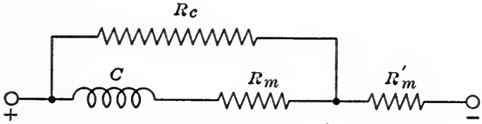


FIG. 40.

resistances to the proper values, the instrument temperature coefficient can be reduced to zero because other temperature errors such as those due to the control springs and the magnets can be compensated for.

83. Calibration of Ammeters.—The calibration of ammeters is effected by adjustment of the resistance of the shunt, the resistance of the millivoltmeter circuit or both. Formerly each instrument and shunt were adjusted together but it is becoming customary to adjust all of the instruments of a given type to deflect full scale with the same potential in millivolts at the terminals. The shunts for these instruments are all similarly adjusted to give the same potential drop, thus making all shunts and instruments of a given type interchangeable.

The shunts should be adjusted by varying the resistance between the potential taps and not by adjusting a resistance wire connected in series with the instrument leads. In calibrating switchboard instruments and the lower-grade portable instruments, the potential terminals are attached to the main current terminals and adjustment is effected by reducing the cross-section of the resistance strips which has been purposely left too large. In instruments of higher grade, the potential terminals are connected to the resistance strips inside the current terminals; approximate adjustment is obtained by trial, and after soldering the tap wire to the resistance strip, final adjustment is obtained by varying the cross-section, or by cutting back a tongue-shaped piece of the strip whose end is soldered to the tap wire.

84. Checking Ammeters.—The accuracy of ammeters of ordinary capacities is determined by connecting them in series with a standard instrument, preferably a potentiometer and a standard resistance. The error is then noted at various points throughout the scale. In instruments of considerable capacity, the effect of heating should be determined by holding the current for some time at two-thirds to full scale value, and then noting any change in the scale errors from the original value. The thermo e.m.f. should be tested for after the shunt is thoroughly heated, by opening the main circuit—leaving the instrument connected to the shunt—and noting the indication of the instrument.

When the rating of the instrument is beyond the facilities at hand, the resistance of the shunt can be accurately measured between the potential taps, and the instrument itself (with its leads) checked separately as a millivoltmeter. For accurate work, the resistance of the shunt should be measured at several temperatures so that when the instrument is in use the true

correction for any given load condition can be obtained by simply noting the corresponding shunt temperature.

Very large capacity ammeters have to be calibrated and checked as a resistance and a millivoltmeter separately as indicated in the last paragraph. In measuring the resistance of large shunts for such a purpose it is very important that the testing current be uniformly distributed throughout the shunt by having proper terminal connections to the test circuit. If the test circuit is connected only to one corner of the terminal blocks, for example, the current distribution will be far from uniform and the drop between the potential taps will not correspond to the true resistance.[1]

The thermo e.m.f. of large shunts can be determined by heating one terminal block only, leaving the other exposed. Thus a temperature difference between the two junctions is produced and the thermo e.m.f. can be measured directly with a potentiometer. If a thermo e.m.f. is found, correction can be made when the shunt is in service by measuring the difference in temperature between the terminals of the shunt.

85. Use of Large Ammeters.—Shunts of several thousand amperes capacity should be provided with long, multiple-leaf copper blocks to insure that the current will be uniform through the shunt and that the distribution will be the same after installation as when it was calibrated. Every time a large shunt is connected to a multiple-leaf bus structure, the contact resistance distribution is changed more or less and if this joint is close to the resistance strips, the current will not have become uniformly distributed through the resistance metal.

Care should be taken so to install a large shunt in a bus structure that the temperature of the two terminal blocks will be equal. This means thoroughly clean and tight connections to the bus structure and removal a sufficient distance from large switch or other connections which are liable to run "hot."

The leads to the millivoltmeter should be twisted and fastened to the switchboard. Otherwise, in a breeze, a pulsating indication might result due to heavy stray fields.

86. Effect of Magnetic Fields.—The discussion of the effect of magnetic fields on voltmeters in Chapter 3 applies also to ammeters.

[1] The resistance of large shunts would ordinarily be measured by a double-bridge method. See Chapter VII.

87. Measurement of Rectifier Currents.—The current delivered by a rectifier is a unidirectional current of a periodically varying magnitude, the number of maximum points being equal to double the frequency of the alternating current supplied to the rectifier. Either of two values of such a current may be required, the average value or the root-mean-square value (see Fig. 41).

In a storage battery or an electrolytic cell, the action which takes place is proportional to the first power of the current. Therefore, where the current fluctuates periodically as in a

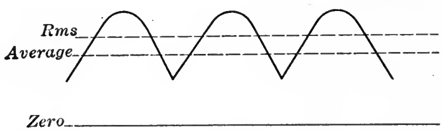


FIG. 41.

rectifier circuit, the average of the instantaneous values is equal to the continuous current which would produce the same chemical effect. Such a value would be indicated by a permanent-magnet type of instrument.

On the other hand, the candlepower of incandescent lamps, and the temperature of electric heating devices are functions of the second power of the current and the root-mean-square value of the current is the important value. Hence, instruments employing thermal principles or the electrodynamometer principle should be employed. Obviously, when the "ripple" becomes sufficiently small, the difference between the two values of the current becomes negligible and either type of instrument may be used.

CHAPTER V

ALTERNATING E.M.F. MEASUREMENTS

88. General.—The several values of an alternating e.m.f. or potential difference are (*a*) instantaneous value or value at any instant during a cycle, (*b*) mean effective value which is the square root of the sum of the squares of the instantaneous values during a cycle (also called effective value and root-mean-square value), (*c*) average value which is the arithmetical average of the instantaneous values during one alternation, (*d*) maximum value which is the maximum of the instantaneous values during a cycle. The relations between these various functions when the wave form is a sine curve (and only under that condition) are as follows:

$$\text{Maximum value} = \sqrt{2} \times \text{effective value.}$$

$$\text{Maximum value} = \frac{\pi}{2} \times \text{average value.}$$

$$\text{Average value} = \frac{2\sqrt{2}}{\pi} \times \text{effective value.}$$

$$\frac{\text{Effective value}}{\text{Average value}} = \frac{\pi}{2\sqrt{2}} = 1.11 = \text{form factor.}$$

$$\frac{\text{Maximum value}}{\text{Effective value}} = \sqrt{2} = 1.414 = \text{crest, peak or amplitude factor.}$$

All of these values are employed in electrical engineering but the effective value is the one ordinarily measured and used. This particular value is used because it is the value of an alternating e.m.f. which, when applied to a circuit containing resistance only, will produce the same heating effect as a continuous e.m.f. of the same magnitude. Also, the effective value is the value indicated by alternating-current instruments used in all ordinary measurements.

The standard of e.m.f. being the standard cell, the measurement of an alternating e.m.f. or potential involves, like continuous e.m.fs. and potentials, a comparison with the standard cell. The comparison may be made by a more or less direct method as in primary measurements or by indirect methods

employing secondary standards which in turn have been calibrated by direct comparison on continuous current.

Primary Measurements

Obviously, an alternating e.m.f. or potential cannot be compared directly with a standard cell because the e.m.f. of the latter is unidirectional and constant. The comparison is made, therefore, by substitution methods.

89. Transfer Method.—In the transfer method, an instrument of the electrodynamometer or electrostatic type is connected to the alternating potential to be measured and the deflection noted. It is then connected to an adjustable continuous potential which is manipulated until exactly the same deflection is obtained. The continuous potential is then compared with the standard cell by means of a potentiometer in the usual manner. A double-throw double-pole switch is usually arranged so that the transfer from alternating current to continuous current can be quickly made, without allowing the deflection to change appreciably, the continuous potential having been previously adjusted to about the proper value. Two readings, direct and reversed, are taken on continuous current at the same reading as the alternating deflection; the average of these two is taken as the true value, thus eliminating the effect of stray magnetic or electrostatic fields. It is obvious that, on account of this indirect method, alternating e.m.f. measurements cannot be made with the degree of precision obtainable in measurements of continuous e.m.fs.

90. Potentiometers.—The principle of an alternating-current potentiometer is the same as that of a continuous-current instrument. In the latter (see discussion of potentiometers in chapter on continuous e.m.f. measurements), the drop along the potentiometer circuit is calibrated in terms of a standard cell by adjusting the current in the circuit until the drop in a certain portion is equal to the e.m.f. of the standard cell. In the alternating-current potentiometer, this same procedure is first followed but, after balance is obtained, the current is measured by an instrument which is suitable for both alternating and continuous current. A source of alternating e.m.f. is then substituted for the battery and the unknown alternating e.m.f. for the standard cell. The alternating current is adjusted to the previous continuous-current value and balance against the unknown obtained as in continuous-current measurements.

In alternating-current measurements there are, however, several points to be considered which do not develop in continuous-current measurements. They are as follows: (*a*) The resistance coils in the potentiometer should be non-inductive and anti-capacitance. (*b*) Provision must be made for detecting both continuous current and alternating current when balancing. Two instruments may be used—a D'Arsonval galvanometer for the continuous-current balance and an alternating-current galvanometer for the alternating-current balance or a separately excited electrodynamometer galvanometer or a synchronous reversing key with a D'Arsonval galvanometer may be used for both bal-

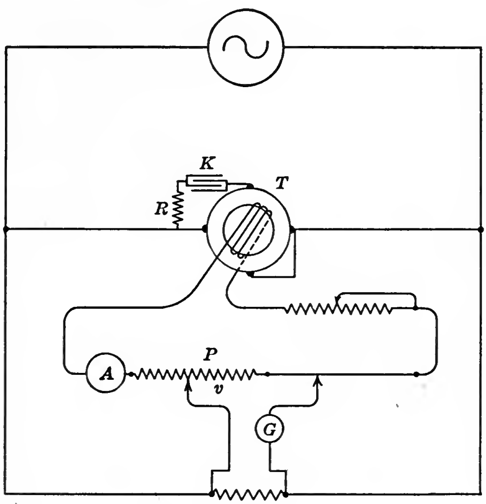


Fig. 42.

ances (see galvanometers). (*c*) To obtain perfect balance on alternating current, the potential differences must not only be equal but the frequency and the phase must be the same. To insure equal frequency the potentiometer current must, therefore, be supplied from the source of the e.m.f. being measured. Phase equality may be obtained by introducing means for shifting the phase of the potentiometer current or the phase angle may be corrected for by noting the quadrature voltage in the detector circuit, as obtained from a mutual inductance, which is necessary to produce balance (see measurement of phase angle of current transformers, par. 130).

91. Drysdale Alternating-current Potentiometer.—The principle of this instrument and a simplified connection diagram are shown in Fig. 42. An ordinary potentiometer circuit, P, is supplied with current from the secondary of a phase shifting transformer T, through a dynamometer-type ammeter, A. The phase-shifting transformer is, in principle, like a two-phase induction motor with a wound rotor—a rotating field being produced from the single-phase supply by the use of a condenser K and a resistance R in series with one of the stator windings. The phase of the potentiometer current and, therefore, the drop along the potentiometer circuit can thus be readily brought into coincidence with that of the voltage being measured by rotating the rotor (see par. 240). The potentiometer is, of course, calibrated on continuous current in the usual way, the potentiometer current as indicated by A being kept the same in the two cases.

The phase-shifting transformer is calibrated so that phase differences can, by proper manipulation, be measured directly to 0.1° and less. A vibration galvanometer is used as the alternating-current detector.

Secondary Standards

92. General.—There is, of course, no secondary standard of alternating e.m.f. in the sense that the Weston saturated cell is a secondary standard of continuous e.m.f. Various forms of carefully constructed instruments employing the electrodynamometer principle are, however, used as secondary or working standards. As explained in the chapter on galvanometers, this principle permits the use of either continuous or alternating current so that these instruments may be calibrated with continuous potential differences. Such secondary standards are usually either wall-type, reflecting-dynamometer galvanometers calibrated as voltmeters, or so-called precision or laboratory voltmeters of the dynamometer type with extra long scales and pointers. The types of dynamometer voltmeters described below are all made in this "laboratory" form.

The Kelvin volt balance (see par. 93) was formerly used to a considerable extent as a secondary standard. It is now practically obsolete because it is difficult to use except where the potential is extremely steady.

VOLTMETERS[1]

93. General.—Alternating-current voltmeters in general use may be classified as dynamometer, soft-iron vane, induction, hot-wire or electrostatic. All of these types are made in the ordinary portable form but secondary standards of alternating e.m.f. such as laboratory standard and precision voltmeters, are usually of the dynamometer type.

94. Dynamometer Voltmeters.—In this type of voltmeter, the deflection is the result of the reaction between one or two movable coils and the field produced by one or more fixed coils, the coils being connected in series and to the e.m.f. or potential to be measured. The movement of the movable system is a measure of the current flowing in the circuit and, therefore, of the potential impressed at the terminals.

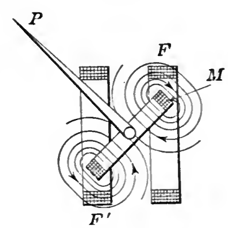


FIG. 43.

In the Weston instruments of this type, a single coil moves within two parallel fixed coils as shown in Fig. 43, where F, F' are the fixed coils and M is the moving coil, to which a pointer P is attached. The deflection is approximately proportional to the square of the current; hence the scale is compressed at the lower end and extended at the upper end.

The General Electric type P_3 voltmeter is similar to the Weston instrument of this class. The Thomson "inclined-coil" voltmeter is also similar, except that the plane of the fixed coils makes an angle of about 45° with the shaft of the moving coil for the purpose of making the scale more uniform.

The Westinghouse type Q voltmeter employs the Kelvin balance principle which is shown in Fig. 44 where there are two coils, MM', attached to opposite ends of a beam which is supported at the middle and free to move. Each coil moves between a pair of fixed coils, FF and $F'F'$, and all of the coils are connected in series in such a manner that the moments of all the forces on the movable system, taken about the beam axis, are cumulative, thus tending to produce rotation. In a Kelvin balance the controlling or opposing force is a weight moved along a graduated

[1] See *Circular* No. 20 (2d edition) Bureau of Standards for an extensive discussion of principles of construction, operation and use of the various types of voltmeters in general use.

scale attached to the beam supporting the movable coils, the whole being in a horizontal plane; the moment of this weight about the beam axis, when the moving system is balanced, varies as the square of the e.m.f. In the Westinghouse instrument

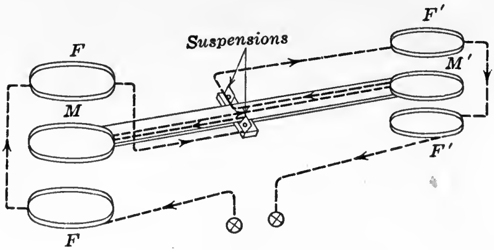


FIG. 44.

(Fig. 45) the coils are arranged vertically and the controlling force is a spiral spring. The amount of compression of this spring necessary to balance the e.m.fs. and bring the movable

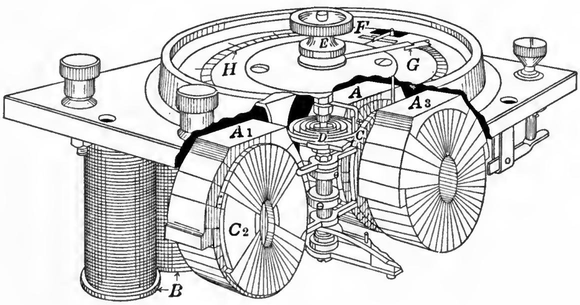


FIG. 45.

system back to zero position, as indicated by a pointer moving over a scale, is a measure of the e.m.f. The important feature of this class of voltmeters is that they can be calibrated on continuous current.

95. Soft-iron-vane Voltmeters.—These instruments utilize the reaction between a temporarily magnetized piece of soft iron and the magnetizing field. In the Thomson "inclined-coil" instrument of this type the plane of the energizing coils, C (Fig. 46), makes an angle with the shaft, S, which carries a member, i, comprising a rectangular piece of very thin, soft iron. This piece of iron is so attached to the shaft that rotation is produced by the tendency of the iron to become parallel with the field

established by the coils. In Weston instruments of this type (model 155), the reaction which produces the deflection takes place between two pieces of soft iron bent in the arc of a circle and placed concentrically, one of which, F' (Fig. 47), is movable, and the other, F, is stationary. When the surrounding coil, M, is energized, the pieces of iron become magnetized in like manner, so that the resulting force is one of repulsion. The stationary piece, F, is made triangular in shape, with the pointed end

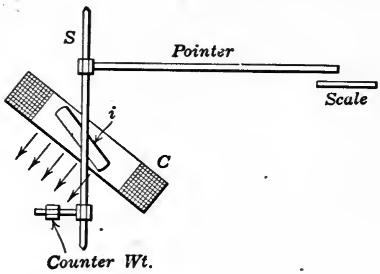


FIG. 46.

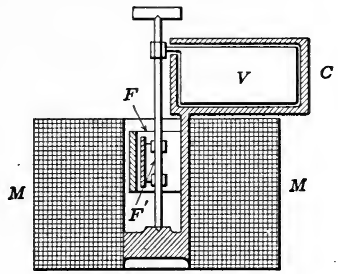


FIG. 47.

in the direction of rotation, for the purpose of making the scale more uniform. Air damping is obtained by means of a light aluminium vane, V, in an enclosing chamber, C.

The advantage of this type of instrument is the low price, ruggedness, open scale and small weight.

96. Induction Voltmeters.—This type of instrument utilizes the principle of induction watt-hour meters (par. 222), or the rotative tendency of a free cup of thin metal when placed within a so-called revolving magnetic field. Actual rotation of the movable element is prevented by an opposing spiral spring, so that the deflections become a measure of the current in the energizing coils. The Westinghouse (type P) voltmeter is an important example of this type, for which is claimed a very high ratio of torque to weight of moving element, rugged and simple construction, extremely long scale and compactness.[1] The arrangement of the circuits is shown in Fig. 48. The primary winding, P, which is connected to the line circuit, induces a current in the secondary winding, S, opposite in phase to the primary current. The secondary current passes through two

[1] "Induction-type Indicating Instruments," P. MacGahan, *Transactions*, A. I. E. E., vol. 31, p. 1565 (1912).

auxiliary coils, AA_1, wound in opposite directions on the poles. The field produced by these coils will be displaced 90° in time phase from the field produced by the primary winding and approximately at right angles thereto in space, thus producing the necessary rotating field to cause the cup, C, to tend to rotate. Incidentally, the inherent frequency error of induction-type instruments is largely neutralized by this combined transformer and induction motor action, the effect of frequency changes being opposite in the two cases.

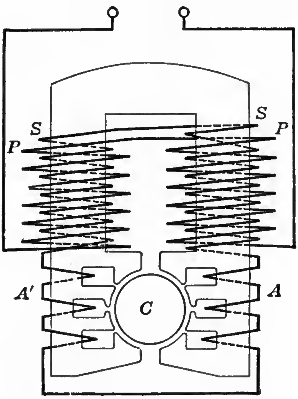


Fig. 48.

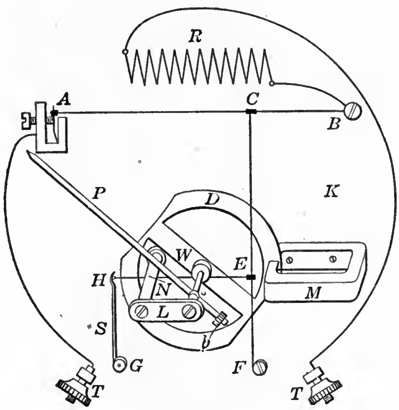


Fig. 49.

97. Hot-wire Voltmeters.—The principle of these instruments is the expansion and contraction of a wire carrying a current proportional to the e.m.f. to be measured. Fig. 49 shows the principal features of the Hartmann and Braun voltmeter, a well-known example of this class of instrument. The current flows through the platinum-silver alloy wire, AB, which is expanded by the heat produced. This expansion reduces the tension on the fine phosphor-bronze wire, CF, which in turn allows the silk fiber, HE, attached to the spring, S, to be pulled to the left. This fiber passes around a small pulley on the shaft of the moving system and thus produces a deflection of the pointer. Damping is effected by the aluminium disc, D, moving between the poles of the permanent magnet, M. The hot wire, AB, is in series with a large non-inductive resistance, R, so that the current is proportional to the e.m.f.

The distinguishing feature of hot-wire voltmeters is their independence of frequency.

This type of instrument is apt to have certain defects, however, such as errors due to change in room temperature, uncertain zero, relatively large power consumption and delicate construction which practically limit it to the laboratory, although some users have found hot-wire instruments very satisfactory.[1]

98. Electrostatic Voltmeters.—These instruments are based on the attractive force between two adjacent, electrically charged bodies of opposite polarity. They are similar to electrostatic galvanometers or electrometers, except that they are designed for measuring larger potentials and are provided with scales which make them direct-reading. They are made in a great variety of forms, for both portable and switchboard use, but are

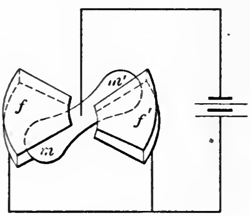


FIG. 50.

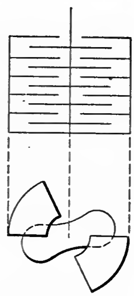


FIG. 51.

used commercially much more in Europe than in this country.

The principle of operation is shown in Fig. 50, in which mm' is a thin aluminium vane suspended or pivoted between two pairs of fixed vanes, ff'. The deflection through moderate ranges is proportional to the square of the potential and is controlled either by a spiral spring or by gravity. Many forms are undamped; in others damping is produced magnetically or by immersing the elements in oil. For ordinary commercial voltages a number of sets of vanes are arranged one above the other in a vertical position and connected in parallel, thus multiplying the effect (Fig. 51). For higher voltages one set of vanes is sufficient and they are usually placed in a vertical plane, as in the General Electric instruments, with the moving element mounted on a horizontal shaft.

In the Westinghouse electrostatic voltmeter for 100,000 volts,

[1] "Hot-wire Instruments," A. W. PIERCE and M. E. TRESSLER, *Transactions*, A. I. E. E., vol. 31, p. 1591 (1912).

the moving system is not connected to the circuit. Fig. 52 shows the arrangement of the parts. When potential is applied to parts A and A', the hollow cylinders C and C' become charged by induction and opposite in polarity to A and A' respectively. The resultant attraction produces a deflection because of the shape of the fixed plates, P and P'. The condensers K and K' are each formed by two flat plates and are connected in series with A and A' to increase the range, but for lower ranges these condensers are short-circuited so that ranges of 30,000, 60,000 and 100,000 volts are available in the same instrument and on one scale. The elements are entirely immersed in oil, which permits, for high voltages, a relatively compact construction.

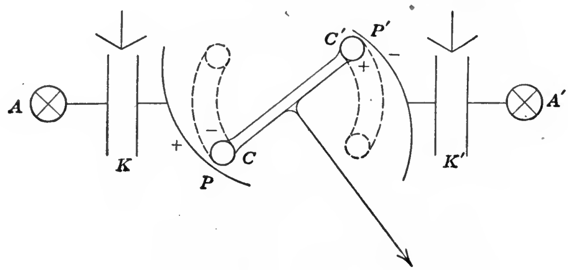


FIG. 52.

The oil also increases the torque, because it has greater specific inductive capacity than air and thus increases the electrostatic charges; it also makes the instrument more nearly dead beat.

The principal advantage of electrostatic voltmeters is that they absorb practically no power. The current required is the charging current to the instrument acting as a condenser and that is extremely small. This feature becomes important at high voltages because direct measurement with any of the electromagnetic instruments, even though the current may be only a few milliamperes, requires power which becomes inconveniently large at the very high voltages. See further discussion in connection with the measurement of high potentials.

99. Switchboard Voltmeters.—All of the types described above are used for switchboard purposes. The construction differs in detail, of course, from that of portable instruments in order to meet the more severe requirements of switchboard service. In this country, however, the dynamometer, induction and soft-iron-vane types are in most general use. The induction principle

probably finds the greatest application in switchboard instruments[1] because it permits the use of a very long scale in a limited space (circular scale) and the inherent frequency error is unimportant because the frequency is normally constant at a switchboard.

100. Ground Detectors.—Ground detectors for ungrounded alternating-current circuits are standard types of voltmeters or special forms of voltmeters. For low-voltage circuits, schemes used for continuous-current systems (par. 69) are employed. On high-voltage circuits electrostatic instruments are used.

Fig. 53 shows (at the left) diagrammatically the principle of the Westinghouse single-phase detector which is typical of

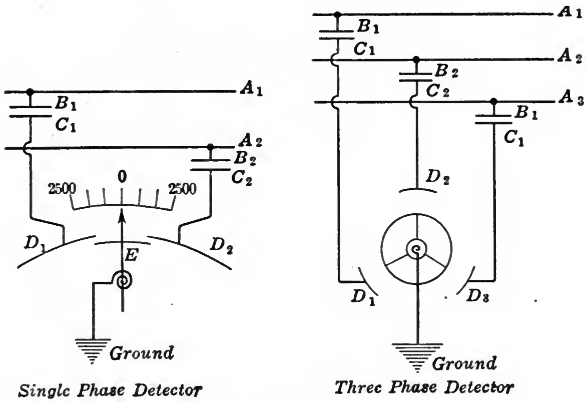


FIG. 53.

this type. It is substantially a differential electrostatic voltmeter with the two fixed vanes, D_1 and D_2, connected through condensers to the two sides of the circuit and the movable vane, E, connected to ground. Normally, the system is in equilibrium and the pointer stands at the center or zero but a ground on either side will cause the movable vane to deflect away from the grounded side. A similar scheme is used for three-phase circuits as indicated at the right of Fig. 53, a ground on any one phase causing the movable spherical vane to move away from the fixed vane connected to the grounded phase.

A luminous type of detector has been developed for potentials

[1] "Induction-type Indicating Instruments," PAUL MACGAHAN, *Transactions*, A. I. E. E., vol. 31, p. 1565 (1912).

from 15,000 to 110,000 volts.[1] It employs the principle of vacuum tube lighting. Two electrodes are suitably spaced in a round glass bulb similar to an incandescent lamp bulb and containing a rarefied gas. One terminal is connected to the line through suitable insulators and the other is connected to ground. With normal potential on the line, the charging current of the capacitances of the insulators causes a luminous discharge to take place and the "glower" is lighted up. The absence of glowing indicates a ground on the line to which the glower is connected.

101. Calibration of Voltmeters.—The dynamometer type of voltmeter gives the same indication on continuous current as on alternating current and may, therefore, be calibrated with continuous currents, direct and reversed readings being taken. The inductance in instruments of commercial ranges is so small that the readings are independent of the frequency within the range of commercial frequencies.

The soft-iron-vane type of voltmeter should theoretically be used only on alternating current because hysteresis occurs to some degree in the vane. Practically, however, the hysteresis is so small that there is very little difference between the respective indications with increasing and decreasing potential. Provided with a steady source of e.m.f., under suitable control, these instruments may be calibrated with continuous current by taking the average of the "up" and "down" potential readings corresponding to the various points. Care should be taken that the potential is increased or decreased only to the desired value and not beyond it. Theoretically, instruments of the soft-iron type are not independent of frequency or wave form; practically, however, the variation is not measurable within the range of commercial frequencies.

Hot-wire voltmeters are theoretically and practically independent of changes in frequency and wave form, and can, therefore, be calibrated with continuous currents. They are especially suitable for high-frequency measurements and conditions where the inductance of electromagnetic instruments is objectionable.

Induction-type voltmeters are affected by changes of frequency. They must, therefore, be calibrated on alternating current of the frequency for which they have been adjusted, by comparison with some secondary standard which can in turn be calibrated with continuous current as described under precision measure-

[1] *Bulletin*, No. 46,022, General Electric Co., February, 1916.

ments. While variations in wave form theoretically affect this instrument, the errors which result from ordinary variations in commercial wave forms can be treated as negligible.

Electrostatic voltmeters are independent of changes both in frequency and in wave form, and may, therefore, be calibrated with continuous currents where the range will permit. High-range instruments must be calibrated with alternating potential, as indicated in par. 107.

102. Effect of Stray Fields.—The effect of external alternating fields is very marked in some types of instruments. The error will vary with the deflection and with the direction of the superposed field. In the case of unshielded, single-coil dynamometer instruments, the error caused by a magnetic stray field of 5 lines per square centimeter may vary from 25 per cent. at quarter scale to 5 or 10 per cent. at three-quarters scale; with a field of 10 lines per square centimeter these figures may become 75 per cent. and 25 per cent. respectively. Recent forms of these instruments are, however, provided with laminated sheet-iron magnetic shields and fields as high as 20 lines per square centimeter produce an error of only 1 or 2 per cent.

Soft-iron-vane instruments are much less affected, a stray field of 10 lines per square centimeter causing about 10 per cent. error. Astatic dynamometer instruments are sometimes slightly affected by strong fields, arising from the fact that the instrument is not truly astatic or that the stray field is not uniform throughout the space occupied by the moving system. Hot-wire and electrostatic instruments are not affected by stray magnetic fields.

Measurement of Small Potentials

103. General.—The single-coil dynamometer-type voltmeter is practically the only one available in portable form for the measurement of potentials of the order of 25 volts and less. Even with a 7.5-volt instrument, the lowest potential that can be measured is about 3 volts because the deflection varies with the square of the potential. However, if the fixed coil circuit is separately excited so that the deflections become proportional to the first power of the potential, the sensitivity at the lower end of the scale is greatly increased. The portable dynamometer galvanometer made by R. W. Paul is arranged for use in this manner, separate binding posts being provided for the two circuits. Full scale deflections can be obtained with about 1 volt.

For measurements of the order of 0.25 volt and less, a reflecting dynamometer-type galvanometer or a potentiometer scheme is usually most convenient and accurate. Alternating-current galvanometers employing thermal principles are also applicable.

A rectifying commutator may be used with a D'Arsonval galvanometer but where the potential is very small, contact resistance and parasitic e.m.fs. due to the rubbing contacts are likely to be troublesome (par. 38). Furthermore, this method measures the average value and, therefore, to get the mean effective value the form factor must be known (see par. 88). This method must, therefore, be employed with caution.

104. Measurements with Dynamometer Instruments.—When making measurements of low voltages with dynamometer instruments, the resistance and the inductance of both the fixed-coil circuit and the movable-coil circuit should be known and as much non-inductive resistance should be used in both circuits as feasible without reducing the sensitivity below the desired value.

When used with both circuits in series as a straight voltmeter, the instrument may be calibrated directly on continuous current, but if the impedance on alternating current is appreciably greater than the resistance ($\sqrt{R^2 + (L\omega)^2}$ greater than R), the calibration must be corrected accordingly. If the fixed-coil circuit is separately excited, the exciting current must have the same frequency as, and be in phase with, the current in the movable-coil circuit. This requirement is most conveniently met by exciting from the same source as that of the potential to be measured through some device which permits shifting the phase of the exciting current (see par. 240). With the exciting current held constant at some convenient value, the phase is shifted until a maximum deflection is obtained and then the two currents are in phase.

The calibration as a voltmeter can be conveniently made by connecting the movable-coil across a variable non-inductive resistance in series with the fixed-coil circuit. With the current in the fixed coil held at a value which will give the desired sensitivity a series of deflections corresponding to known potentials is obtained. Care should be taken not to disturb the phase adjustment.

Obviously, if the voltage being measured is the potential drop in part of an inductive circuit, any component of the potential can be measured by properly adjusting the phase of the exciting current. For instance, the power component can be measured

by first connecting the movable-coil circuit across a non-inductive resistance in the circuit and adjusting the phase of the exciting current to maximum deflection. Transferring the movable-coil circuit back to the potential to be measured, the deflection will be proportional to the power or alternating-current resistance component. Similarly, the inductive component can be measured by first adjusting for maximum deflection with a condenser in series with the movable coil while connected across a non-inductive resistance.

It should be noted that when measuring components, the galvanometer circuits should preferably be practically non-inductive, otherwise the movable-coil currents will not be in phase with the potential being measured and a correction would have to be made for the inductance (or compensated, see par. 197).

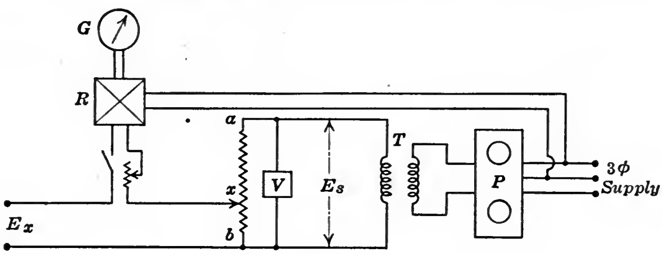


FIG. 54.

The possible effect of variation in temperature of the instrument circuit should also be kept in mind. When the series resistance, which is usually low temperature-coefficient material, is decreased in order to gain sensitivity, the proportion of copper resistance may become sufficiently great to make the temperature coefficient of the circuit appreciable. This is particularly important where the coils are close together as in a sensitive reflecting instrument and where heating of the fixed coil due to high excitation may heat up the moving coil.

105. Potentiometer Method.—The principle of the potentiometer method is indicated in Fig. 54. A potential, E_s, sufficiently high to be conveniently and accurately measured is applied to a non-inductive resistance, *ab*. The potential to be measured, E_x, is connected in opposition across a portion of this resistance such that the drop is just equal to E_x. Then

$$E_x = \frac{bx}{ab} E_s$$

The potential E_s must be from the same source as E_x and it must also be in phase with E_x which necessitates some means of shifting the phase of E_s such as a phase shifter, P. The transformer T is necessary for insulating the two circuits even if not required to get a convenient value for E_s.

If the detector is a dynamometer instrument or a synchronous reversing key, phase balance as well as voltage balance can be readily made. The conditions existing are indicated vectorially in Fig. 55 where E_s represents the standard voltage and E_x the unknown. With angle ϕ between the two voltages, there will be an unbalanced voltage, ab. If the detector is a dynamometer instrument and its excitation is in phase with E_s, zero indication and, therefore, a balance will be obtained by shifting x (Fig.

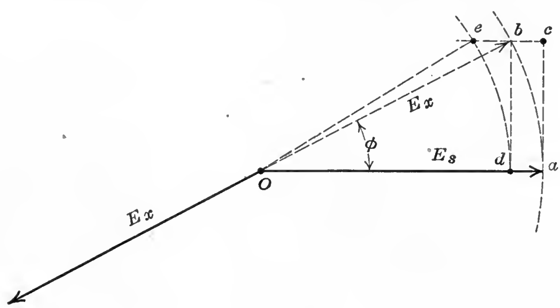


FIG. 55.

54) to a point corresponding to d (Fig. 55). The two voltages are not balanced, however, by the amount ad. The excitation is now shifted through 90° and the indication becomes proportional to the component ac. The phase of E_s is shifted until the galvanometer shows zero deflection (to position 0_e). The first operation is then repeated because, in shifting E_s, the first balance was destroyed by the amount, eb. Complete balance both as to phase and potential has now been obtained. A synchronous key with a continuous-current galvanometer is used in a similar manner, first balancing for voltage, then for phase angle and again for voltage (see par. 111c).

MEASUREMENT OF HIGH POTENTIALS

106. General.—Theoretically, any high voltage can be measured with either of the types of electromagnetic voltmeters that

have been described, if sufficient resistance is connected in series with the instrument as a multiplier. Practically, however, this method is applicable only up to a few thousand volts and then only for testing purposes. The difficulties are the large power consumption, insulation of the resistance and the fact that the inductance and capacity may be sufficient to make the impedance on alternating current appreciably greater than the resistance. Also on very high potentials, there may be distributed electrostatic capacitance to earth along the resistance units and the charging current to this capacitance will introduce an error.

Electrostatic voltmeters are made for potentials as high as 300,000 volts. They can probably be made for any voltage but the size becomes excessive and the insulation problem becomes serious. Adding condensers as a multiplier may introduce an error due to the capacitance to ground.

Fortunately, in ordinary measurements in connection with generation and transmission, it is not often necessary to make direct measurements of potentials higher than about 13,000 volts. In this country such measurements are almost universally made with potential transformers, a type of transformer similar to the ordinary power transformer but relatively small because the only load is instruments especially designed to give an actual ratio very close to the theoretical ratio.

107. Methods.—The measurement of potentials higher than 13,000 volts is usually required only in connection with high-voltage tests, that is, tests of insulating materials, insulators and so forth. The various methods of making such measurements are as follows:

(*a*) *Ratio of the testing or power transformer,* in connection with an ordinary voltmeter on the low-tension side. This method requires an accurate knowledge of the transformer ratio under various conditions of load and potential, information which is often difficult to obtain.

(*b*) *Step-down instrument transformer* with an ordinary voltmeter. This method requires an accurate knowledge of the transformer ratio at various potentials with the voltmeter as the secondary load. The method is simple, convenient and accurate, but the power consumption and the cost of the transformer become prohibitive at very high potentials.

(*c*) *Electrostatic Voltmeter.*—Commercial instruments are available up to about 200,000 volts. They require no appreciable

power and are quite satisfactory. The principal objections are the high cost of large sizes and, in some forms, the lack of dead-beat qualities.

(*d*) *Test Coil.*[1]—Where the source of the high potential to be measured is a testing transformer, an ordinary low-reading voltmeter can be connected to a few turns of the high-tension winding brought out to separate terminals. These turns should be at the grounded end of the winding. The ratio, under all conditions, will be that of these turns to the total turns in the high-tension winding if the transformer has been properly designed. This method is accurate and convenient.

(*e*) *Spark Gaps.*—The sparking distance between two terminals in air is a standard method of measuring high potentials. The distinguishing feature of this method is that the maximum value of the applied potential determines the break down of the air and not the effective value which is the value indicated by the other methods. The spark gap really measures, therefore, maximum values but is usually calibrated in corresponding sine-wave effective values.

108. Spark Gaps.—The needle-point spark has for many years been the standard method of measuring high voltages, but it is unsatisfactory for very high potentials because of variations due to atmospheric pressure, humidity, proximity of surrounding objects and sharpness of the needle points. It has been found that a gap with spheres which have been carefully machined and polished gives very reliable and consistent results[2] due probably to the fact that the gap breaks down before corona forms and perhaps also to the smaller dielectric spark lag.[3] Furthermore, such a gap is more convenient to use because it does not require attention after each arc-over whereas the needles in a needle gap have to be renewed after each discharge. The 1916 A. I. E. E. Standardization Rules recommend the use of the needle gap from 10,000 to 50,000 volts and the sphere gap above 50,000 volts but preferably above 30,000 volts.

[1] "The Voltmeter Coil in Testing Transformers," A. B. Hendricks, Jr., *Proceedings*, A. I. E. E., February, 1916, p. 138.

"Notes on the Measurement of High Voltage," W. R. Work, *Proceedings*, A. I. E. E., February, 1916, p. 203.

[2] "The Sphere Spark Gap," S. W. Farnsworth and C. L. Fortescue, *Transactions*, A. I. E. E., vol. 32, p. 301 (1913).

[3] "Effect of Dielectric Spark Lag on Spark Gaps," J. P. Minton, *General Electric Review*, vol. 16, p. 514 (1913).

Spark gaps are ordinarily used only as secondary standards to calibrate indicating instruments which are used in the actual high-voltage measurements. The following table gives the calibration data for needle and sphere gaps as prescribed by the A. I. E. E. Standardization Rules. These rules should be consulted for further information in regard to the construction and use of spark gaps, and for the correction to be applied where the barometric pressure departs materially from the standard of 760

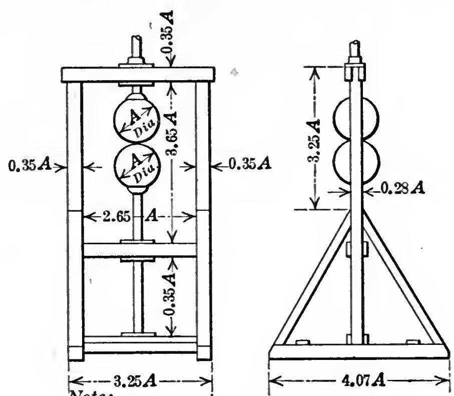


FIG. 56.

mm.[1] Fig. 56 shows in outline an acceptable form of construction of sphere gap.

NEEDLE-GAP SPARK-OVER VOLTAGES

(A. I. E. E. Standardization Rules, October, 1916)
25°C. temperature and 760 mm. barometer
Sinusoidal wave, No. 00 sewing needles

R.m.s. kilovolts	Millimeters	R.m.s. kilovolts	Millimeters
10	11.9	35	51
15	18.4	40	62
20	25.4	45	75
25	33.0	50	90
30	41.0	....	....

[1] See also "A Source of Error When Using the Sphere Gap," R. H. MARVIN, *Electrical World,* March 18, 1916, for effect of condition of surface of spheres on the arc-over potential.

Sphere-gap Spark-over Voltages

(A. I. E. E. Standardization Rules, October, 1916)
Sinusoidal wave, 25°C. temperature and 760 mm. barometer

R.m.s., kilovolts	Sparking distance in millimeters							
	62.5-mm. spheres		125-mm. spheres		250-mm. spheres		500-mm. spheres	
	One sphere grounded	Both spheres insulated	One sphere grounded	Both spheres insulated	One sphere grounded	Both spheres insulated	One sphere grounded	Both spheres insulated
10	4.2	4.2	.....	.....	.....	.....	.....	.....
20	8.6	8.6	.....	.....	.....	.....	.....	.....
30	14.1	14.1	14.1	14.1	.....	.....	.....	.....
40	19.2	19.2	19.1	19.1	.....	.....	.....	.....
50	25.5	25.0	24.4	24.4	.....	.....	.....	.....
60	34.5	32.0	30.0	30.0	29	29	.....	.....
70	46.0	39.5	36.0	36.0	35	35	.....	.....
80	62.0	49.0	42.0	42.0	41	41	41	41
90	.....	60.5	49.0	49.0	46	45	46	45
100	.....	.....	56.0	55.0	52	51	52	51
120	.....	.....	79.7	71.0	64	63	63	62
140	.....	.....	108.0	88.0	78	77	74	73
160	.....	.....	150.0	110.0	92	90	85	83
180	.....	.....	.....	138.0	109	106	97	95
200	.....	.....	.....	.....	128	123	108	106
220	.....	.....	.....	.....	150	141	120	117
240	.....	.....	.....	.....	177	160	133	130
260	.....	.....	.....	.....	210	180	148	144
280	.....	.....	.....	.....	250	203	163	158
300	.....	.....	.....	.....	.....	231	177	171
320	.....	.....	.....	.....	.....	265	194	187
340	.....	.....	.....	.....	.....	.....	214	204
360	.....	.....	.....	.....	.....	.....	234	221
380	.....	.....	.....	.....	.....	.....	255	239
400	.....	.....	.....	.....	.....	.....	276	257

Note.—The sphere gap is more sensitive than the needle gap to momentary rises of voltage and the voltage required to spark over the gap should be obtained by slowly closing the gap under constant voltage, or by slowly raising the voltage with a fixed setting of the gap. Open arcs should not be permitted in proximity to the gap during its operation, as they may affect its calibration.

109. Corona Standard for High-voltage Measurements.—When a potential is applied to two conductors separated by air and gradually raised, a discharge suddenly begins at a definite, critical voltage. This discharge, called corona, always begins at the same voltage if the conditions remain constant and this fact has been utilized by Prof. Whitehead[1] in the development of a standard for direct measurement of high alternating potentials.

The point of initial corona formation varies with the potential gradient at the surface of the conductors, the temperature and the pressure of the air. In the Whitehead instrument one conductor or electrode is a straight wire accurately centered in a metal cylinder which forms the other electrode. This cylinder is then enclosed in an outer air-tight cylinder provided with bushings for the leading-in wires to the two electrodes. Provision is made for varying the pressure and the accurate determination of the pressure, temperature and point of corona formation.

The beginning of corona can be detected with an electroscope, a galvanometer or a telephone. The first two methods utilize the fact that at the instant of corona formation, the air adjacent to the wire electrode becomes ionized and, therefore, conducting. The telephone method is the simplest, a transmitter being inserted in the outer cylinder and connected to a receiver which enables the observer readily to detect the peculiar hissing sound that accompanies the beginning of corona.

The law of this scheme is

$$V_c = 32\left\{\left(\frac{3.92p}{273+t}\right) + 0.296\sqrt{\frac{\left(\frac{3.92p}{273+t}\right)}{r}}\right\} r \log_\epsilon \frac{R}{r}$$

where V_c = voltage in kilovolts at the instant of corona formation, p = air pressure in centimeters of mercury, t = air temperature in degrees C., r = radius of wire electrode in centimeters and R = radius of inner surface of cylindrical electrode in centimeters.

110. High-voltage Potentiometer.—Prof. Ryan has described[2] a very simple and easily devised apparatus for investigating the distribution of potential over, for example, the surface of an insulator bushing, a pin-type insulator or along a string of suspension insulator units.

[1] "Corona as a Standard for Measuring High Voltage," J. B. WHITEHEAD, *Electrical World*, June 17, 1916, p. 1405.

[2] "A High-voltage Potentiometer," HARRIS J. RYAN, *Proceedings*, A. I. E. E., August, 1916, p. 1187.

The scheme is to connect a sectionalized resistor of very high resistance, in the form of a stream of water, across the circuit and measure the potential at any point on the insulator by locating the point on the stream of water which is at the same potential.

A plain rubber garden hose is cut into equal, short sections which are connected together again with metallic connectors or "hose-menders." One end is connected to the ordinary water supply system of the building and to the grounded end of the transformer. The other end of the hose terminates in a sprinkler head supported several feet from the floor and connected to the "high" end of the transformer. Thus a stream of water flowing through the hose is effectually broken up by the sprinkler head in small drops and thereby insulated from ground.

The detector is a sharply pointed electrode at one end of a piece of wire, the other end of which is attached to the point on the insulator the potential of which is to be determined. The electrode is then held close to each hose connector in turn until one is located where no spark passes. If a minimum spark passes to two adjacent connectors, the balancing point is obviously between them.

Prof. Ryan used a $\frac{3}{4}$-in. hose, 75 ft. long, divided into 50 equal sections and formed into a cylindrical helix supported by strain insulators. With a slow stream of water a drop of about 1,000 volts per foot (root-mean-square value) was obtained with a current not exceeding 50 milliamperes. Potentials were measured to 2 or 3 per cent. of the true value.

111. Potential Transformers.—As previously stated, commercial measurements of high voltages are practically always made with potential transformers. When it is desired to make the measurement with an accuracy greater than 2 or 3 per cent., as in power and energy measurements, it is necessary to determine the ratio by direct measurement because the true ratio may deviate from the nominal ratio by at least that amount. Also, for accurate power and energy measurements the phase angle must be measured.

The following are the more commonly employed methods that have been developed for the measurement of ratio and phase angle of potential transformers.

(a) *Ratio by direct measurement* of the high and low voltages preferably with two similar voltmeters. Fig. 57 shows diagrammatically the connections for this method. The voltmeters,

V_1 and V_2, are similar, and the resistance, R, is adjusted until the two deflections are about alike. The two voltmeters are then connected in parallel on the secondary of the transformer and the indication of V_2, corresponding to the previous indication of V_1, is noted. The ratio is equal to

$$\left\{\frac{R + r_v}{r_v}\right\} \frac{X_1}{X_2}$$

wherein R = resistance in series with V_2; r_v = resistance of voltmeter V_2; X_1 = first reading of V_2; X_2 = second reading of V_2.

(*b*) *Ratio by Opposition Method.*—The low-tension voltage is reversed and connected in opposition to a part of the high-tension voltage. The scheme is shown in Fig. 58. The resistance, r, is adjusted until the detector, T, indicates zero. If the phase

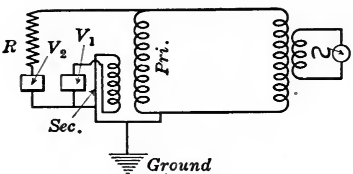


FIG. 57.

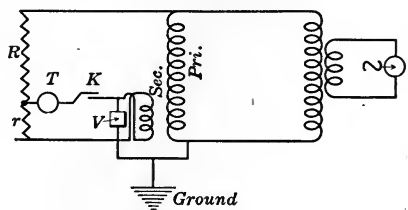


FIG. 58.

angle is negligibly small as is usually the case, the ratio is equal to

$$\frac{R + r}{r}.$$

The detector may be a telephone receiver, a dynamometer instrument or a synchronous reversing key connected to a continuous-current galvanometer. A telephone receiver is not sensitive at commercial frequencies, and if harmonics are present, the precise balance point for the fundamental frequency is difficult to locate. When using a dynamometer, the fixed coils are connected in series with R and its resistance added thereto. It should be connected in series with R and not r because a change in resistance due to heating will introduce less error. The moving coils are connected in place of T. When a reversing key is used, it is connected directly in place of T.

(*c*) *Phase Angle and Ratio by Opposition Method.*—The last method does not give the exact ratio because it assumes that the

high-tension and low-tension voltages are displaced by exactly 180°. Referring to Fig. 59, E_p represents the primary voltage and E_s the secondary voltage. The point of true balance on the resistance is at m and if there were no phase angle, θ, this point would be indicated by zero current in the detector. When the angle θ differs from zero, a telephone detector would not locate a point of absolute silence but one of minimum sound, that is, a (Oa being the shortest distance between point O and vector E_p). A dynamometer detector with the fixed coils in series with R and the moving coil between O and the resistance would respond only to that component of the potential across the moving coil which is in phase with E_p and zero indication would be shown at the point a as before. Thus an error in the ratio would be introduced corresponding to the difference between a and m in the resistance r. By using a detector which will respond to currents 90° from E_p, it is evident that the com-

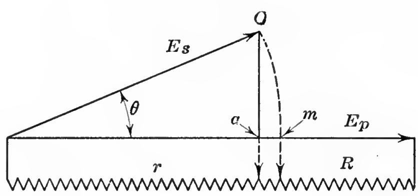


FIG. 59.

ponent Oa can be measured and a correction made to give the true ratio. Also, the value of the angle can be calculated.

If the detector is a dynamometer, this 90° component is measured by shifting the excitation 90°. This can be done by connecting a condenser in series with the fixed coils and across E_p or by connecting the fixed coils to a voltage 90° from E_p obtained with a phase-shifting device or from a two-phase circuit. The deflection is calibrated with the excitation in phase with E_p and noting the deflection produced by a given change in r. The voltage corresponding to the latter can be calculated if E_p or E_s is known.

If the detector is a synchronous reversing key, the contacts of the reversing key are shifted to the angular position where the reversals of the current to the galvanometer occur simultaneously with the zero points of the E_p potential wave, and zero deflection will be obtained at the position corresponding to a as before.

By shifting the contacts through 90 electrical degrees, the indication becomes proportional to *Oa*. Calibration is made in the resistance position by noting the deflection corresponding to a given change in *r* as with the dynamometer.

In a modification of this method,[1] the primary of an adjustable mutual inductance, *P*, Fig. 60, is connected in series with the resistance, *R*, and the secondary, *S*, is connected in series with the detector circuit. Thus an e.m.f. is introduced into the detector circuit which is 90° from the primary voltage. With the contacts of the reversing key in the "90°" position, and the

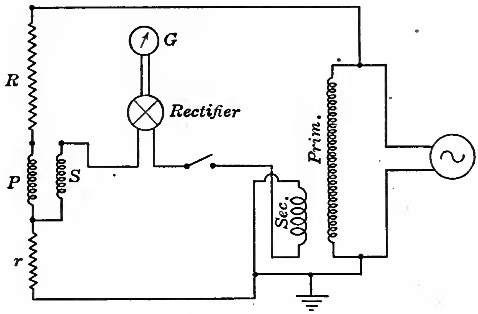


Fig. 60.

mutual inductance secondary e.m.f. in opposition, the indication is reduced to zero by manipulation of the secondary coil with respect to the primary. The ratio is then

$$X = \frac{R + r}{r}\sqrt{1 + \left\{\frac{2\pi f M}{r}\right\}^2}$$

and the phase angle is

$$\phi = \tan^{-1}\frac{2\pi f M}{r} \qquad \text{(degrees)}$$

where X = ratio, R and r are in ohms, ϕ = phase angle, f = frequency in cycles per second, M = mutual inductance in henrys. When the phase angle is not over 2°, the correction factor in the ratio formula (expression under the radical) may be neglected.

[1] "Recent Progress in Exact Alternating-Current Measurements," C. H. Sharp and W. W. Crawford, *Transactions*, A. I. E. E., vol. 29, p. 1517 (1910). The mutual inductance is also proposed by P. G. Agnew and E. B. Silsbee, "The Testing of Instrument Transformers," *Transactions*, A. I. E. E., vol. 31, p. 1635 (1912). In their method the detector is a vibration galvanometer.

The phase angle may be measured with two dynamometers. The fixed coils are connected in series and to a source whose phase can be adjusted. The movable coil of one is connected to a portion of a non-inductive resistance across the high-tension side of the transformer and the other movable coil is connected to the low-tension side. If the phase of the exciting current is adjusted so that the high-tension dynamometer shows no deflection, the low-tension dynamometer will deflect in proportion to the component of the low-tension voltage which is 90° from the high-tension voltage and the indication will, therefore, be proportional to the sine of the phase angle between the two voltages.[1]

(*d*) *Phase Angle and Ratio, Wattmeter and Watt-hour Meter Methods.*—The preceding methods are essentially laboratory methods, those under (*c*) giving results of high precision. They are not, therefore, readily applicable to transformers in service. As it is not always possible to remove transformers from service and test them in the laboratory, the two following methods have been devised to permit testing without removing the transformers from the circuit. These methods give results which, though not rigidly exact, are sufficiently accurate for all practical purposes. One method employs wattmeters and the other watt-hour meters. Both can be used with fluctuating voltage.

The wattmeter method was first proposed by Brooks[2] and involves the use of the ordinary 5-amp. 150-volt portable wattmeter and a standardized transformer having the same nominal value as the one being tested. As indicated diagrammatically in Fig. 61, the low-voltage windings are connected in opposition through the potential circuit of the wattmeter and the current coil is excited at about 5 amp. from the same circuit using an auxiliary transformer if necessary. The wattmeter is thus used as a separately excited dynamometer. The indicated watts are very nearly equal to the product of the current and the difference between the low-tension voltages. The ratio of the test transformer in terms of the standardized transformer can then be calculated. To measure the phase angle, the ratios are first

[1] "Electrical Measurements on Circuits Requiring Current and Potential Transformers," L. T. Robinson, *Transactions*, A. I. E. E., vol. 28, p. 1005 (1909).

[2] "Testing Potential Transformers," H. B. Brooks, *Bulletin*, Bureau of Standards, vol. 10, p. 419 (1914). "Testing Shunt Instrument Transformers," *Electrical World*, Nov. 1, 1913, p. 898.

made equal by loading the higher-voltage transformer. The excitation is then shifted, preferably through 90°, and the wattmeter indication becomes practically proportional to the sine of the angle between the two low-tension potentials from which the

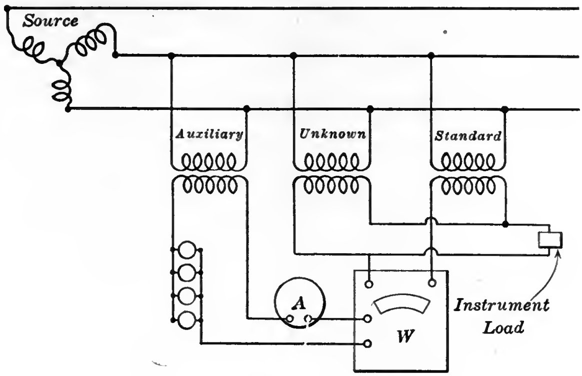


FIG. 61.

angle of the test transformer can be determined. If the excitation is shifted through 60°, for example, instead of 90° (which would be the case when shifting to another phase on a three-phase system) the indication can be corrected to that correspond-

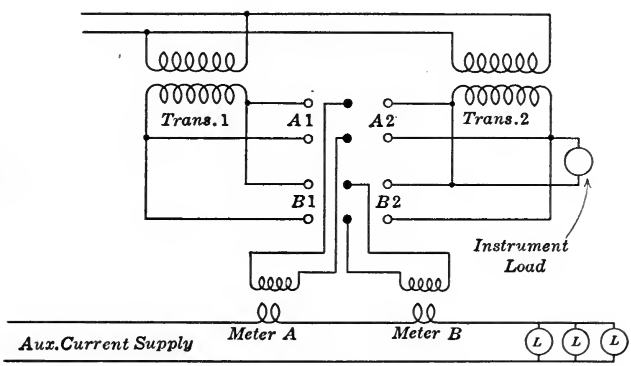


FIG. 62.

ing to 90°. To determine which transformer has the greater angle, use can be made of the fact that increasing a non-inductive load on a voltage transformer tends to increase the lag of the low-tension voltage.

The watt-hour meter method proposed by Agnew[1] is similar to the wattmeter method. Two watt-hour meters (preferably rotating standards) are used as indicated diagrammatically in Fig. 62. The current coils are excited from a common circuit and the voltage coils are connected to the two transformers, the characteristics of one being known. If the rates of the two meters are exactly equal, the difference in ratio will be proportional to the difference in revolutions in a given time. If the meter rates are not equal, any difference may be eliminated by interchanging the meters. The phase angle is obtained by making similar observations with a known large angle between the auxiliary current and the supply voltage. The working formulas are:

For ratio, $$\frac{R_2 - R_1}{R_1} = \frac{1}{2}\left\{\frac{a_1 - a_2}{a_2} - \frac{b_1 - b_2}{b_2}\right\}.$$

For phase angle,

$$\phi_2 = \phi_1 + \frac{3438}{\tan\,\theta}\left[\frac{a_1 - a_2}{2a_1} + \frac{b_1 - b_2}{2b_1} - \frac{R_2 - R_1}{R_1}\right] \text{ (degrees)}$$

where $R_2 - R_1$ = difference between the ratios; R_1 = ratio of standard transformer(1); a_1 and a_2 = revolutions of meter A when connected to transformers 1 and 2 respectively; b_1 and b_2 = same for meter B; ϕ_1 and ϕ_2 = phase angles of transformers 1 and 2 respectively; $\cos\,\theta$ = power-factor.

112. Polarity of Potential Transformers.—When several instrument transformers are to be connected to one instrument or apparatus it is frequently necessary, or at least desirable, to know which terminals of the two windings have the same polarity at the same instant. The most convenient way of determining this in the case of potential transformers is to connect the two windings in series, the high-tension winding being excited at any convenient voltage such as 110 or 220 volts. The potential across the two free ends is measured and if it is greater than the exciting voltage the low tension winding voltage is being added to the line voltage and the two ends connected together are of opposite polarity. Conversely, if the total potential is less than the line voltage, the windings are "bucking" and the two ends connected together are of the same polarity.

[1] "A Method of Testing Instrument Transformers," P. G. AGNEW, *Electrical World*, Nov. 21, 1914, p. 1004; "A Watt-hour Meter Method of Testing Instrument Transformers," *Bulletin*, Bureau of Standards, vol. 11, p. 347 (1914–1915).

Crest Voltage Measurements

113. General.—In the majority of alternating-potential measurements, the mean effective value is the value which is measured. In certain cases, however, other values are required. For example, in the testing of insulation materials for dielectric strength or when subjecting such materials in a machine to a high voltage test, the maximum or crest voltage is the important value because it, rather than the mean effective, determines the breakdown voltage. While high-voltage tests are stated in terms of the root-mean-square value, a sine-wave form is always inferred. If the test wave differs from a sine-wave curve, either the maximum value, the crest factor (ratio of maximum value to root-mean-square value) or the sine-wave root-mean-square value corresponding to the actual maximum value is stated.

The crest voltage may, of course, be determined from the plotted wave obtained with a wave meter or similar means, or from an oscillogram. These methods are not only laborious but are not applicable where the wave form changes with the test conditions, the character and amount of the load, and so forth. These limitations exist just where the crest voltage is most important, as, for example, in testing lead-covered cable and similar material which has electrostatic capacity.

The spark gap may be used as a measure of crest voltage but it is not always applicable principally because surges are apt to be set up when the gap breaks down which might injure the apparatus or material being tested.

114. Instrumental Methods.—Several methods have been developed for the direct measurement of crest voltage. About the simplest scheme is that proposed by Sharp and Doyle[1] in which an electric valve is used in series with an electrostatic voltmeter connected across the voltage to be measured. The kenotron acts as a valve through which charging current to the voltmeter can flow in only one direction, consequently the indications of the voltmeter are strictly proportional to the crest voltage. The connections are indicated in Fig. 63 where the upper diagram shows the arrangement when the cathode of the kenotron is heated by a battery current and the lower diagram shows the arrangement for heating with a step-down transformer. The

[1] "Crest Voltmeters," C. H. Sharp and E. D. Doyle, *Proceedings*, A. I. E. E., February, 1916, p. 129.

root-mean-square value is measured by simply closing the short-circuit switch, *s*, hence the crest factor is very easily determined. The condenser, *C*, is connected to the voltmeter, *V*, in order to

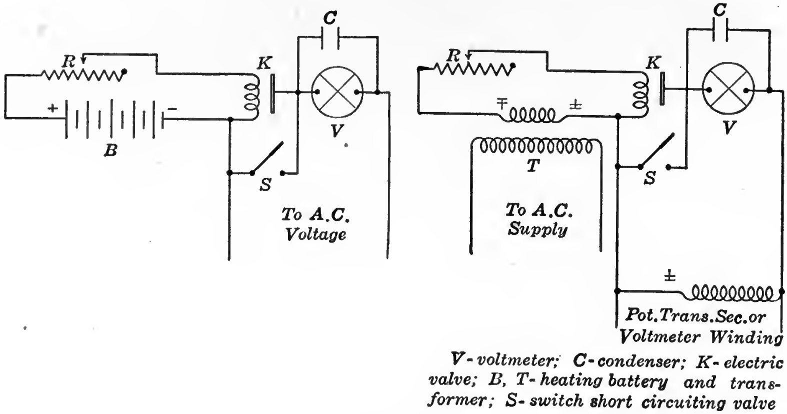


FIG. 63.

increase the capacitance and eliminate the effect of leakage. The apparatus would ordinarily be used on a voltmeter coil in the high-tension winding of the high-voltage transformer but may be

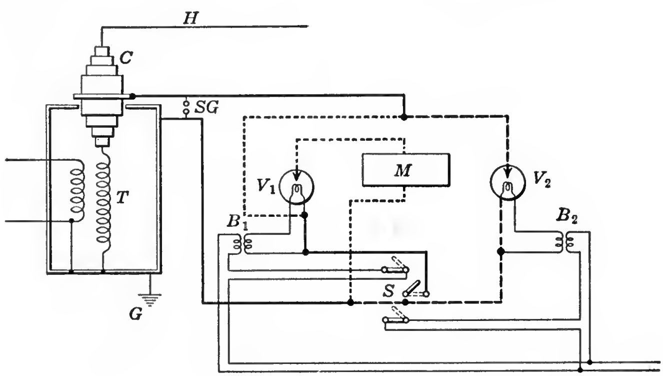


FIG. 64.

connected directly across the high-tension circuit up to limiting voltage fixed by the kenotron and the voltmeter.

The Chubb crest voltmeter[1] also employs the hot-cathode-valve

[1] "The Crest Voltmeter," L. W. CHUBB, *Proceedings*, A. I. E. E., February, 1916, p. 121.

principle, the indicator being a permanent-magnet-type instrument which measures the charging current to a condenser every other half cycle. The arrangement of the circuits is shown in Fig. 64, where T is the high-voltage transformer, V_1 and V_2 two hot cathode valves or kenotrons, M is the indicating instrument and C the condenser which in this case is a condenser-type terminal of the transformer. The cathodes of the valves are heated by the small bell-ringing-type transformers, B_1 and B_2. The operation is as follows: The condenser charging current is zero at the two maxima of the voltage wave and the average value of the current flowing between two successive maxima is a measure of the difference between the two maximum voltages and, therefore, of the crest voltage. In the diagram it will be noted that the current in one direction passes through the instrument valve, V_1, and the instrument, M (heavy dotted line), while current in the opposite direction passes through V_2 without passing through the instrument (heavy broken line). The instrument indications, being proportional to the average current, are a measure of the crest voltage. Calibration is obtained with a sphere spark gap. The indications vary directly with the frequency and are not correct when there is more than one maximum and one minimum value per cycle.

Sharp and Farmer[1] proposed the use of a synchronously driven instantaneous contact apparatus which momentarily connected an electrostatic voltmeter (in parallel with a condenser to eliminate the effect of leakage) to the circuit, the contact being shifted until the maximum indication which corresponds to the crest voltage is reached.

The Simplex Wire and Cable Co.'s peak-reading voltmeter manufactured by the Leeds and Northrup Co. is practically an oscillograph element. The amplitude of the deflections, which is proportional to the crest voltage, is measured by the width of the band of light produced by the reflection of an incandescent lamp filament from the mirror mounted on the bifilar suspension to the scale.

Average Voltage Measurements

115. General.—It is sometimes necessary to know the average value of an alternating e.m.f. as well as the mean effective and

[1] "Measurements of Maximum Values in High-Voltage Testing, C. H. Sharp and F. M. Farmer, *Transactions*, A. I. E. E., 1912, vol. 31, p. 1617.

maximum values. For example, in making measurements of iron losses the form factor of the applied e.m.f. wave must be known. Since the form factor is the ratio of the mean effective value to the average value, it is necessary to determine the average value.

The average value may, of course, be determined if the wave shape is known by averaging the instantaneous values of one-half of the cycle or by integrating a half cycle plotted on rectangular coördinates. The simplest method of directly measuring the average value is to use a rectifying commutator (see par. 38) in conjunction with a continuous-current voltmeter. The maximum value which is obtained by shifting the commutator brushes is the average value.

The exact point of maximum deflection may not be sharply defined, especially with a flat-top wave, in which case greater accuracy may be attained by first adjusting for zero deflection with a condenser between one slip ring and the line. The condenser is then short-circuited and the resulting indication of the voltmeter is the average voltage.

CHAPTER VI

ALTERNATING-CURRENT MEASUREMENTS

116. General.—Alternating current, like alternating potential, cannot be measured directly in terms of the unit of current. Both primary and secondary measurements are made by employing the principle discussed in the chapter on alternating e.m.fs., primary measurements being made by substitution methods and secondary measurements with secondary standards which have been calibrated on continuous current.

The mean effective value is the value of an alternating current which is practically always measured. Average and maximum values of current are rarely required in commercial measurements.

Primary Measurements

117. Transfer Method.—This method employs an instrument which indicates the same on alternating as on continuous current. With an electrodynamometer, the measurement is made in the manner described for potential measurements. The deflection corresponding to the alternating current to be measured is noted and then, with continuous current substituted for the alternating, the current is adjusted until the same deflection is obtained. The continuous current is then measured in terms of the ampere in the usual manner.

Electrodynamometers for precision current measurements are usually of the suspension, reflecting type which are read with a telescope and scale or a lamp and scale. They are made astatic by having two sets of fixed and moving coils, one above the other, with the two moving coils on a common suspension. In instruments of large capacity, the fixed coils are wound with conductors made up of very fine wires which are both stranded and braided in order to eliminate eddy-current errors and insure the same distribution of the current for both alternating and continuous current. The moving coils are wound with relatively fine wire and are connected across resistors or "shunts" which are

in series with the fixed coils. In careful measurements switching arrangements should be provided so that the change from alternating to continuous current and *vice versa* can be made so quickly that the deflection does not drag back very much, thus avoiding errors due to changes in tension, temperature and so forth. Direct and reversed readings should be taken with both currents, for, even in astatically arranged instruments, perfect astaticism is rarely attained.

For very large currents, tubular conductors through which water can be circulated are used for the "fixed coil." In a dynamometer developed by Agnew[1] for 5,000 amp., two straight tubes

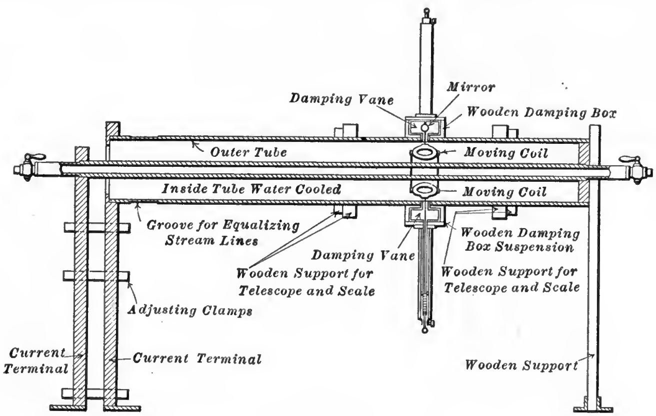


FIG. 65.

are used, one inside of and exactly concentric with the other. The tubes are connected at one end so that the current flows in one direction in one tube and in the opposite direction in the other tube. The moving coil is suspended in the annular space between the two tubes. The general arrangement is shown in Fig. 65.

While the inductance of the shunt should be small, it does not introduce an error because, on alternating current, the deflection is proportional to the product of root-mean-square value of the main current through the fixed coils and the root-mean-square value of the resistance component only, of the shunted current through the movable-coil circuit. The deflection is the same,

[1] "A Tubular Electrodynamometer for Heavy Currents," P. G. AGNEW, *Transactions*, A. I. E. E., vol. 31, p. 1483, 1912. *Bulletin*, Bureau of Standards, vol. 8, p. 651 (1912–1913).

therefore, as that produced by an equivalent continuous current. The inductance of the movable-coil circuit, however, does introduce an error because it throws the current out of phase with the drop across the shunt. Where there is sufficient sensitivity, this error can be made negligible by introducing non-inductive resistance in the circuit. Otherwise, it can be corrected for if known, or eliminated by compensating with capacitance in the manner indicated in par. 197.

Primary measurements of moderate currents can be made with fair accuracy where steady current is available by noting the current consumption of a non-inductive load when the voltage is held at a fixed value on both continuous and alternating current by means of a suitable voltage-measuring instrument. A bank of incandescent lamps with heavy connections between lamps and a dynamometer or electrostatic-type voltmeter connected at the "center of distribution" makes a simple and satisfactory arrangement. Tungsten lamps operating at about normal voltage are preferable because of the voltage and current relation, about 5 per cent. change in current corresponding to 10 per cent. change in voltage. Thus, if the voltage can be controlled within 0.1 per cent. the current is constant within 0.05 per cent. Obviously, the switching arrangements should be such that transfer can be made rapidly and both direct and reversed readings taken.

118. Potentiometer Method.—Alternating-current potentiometers which are described in par. 90 may be used for current measurements in exactly the same way that continuous-current potentiometers are employed for continuous-current measurements. Particular care is taken in the construction of the resistors to eliminate inductance (see par. 304).

Secondary Standards

119. General.—Indicating instruments which are particularly designed and constructed for high-accuracy measurements under laboratory conditions and which can be calibrated on continuous current may be classed as secondary standards. The electrodynamometer principle is the most commonly employed for such instruments.

120. Siemens Dynamometer.—This was one of the earliest forms of dynamometer instruments but is now practically obso-

lete. It consists of two stationary coils connected in series, with a moving coil suspended between them by a silk fiber. The deflecting force or moment is opposed by a spiral spring attached to a so-called torsion head which is twisted by hand until the moving coil is brought back to the zero position. The amount of this twist, as indicated on a scale over which moves a pointer attached to the torsion head, is a measure of the current. The final relation of the fixed and moving coils is, therefore, always the same. The current in amperes is

$$I = k\sqrt{L}$$

where L = twist of torsion head in degrees and k = a constant determined by calibration on continuous current.

While this instrument was used to a great extent in the early days of the application of alternating current, the errors due to stray fields to which the instrument was very sensitive and to temperature changes and other causes, rendered it unsatisfactory as a standard when higher precision was demanded as the art advanced.

121. Kelvin Balance.—The principle of this instrument is described in par. 94. Instruments for current measurements are constructed in the same manner as those for potential measurements except for the large size of the conductors and smaller number of turns per coil.

These instruments have been constructed for currents as large as 5,000 amp. While water-cooling and modern refinements in construction such as the use of braided and stranded conductors have greatly reduced, in recent instruments, the errors which formerly existed, Kelvin balances are used only to a very limited extent in this country. They are slow and troublesome to use except where the current is particularly steady, a condition which rarely exists with alternating currents. Consequently, they have been superseded by more reliable and convenient types of dynamometer instruments.

122. Westinghouse Precision Ammeter.—This instrument is practically a semiportable Kelvin balance of relatively small dimensions and with the coils in a vertical plane instead of a horizontal plane. The forces exerted between the fixed and movable sets of coils is opposed by a helical spring which is twisted by means of a torsion head as in the Siemens dynamometer until the movable system is brought back to the zero position. The

current is equal to the square root of the angular movement of the torsion head multiplied by a constant (see par. 120).

123. Alternating-current Comparator.—This instrument devised by Prof. E. F. Northrup[1] utilizes the expansion of wire under the heating action of a current which traverses it. Two wires are arranged side by side, as in a bifilar suspension, and are placed under tension in such a manner that if one is heated more than the other, a small mirror attached to the center of both wires is deflected. The alternating current to be measured is passed through one of the wires, while continuous current is passed through the other and adjusted until there is no deflection. The continuous current is then measured. For large currents, the alternating-current wire is energized from a non-inductive shunt.

The instrument has not come into very extended use probably because it is slow and, for reasonable accuracy, a very steady source of alternating current is required.

124. Mercury Constriction Ammeter.—The principle of this instrument which was also developed by Prof. Northrup[2] is the constriction effect (also called pinch effect) which takes place at the center of a column of mercury when it is carrying current. This effect is due to the controlling force exerted by the surrounding magnetic field. The construction of the instrument is such that the effect is multiplied sufficiently to provide a means of measuring the current. The indicator is a column of colored liquid in a vertical capillary tube, the column moving up and down with the apparent expansion and contraction of the mercury at the center of the column. The movement is proportional to the square of the current.

The particular application of this instrument is where a secondary standard of comparatively rugged construction for very large currents of a limited range is desired. But the need for the direct measurements of large alternating currents in terms of continuous currents is becoming less as methods of measuring with strictly alternating-current instruments become more refined. With properly designed current transformers which have been carefully tested for ratio and phase angle, the largest alternating

[1] "A New Instrument for the Measurements of Alternating Currents," E. F. Northrup, *Transactions*, A. I. E. E., vol. 24, p. 741 (1905).

[2] "Ammeter for Accurate Measurements of Large Alternating Currents," E. F. Northrup, *London Electrician*, Oct. 1, 1909.

currents can be measured with an accuracy sufficient for most commercial purposes.

125. Weston Ammeter, Model 326.—This secondary standard is a shielded dynamometer indicating instrument similar to the ordinary portable ammeters of the same type except that it is much larger, has a much longer scale and pointer, and is more carefully constructed. The two fixed coils carry the total current while the movable coil is connected across a shunt in series with the fixed coils. The Siemens and Halske precision ammeter is of the same type but the scale and pointer are only slightly larger than in portable instruments, and the instrument is not magnetically shielded.

These instruments are very satisfactory secondary standards because the continuous-current calibration is accurate for alternating current; they are free from frequency and wave-form errors and are convenient to use.

Ammeters[1]

126. General.—Alternating-current portable ammeters are usually of one of the following types; dynamometer, soft-iron-vane, induction or hot-wire. The principle employed in each type is the same as that in the corresponding voltmeter, descriptions of which will be found in Chapter 5. The construction differs somewhat, however. In some modern dynamometer instruments over 0.5- or 0.75-amp. range, the fixed coils carry the total current to be measured while the movable coil is connected across a shunt which is in series with the fixed coils. Soft-iron-vane and induction ammeters differ from voltmeters of the same types only in the size of wire used. Hot-wire ammeters of more than 1- or 2-amp. range are usually small-current instruments connected to shunts through which the current to be measured is passed, the shunt being either external to the instrument or mounted within the instrument case.

The majority of modern switchboard ammeters are of the soft-iron-vane and induction types. The principle of this type permits simple and rugged construction which is essential for switchboard service. While dynamometer instruments are em-

[1] "Electrical Measuring Instruments," *Circular*, Bureau of Standards, No. 20, 1915 (2d edition). An extensive discussion of the principles of construction and operation of the various types of ammeters in general use.

ployed, they are more expensive, less rugged and more affected by stray magnetic fields.

The discussion given in connection with alternating-current voltmeters in regard to calibration, use, effect of stray fields and so forth, also applies, in general, to alternating-current ammeters.

Measurement of Large Currents

127. Shunts.—The only type of ordinary ammeter which is applicable to the direct measurement of large alternating currents is the hot-wire instrument because it can be used with shunts. Such instruments are made for ranges as high as 2,000 amp. but great care is required in the design and construction of the shunts to eliminate any drop other than that due to the resistance, and any difference between the distribution with continuous current and that with alternating current. This means that the shunts must be free from inductance, eddy currents and "skin" effect. The "hot-wire" method of measurement is particularly applicable to high-frequency currents or to currents with a badly distorted wave form, a condition which is frequently found, for example, where a large current is supplied by an improperly designed low-voltage transformer or generator.

128. Current Transformers.[1]—In all ordinary measurements the most common method of measuring large alternating currents is to use current transformers to step down the current to a small value which can be easily measured with standard instruments. The ratios are usually such that the normal secondary capacity is 5 amp. Current transformers are also employed to insulate instruments and apparatus from high-voltage circuits.

They are similar to so-called power transformers, except that the latter are connected in shunt across the line and the secondary potential remains substantially constant irrespective of the

[1] For a discussion of the theory and characteristics of current transformers see the following:

"Characteristics and Limitations of the Series Transformer," A. R. Anderson and H. R. Woodrow, *Bulletin* No. 61, University of Illinois, October, 1912.

"Theory and Design of the Current Transformer," A. P. Young, *Journal Institute E. E.*, vol. 45, p. 670 (1910).

"The Current Transformer," K. L. Curtis, *Proceedings*, A. I. E. E., vol. 25, p. 715 (1906).

"The Series Transformer," E. S. Harrar, *Electrical World*, vol. 51, p. 1044 (1908).

connected load. Series transformers are connected in series with the primary line and the secondary current remains substantially constant for a wide range of loads, the "load" being only instruments or other devices which are connected directly in series with the secondary winding.

Well-designed current transformers may, when the ratio and phase-angle characteristics are known, be used for very precise measurements of large currents. Unless the secondary volt-ampere load is excessive, the wave form of the secondary current may be considered the same as that of the primary current because the distortion within the transformer is ordinarily negligible. The effect of wave-form variation on the characteristics of the transformer is also usually negligible.

129. Use of Current Transformers.—Where the highest accuracy is desired, no load, other than the instruments to be used in the measurement, should be connected to current transformers. Where a number of instruments are to be used, it may be desirable to use separate transformers for the wattmeters because they are ordinarily the most important instruments. Even for permanent switchboard work, the manufacturers recommend that instruments be connected to separate transformers and not to those being used for trip coils, relays and other devices requiring a relatively large amount of power.

Care should always be taken not to open the secondary circuit of a current transformer when it is excited, not only because of the possibility of an accident due to the high voltage which may be induced when on open circuit, but the high magnetization which takes place may change the characteristics of the transformer, particularly the phase angle. It is thus advisable to "demagnetize" a transformer before determining its ratio and phase angle by gradually increasing the resistance in the secondary circuit to a relatively high value (at least 20 or 30 ohms) and then decreasing to short-circuit, the primary carrying normal current in the meantime.

In so-called portable-current transformers of the cable type, the arrangement of the primary conductors has no effect on the characteristics. Each time the primary conductor passes through the core, it counts as one turn and the position of the conductor in the hole or the size and disposition of the loop between turns makes no difference whatever. Transformers of the split-core type should, however, be used with caution, particularly

for power measurements because of the large phase angle due to the high reluctance in the magnetic circuit.

130. Ratio and Phase Angle of Current Transformers.—Theoretically, the ratio of transformation should be the same as the ratio of the secondary turns to the primary turns, and the secondary current should be in exact phase opposition to the primary current. Actually, neither of these conditions exists because of the current required to excite the core and supply the losses.

In Fig. 66, I_p represents, vectorially, the primary ampere-turns and E the corresponding impressed voltage; I_e represents the exciting ampere-turns made up of magnetizing ampere-turns, I_m and the ampere-turns, I_h, which supply the eddy-current and hysteresis losses. The resultant or working primary ampere turns are represented by I'_p. The secondary ampere-turns, I_s, will be exactly equal to and 180° from I'_p. It will be seen that

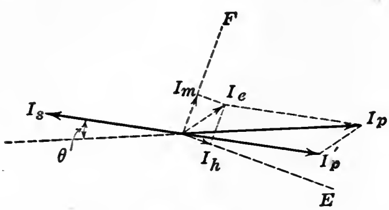


Fig. 66.

I_s is smaller than it should be by the difference between I_p and I'_p and the actual ratio of transformation will be larger than the theoretical value. Also I_s is less than 180° from I_p by the *phase angle, θ.* Furthermore, these errors vary with the magnitude of the primary current and with the impedance of the secondary circuit because of the corresponding variation in the flux density.

In many commercial measurements the errors can be neglected, but in accurate measurements of current, power and energy, the true ratio should always be known. In power and energy measurements, the phase angle should also be known.

Typical phase-angle and ratio curves are shown in Fig. 67. The phase angle in minutes and the ratio in per cent. of the nominal ratio are plotted against current expressed as per cent. of the full-load current. In all cases, the secondary load had a negligible inductance and a resistance less than 0.2 ohm. In transformer No. 1 the phase angle is very small but the ratio is nearly

2 per cent. low although it is uniform above 25 per cent. load. In transformer No. 2 the ratio error is only 0.5 to 0.75 per cent. but the phase angle is abnormally high. Transformer No. 3 has an extremely small phase angle. Transformer No. 4 has a ratio error of 3 per cent. at full load.

The following are the principal methods of measuring ratio and phase angle.

(*a*) *Ratio from Primary and Secondary Currents.*—The ratio may obviously be determined by measuring the primary and

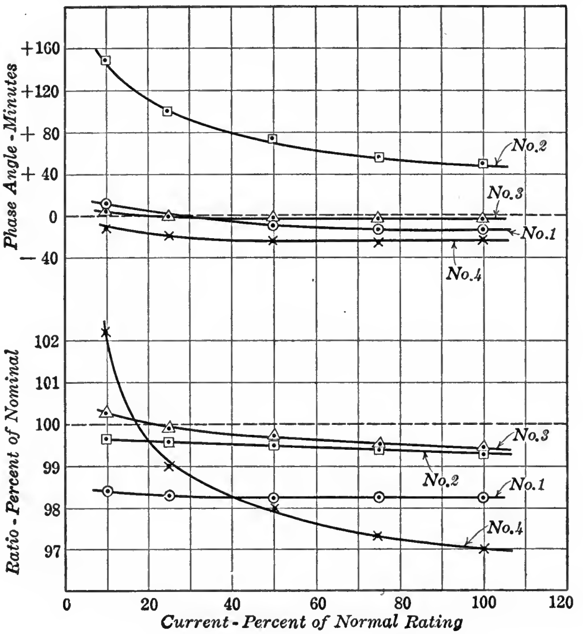


Fig. 67.

secondary currents directly with current-measuring instruments but such a method is less accurate than the null methods described below.

(*b*) *Ratio and Phase Angle by Opposition and Null Methods.*—The principle of these methods is that of the potentiometer and is similar to the corresponding method for determining the ratio and phase angle of potential transformers, to which reference should be made (par. 111*c*). A non-inductive resistance in the secondary circuit is adjusted until the drop across it is equal to that

across a non-inductive resistance in the primary to which it has been connected in opposition. The ratio of the two resistances when balance has been attained is equal to the ratio of transformation. The differences between the various methods employing this scheme which have been developed are largely in the manner of determining balance and of measuring the phase angle.

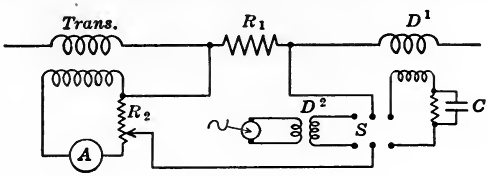


Fig. 68.

A method used at the Bureau of Standards[1] and indicated in Fig. 68 employs a separately excited, compensated (see par. 197) reflecting dynamometer as the detecting instrument. The resistances R_1 and R_2 are in the primary and secondary circuits, respectively. The fixed coil of a dynamometer, D_1, is connected in series with the primary; then, with the switch S thrown to the right, R_2 is adjusted until zero deflection is obtained. The component of the potential drop in R_2 which is in phase with that in R_1, is thus equal in magnitude to the drop in R_1. Since the phase angle is always very small, the ratio of R_2 to R_1 may be taken as the transformer ratio. The phase angle is then determined by measuring the component of the R_2 drop which is 90° from the R_1 drop, by means of another dynamometer, D_2, the fixed coils of which are excited by a current displaced 90° in phase from the primary current. The deflection is proportional to the sine of the phase angle.

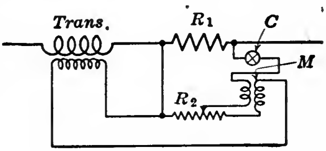


Fig. 69.

A method developed at and used by the Electrical Testing Laboratories[2] is shown schematically in Fig. 69. The primary

[1] "Determination of the Constants of Instrument Transformers," P. G. Agnew and T. T. Fitch, *Bulletin*, Bureau of Standards, vol. 6, p. 281 (1909–10).

[2] "Recent Progress in Exact Alternating-current Measurements," C. H. Sharp and W. W. Crawford, *Transactions*, A. I. E. E., vol. 29, p. 1517 (1910).

and secondary resistances are R_1 and R_2 respectively, and C is the detector which in this case is a synchronously driven reversing key connected to a Paul "unipivot" galvanometer. The secondary of a mutual inductor, M, is in the detector circuit and the primary of the inductor is in series with the secondary circuit of the transformer. After the value of R_2 has been found which will make the "in-phase" potential drops balance each other, the quadrature drop in R_2 is balanced by the e.m.f. in the secondary of the variable mutual inductor (see par. 111*c* for further dis-

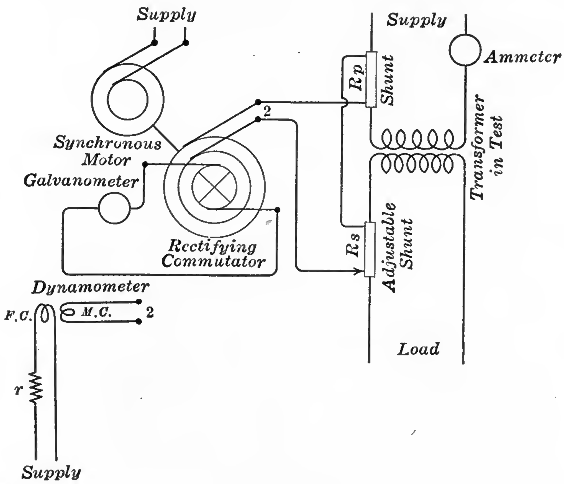


FIG. 70.

cussion of this method). The value of the ratio and phase angle are calculated from the following formulas:

$$X = \frac{R_2}{R_1}\sqrt{1 + \left\{\frac{2\pi f M}{R_2}\right\}^2}$$

and

$$\phi = \tan^{-1}\frac{2\pi f M}{R_2} \qquad \text{(degrees)}$$

where the symbols have the same significance as in par. 111*c*.

The term under the radical which is the correction factor amounts to only 0.02 per cent. with a phase angle of 1° and to 0.06 per cent. with an angle of 2°. It can, therefore, be neglected in many cases so far as the ratio is concerned.

In the two-dynamometer method,[1] the ratio is obtained with a reflecting dynamometer in both primary and secondary circuits used simply as precision ammeters. The phase angle is measured as described in par. 111*c*, the connections being as shown in Fig. 70.

Sharp and Crawford[2] proposed a method and Fortescue[3] developed an apparatus in which the resistances R_1 and R_2 were replaced by mutual inductors. The principle is exactly the same as with resistors except that the two potentials which are made equal and opposite are 90° in phase from their respective currents instead of being in the same phase and the small e.m.f. introduced

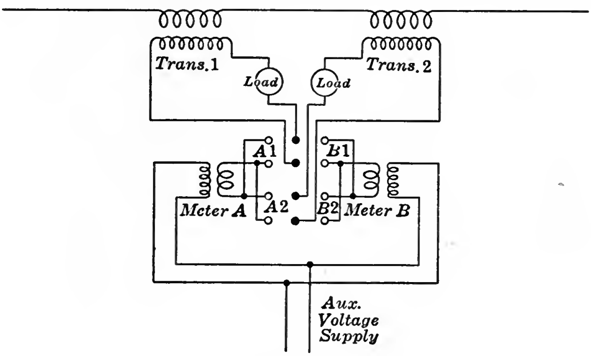


FIG. 71.

to compensate for the phase angle is obtained from a resistor in the secondary circuit instead of a mutual inductor.

(*c*) *Ratio and Phase Angle, Watt-hour Meter Method.*—The method described in par. 111*d* for checking potential transformers in service is equally applicable to current transformers. The connections are arranged as indicated in Fig. 71. The test is made and results calculated as indicated in par. 111*d*.

131. Polarity of Current Transformers.—When more than one transformer is being connected to an instrument or apparatus, it

[1] "Electrical Measurements in Circuits Requiring Current and Potential Transformers," L. T. ROBINSON, *Transactions*, A. I. E. E., vol. 28, p. 1005 (1909).

[2] "Recent Progress in Exact Alternating-current Measurements," C. H. SHARP and W. W. CRAWFORD, *Transactions*, A. I. E. E., vol. 29, p. 1517 (1910).

[3] "The Calibration of Current Transformers by Means of Mutual Inductance," CHARLES FORTESCUE, *Proceedings*, A. I. E. E., June, 1915, p. 1199.

is convenient to know the polarity of the transformers or at least that the polarities are all alike. If the polarity of the transformers has been determined, the correct connections can be made once for all instead of being determined by trial.

A convenient and simple method of determining the polarity of current transformers is to connect the primary or larger current winding to one or two dry cells through a resistance of 40 or 50 ohms and a suitable switch, noting which terminal of the winding is positive. The voltmeter is then connected to the terminals of the secondary winding and the terminal which is positive when the battery circuit is suddenly *closed* is noted. Then on alternating current, this secondary terminal will have the same polarity at any instant as the primary terminal which was positive.

132. Measurement of Small Currents.—Portable ammeters of the soft-iron-vane type are available in capacities down to about 50 milliamperes full scale. Portable instruments of the dynamometer type are made with full scale ranges as low as 15 milliamperes. With the latter, 5 milliamperes is about the minimum which can be measured with fair accuracy because the deflections are proportional to the square of the current. This minimum can, of course, be lowered by separately exciting the fixed coil so that the deflections become proportional to the first power of the current. Obviously, alternating-current voltmeters may be used as milliammeters where the potential is sufficiently high, the full scale current being the voltage at full scale divided by the impedance of the instrument.

Portable instruments of the ordinary types can only be used for measuring small currents where the potential is relatively high because of the high impedance of the instrument. For instance, a 75-milliampere ammeter of the soft-iron-vane type will have an impedance of about 100 ohms on 60 cycles and a 15-milliampere instrument of the dynamometer type will have an impedance of about 1,500 ohms. Portable "unipivot" dynamometer milliammeters are made by R. W. Paul down to 10 milliamperes full scale with an impedance of about 150 ohms.[1]

Where the potential is very small, reflecting alternating-current galvanometers (see Chapter II) are most convenient for measur-

[1] In these and the ordinary portable milliammeters, the inductance is usually so small, relatively, that the impedance is practically equal to the resistance.

ing very small currents. When a separately excited dynamometer instrument is used, however, care must be taken that the wave form of the exciting current is the same as that of the current being measured. When measuring the exciting current of a magnetic circuit at high densities, this may not be the case.[1] Also, the two currents must be in phase (see par. 104).

Measurements of High-frequency Currents

133. General.—It is obvious that instruments which operate on an electromagnetic or an electrostatic principle could not be used for the measurement of high-frequency currents because of the large inductance and capacitance errors that would exist. Practically all methods for measuring current where the frequency is greater than about 500 cycles utilize the heating effect of the current.

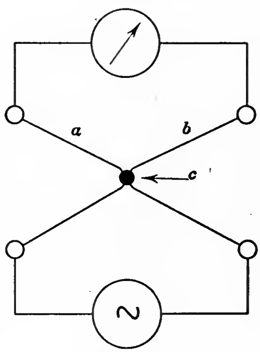


Fig. 72.

134. Small Currents.—The Duddell thermogalvanometer described in par. 36 is particularly suited to the measurement of small high-frequency currents. This same principle is, however, applied in a number of ways. In one form of high-frequency galvanometer as made by R. W. Paul, a thermo junction is formed at *c* (Fig. 72), the center of two wires, *a* and *b*. These wires may be iron and manganin for example. The current to be measured is passed through the junction between one pair of terminals as indicated, and the thermo e.m.f. produced by the resulting rise in temperature is measured with a continuous-current galvanometer connected to the other pair of terminals. With the circuits arranged as indicated, there may be an error when calibrating on continuous current, due to the Peltier effect caused by the current flowing through the junction. However, this source of error is eliminated in later forms by making the current circuit one continuous wire of the same material, with the thermo junction welded to it at the center.

In another similar arrangement, the thermo junction is not

[1] "Measurements of Alternating Current of Low Value," M. G. Newman, *Transactions*, A. I. E. E., vol. 31, p. 1489 (1912).

connected to the circuit through which the high-frequency current flows, but is placed very close to the so-called "heater" wire. This arrangement permits the convenient interchanging of heater elements for a wide range of currents. On the other hand, the first arrangement can be made more sensitive for very small currents because it can be readily mounted in a small exhausted glass bulb. Commercial instruments of these types have sensitivities ranging from 4 or 5 to 500 milliamperes per millivolt thermo e.m.f.

In the method indicated in Fig. 73, a and a_1 are two fine wires of different materials stretched between two terminal blocks. The wires to the galvanometer are of the same materials but so connected that a_1 and b are alike and a and b_1 are alike, thus

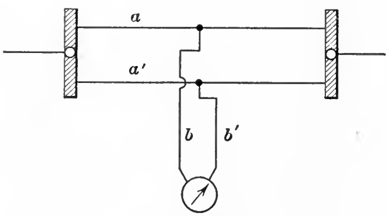


FIG. 73.

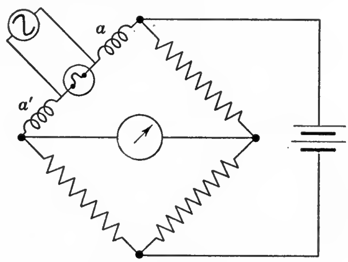


FIG. 74.

making two thermocouples in series with the galvanometer. Equality of potential is obtained by adjusting with direct and reversed continuous current. This scheme eliminates the objections to the form shown in Fig. 72.[1]

The principle of Wheatstone-bridge methods is shown in Fig. 74. The high-frequency current is measured by the change in the resistance of a very fine wire having a high temperature coefficient, which is caused by the passage of the current. The inductors a and a^1 prevent the high-frequency current from flowing through the bridge. For large currents carbon incan-

[1] Much information and data on the construction of these thermo-junction instruments, their relative merits, methods of standardization and so forth, will be found in an article entitled "On the Use of Thermo Junctions for High-frequency Current Measurements," C. M. DOWSE, *The Electrician* (London), vol. 65, p. 765 (Aug. 19, 1910).

An extended discussion and investigation of high-frequency ammeters is given in an article entitled "High-frequency Ammeters" by J. H. DELLINGER, *Bulletin*, Bureau of Standards, vol. 10, p. 91 (1914).

descent lamps may be used, but for very small currents resistors called "'barretters" are employed. These consist of platinum or gold wire as small as 0.002 mm. diameter and 5 mm. long, mounted on a suitable support in a glass tube and at atmospheric pressure.[1] Sensitivities of the order of 10^{-10} amp. high-frequency

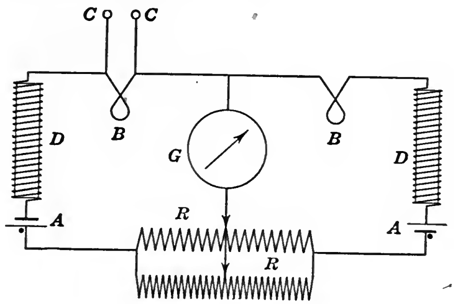


FIG. 75.

current can be obtained with such a barretter and a high-sensitivity galvanometer. In the barretter set devised by Gati, a compensating bridge arrangement indicated in Fig. 75 is used. B_1B are two barretters, DD are choke coils, RR variable resistances, AA storage cells and G a galvanometer. The resist-

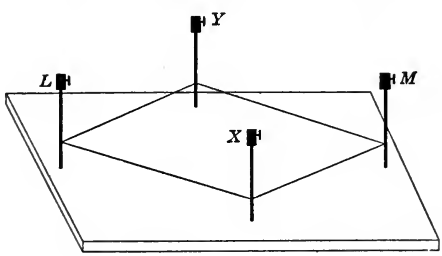


FIG. 76.

ances are first adjusted until no deflection is indicated, then any high-frequency current through one of the barretters will produce a deflection proportional to the current. By using a pointer type of galvanometer, the whole instrument can be made readily portable.

[1] For a method of construction of a high-sensitivity barretter see paper by Dr. A. E. KENNELLY, "High-frequency Telephone Circuit Tests," International Electrical Congress, St. Louis, 1904; *Transactions*, vol. 3, p. 418.

Dellinger[1] used the arrangement indicated in Fig. 76 as a standard for testing commercial ammeters at 1 to 10 amp. at wireless telegraph frequencies. A copper wire about 0.1 mm. diameter was soldered about 3 cm. above the base to four vertical copper studs at the corners of a rhombus about 10 cm. on a side. The current to be measured was introduced at L and M and the change in resistance due to the heating caused by the current was measured with a Wheatstone bridge connected to X and Y.

135. Large Currents.—The methods described above which employ a hot wire and thermocouple may be used for larger currents by making the "heater" wire larger or of lower resistance metal. In the Fleming thermo-electric ammeter for relatively large currents, a number of separate, fine, copper wires are connected in parallel between two terminal blocks. Each wire carries about 2.5 amp. and additional wires may be added to give as high as 30 amp. capacity. The thermo junction is soldered to the center of the middle wire. Dellinger[1] finds large errors at high frequencies (over 100,000 cycles) due to mutual inductance and particularly to change in the distribution of the current. He suggests using high-resistance wires arranged around the periphery of a cylinder to give a symmetrical distribution, with a thermocouple at the center of each wire, these couples being connected alternately reversed so as to give an average thermo e.m.f. which would eliminate the effect of slight inequalities in the currents in individual wires.

Brown[2] proposes a heat detector in the form of a sensitive resistance thermometer consisting simply of a small globule of lead oxide in which the ends of the wires to the bridge are imbedded with a thin layer of oxide between. Such a detector is stated to be sensitive to a temperature change of 0.001°C.

Campbell and Dye[3] investigated methods of measuring high-frequency currents using thermal instruments immersed in oil, air-core transformers and iron-core transformers. They conclude as a result of much experimental work: "It is evident from our

[1] "High-frequency Ammeters," J. H. Dellinger, *Bulletin*, Bureau of Standards, vol. 10, p. 91 (1914).

[2] "Measuring Electric Currents by Their Heating Effect," S. L. Brown, *Physical Review*, March, 1916.

[3] "Measurement of High-frequency Alternating Currents," Albert Campbell and D. W. Dye," *The Electrician* (London), March 19, 1915.

experiments that properly designed air-core transformers when used with care in conjunction with thermal ammeters afford a simple means of measuring currents of the order of 1 to 50 amp. with good accuracy at frequencies from 50,000 to 2,000,000 cycles per second. Iron-core transformers, which have some advantages in ease of construction can also be designed to fulfil the same purpose and to give satisfactory results. To a limited extent both types of transformers are useful for measuring very small high-frequency currents" (that is, by using them inverted). Some design data on transformers for high-frequency measurements are given in the original article.

136. Telephone Currents.—Telephone talking currents are of constantly varying amplitude and frequency so that their quantitative measurement, in the sense that the mean effective value of a sine-wave current is measured, is impossible. Tests and measurements with generated currents of telephonic frequencies and intensities are made with the various methods described above. Talking current intensities are usually compared by ear with the telephone and standardized artificial cables. When quantitative data are required, recourse is had to a high-sensitivity oscillograph.[1]

A simple method of measuring the sensitivity of telephone receivers is as follows: Connect the receiver in series with a fixed non-inductive resistance of several megohms and connect this circuit as a shunt through movable contacts to a low, non-inductive resistance carrying a relatively large and known current of the desired frequency.[2] The contacts are adjusted until the desired sensitivity is obtained when the receiver-circuit current can be calculated from the values of the two resistances and the main current. A high resistance of 8 or 10 megohms can be made by filling a small glass tube, 6 cm. long, with alcohol. The low resistance is conveniently made with a piece of manganin wire doubled back on itself forming a single, narrow loop which is practically non-inductive. Adjustment is obtained by means of a short-circuiting movable contact across the two wires of the loop. With this apparatus currents from 5 to $10{,}000 \times 10^{-10}$ can be measured at frequencies of 100 to 2,500 cycles.

[1] Report of Second International Conference, European Telephone and Telegraph Administration, Beli Gati, 1910.

[2] For sources of high-frequency currents for testing purposes, see Chapter 11, par. 293.

CHAPTER VII

RESISTANCE, REACTANCE AND IMPEDANCE MEASUREMENTS

137. Absolute Unit of Resistance.—The fundamental unit of resistance is expressed in the electromagnetic system in terms of the centimeter and second. Several methods have been devised and used for the absolute measurement of resistance but probably the best known is the Lorenz method. In this method, a circular copper disc is rotated at an accurately known speed in a uniform field and the e.m.f. induced in the disc is balanced against the fall of potential produced in the resistance to be measured. The field is produced by a helix or by two coaxial coils accurately made and placed with respect to the disc. A steady current is passed through the field coils and the resistance in series. The induced e.m.f. is calculated from the dimensions of the coil or coils, the area of the disc and the speed of the disc.[1]

138. Practical Unit and Standard.—The practical unit of resistance is the international ohm. The primary standard representing this unit is the resistance of a column of mercury of uniform cross-section, 106.300 cm. long, 14.4521 grams mass and at the temperature of melting ice. This standard is necessarily difficult to construct, maintain and use so that in actual measurements metallic resistors which have been standardized by comparison with the primary standard are employed. The primary standard will in fact be found only in the laboratories of the custodians of the electrical standards in the several countries.

Secondary Standards

139. General.—Secondary standards are made with metal of high specific resistance in the form of wire or ribbon. Manganin, a copper-nickel-manganese alloy, is generally used because, when properly treated and aged, it meets the necessary requirements. These requirements are: permanent electrical and physical characteristics; low thermo e.m.f. against copper; small

[1] "Absolute Measurements in Electricity and Magnetism," Alexander Gray, vol. 2, p. 580.

temperature coefficient of resistance; relatively high specific resistance. The completed standard must, in addition, be unaffected by immersion in oil, or by changes in atmospheric conditions.

In general, resistance standards may be divided into two classes: standards of resistance, or those used primarily for the measurement of resistance; and current standards or those intended primarily for the measurement of current.

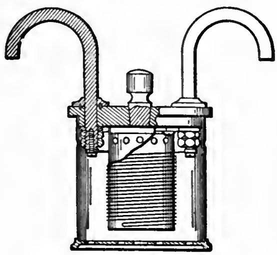


Fig. 77.

140. Standards of Resistance.—Standards of resistance are usually made with very small current-carrying capacity. There are two forms in general use, the Reichsanstalt and the N. B. S. (National Bureau of Standards).[1] The Reichsanstalt form is shown, partially in section in Fig. 77. The N. B. S. form is shown in Fig. 78. The distinctive features of the latter form are that it is immersed in oil and hermetically sealed. This prevents the absorption of moisture by the shellac, with which the resistance wire or strip is coated, and the consequent elongation of the fine wire used in the higher resistors.[2] Both forms are intended to be hung from mercury cups by means of lugs, and may be suspended in an oil bath in order to measure the temperature more accurately. The N. B. S. form is made only in sizes higher than 1 ohm.

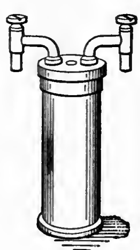


Fig. 78.

141. Current Standards.—Current standards are made in two types, the Reichsanstalt and air-cooled. The Reichsanstalt standards are made in two forms, the small pattern for moderate currents, and the large pattern (in low resistances) for large currents. The small-pattern form is similar in appearance to Fig. 77 except that, for 1 ohm and less, separate potential taps are provided. They are suspended from mercury cups in an oil bath for cooling purposes. Obviously, this type may be used also as a resistance standard. The following table shows the current rating assigned by Otto Wolff, and the Leeds and Northrup Co.

[1] "A New Form of Standard Resistance," E. B. Rosa, *Bulletin*, Bureau of Standards, vol. 5, p. 413 (1908).

[2] "The Variation of Resistances with Atmospheric Humidity," E. B. Rosa and H. D. Babcock, *Bulletin*, Bureau of Standards, vol. 4, p. 121 (1907–1908).

Current Rating of Standard Resistors, Reichsanstalt Type
Permissible Current Capacity in Amperes[1]

Nominal resistance, ohms	When used for resistance measurements (accuracy 0.01–0.02 per cent.)		When used for current measurements (accuracy 0.02–0.04 per cent.)	
	Still air	Still oil	Still air	Still oil
100,000.0	0.002	0.003	0.005	0.01
10,000.0	0.005	0.01	0.015	0.03
1,000.0	0.015	0.03	0.05	0.1
100.0	0.05	0.1	0.15	0.3
10.0	0.15	0.3	0.5	1.0
1.0	0.5	1.0	1.5	3.0
0.1	1.6	3.0	5.0	10.0
0.01	5.0	10.0	15.0	30.0
0.001	16.0	30.0	50.0	100.0
0.0001	55.0	110.0	165.0	330.0

The large pattern form as made by Wolff, in 0.001-ohm, 0.0001-ohm and 0.00001-ohm sizes is intended for very large

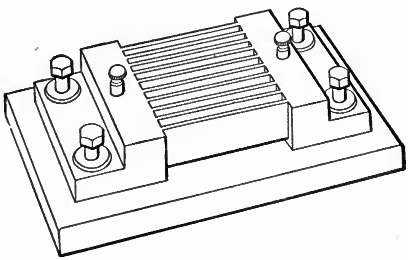


Fig. 79.

currents. The resistance element is permanently mounted in a tank of oil, which is water-cooled. Connection with the source of current is made directly to heavy copper blocks. The potential terminals are separate and tap directly on to the resistance metal. Large pattern standards of low resistance have capacities from 100 watts to 2,500 watts and over.

Air-cooled current standards employ sufficient surface to permit of use in air without excessive temperature rise. While they are not as accurate or as reliable as the Reichsanstalt form,

[1] Data for current measurements based on 10°C. temperature rise, 2.5 and 10 watts capacity in still air and oil, respectively. Data for resistance measurements based on 0.3 and 1.0 watt for still air and oil, respectively.

they are amply satisfactory for much commercial work and are especially convenient where oil baths would be inconvenient. Fig. 79 shows a Leeds and Northrup Co. 0.00002-ohm resistor of 2,000 amp. capacity for which an accuracy of 0.04 per cent. is claimed.

142. Precautions in the Use of Standard Resistors.—Care should be taken that standard resistors are not overheated and that the oil used is free from acid. Both the current terminals and the potential terminals should be kept perfectly clean. Poor contact at the current terminals of large-capacity standards will produce excessive heating which may permanently change the resistance. Poor contact at the potential terminals will introduce resistance in the measuring circuit.

In resistors smaller than 10 ohms used for current measurements, the potential drop should be measured between points some distance inside of the current terminals. This permits more convenient adjustment of the resistance, eliminates the joint resistance between the terminal lug and the resistance metal, and insures uniform distribution of the current at all values in that part of the resistance metal across which the potential is measured.

Resistance of Conductors

143. General.—There is no sharp distinction between materials commonly called conductors and those called insulators. Resistances of the former class may, however, be relatively high or relatively low and certain methods of measurement are especially applicable to each.

144. Fall-of-potential Method.—The fall-of-potential method consists simply in noting the voltage drop with a known current flowing through the resistance, and calculating the resistance from Ohm's law, $R = E/I$. This method is not suitable for very high or very low resistances and the accuracy depends upon the measurement of the two unknown quantities with indicating instruments. Furthermore, the current required to give a readable drop may cause overheating. The method should therefore be used with caution and only where accuracy is subordinate to simplicity and convenience. The potential should be measured, when possible, between points well within the current connections especially when the resistance is low and the current is high.

Greater accuracy can be obtained by substituting a standard

resistor in place of the ammeter, and noting the drop across it and across the unknown resistance in rapid succession. If a voltmeter is used, its error need not be known because the unknown resistance is simply equal to the ratio of the two readings multiplied by the standard resistance. Similarly, if the drops are measured with a potentiometer, any steady source of current can be used as the "standard cell." The accuracy will be greatest when the two resistances are nearly equal.

145. Bridge Methods.—Bridge methods are the most accurate for resistance measurements because they are zero methods and also because the comparison is made directly with standardized resistances, the accuracy of which can be made very high. The

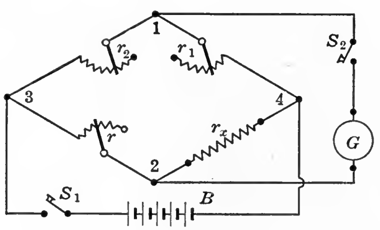


FIG. 80.

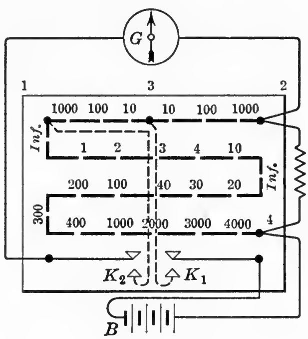


FIG. 81.

principal types of bridges are known as Wheatstone, Carey-Foster, and Kelvin.

(*a*) *Wheatstone Bridge.*—The Wheatstone bridge is the most generally used form for the measurement of all but the highest and the lowest resistances. Fig. 80 shows the theoretical arrangement of a Wheatstone bridge where r, r_1 and r_2 are accurately known resistances and r_x is the resistance to be measured. When using the bridge, the various resistances are adjusted until the galvanometer, G, shows no current flowing; then

$$r_x = \frac{r_1}{r_2} \times r.$$

The battery switch, S_1, should always be closed before the galvanometer switch, S_2, in order to protect the galvanometer from the momentary rush of current. The galvanometer and the battery may be interchanged without affecting the result.

Wheatstone bridges are made in a variety of forms. In most forms the resistances, r, r_1 and r_2, consist of a number of resistance coils or units carefully adjusted to various multiples of 10 and so arranged that they can be conveniently connected in and out of the circuit by means of plugs or switches. The resistances, r_1 and r_2 (Fig. 80), are commonly called the ratio arms and r the rheostat arm. A very early form which is still in use in small portable sets is the Postoffice pattern, shown diagrammatically in Fig. 81. Coils are cut out by short-circuiting them with plugs, so that there may be several plug-contact resistances of an unknown and variable amount in a given arm. In the decade form, shown diagrammatically in Fig. 82, this objection is overcome by arranging the coils of the rheostat arm on the "decade" plan, in which there are nine 1-ohm coils in the "units" division, nine 10-ohm coils in the "tens" division, and so forth. Any number of coils in a given division can be connected in circuit by changing only one plug. In many later types, the ratio-arm coils are also connected on the decade plan, which in addition to eliminating plug-contact resistance errors, permits interchecking the coils. Furthermore, the decade arrangement permits the use of sliding-brush or dial construction instead of plugs.

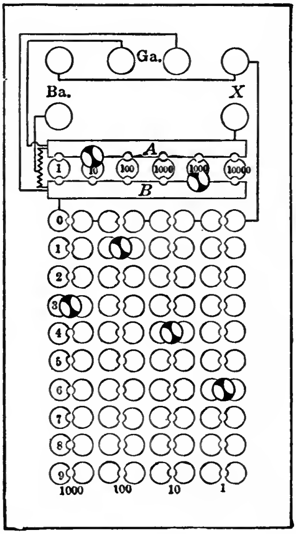


FIG. 82.

The Wheatstone type of bridge as ordinarily arranged is best suited to measuring resistances of the order of about 1 ohm to 100,000 ohms. Accuracies of the order of 0.05 per cent. are obtainable with a first-class bridge if the unknown resistance is intermediate in value between the limits stated. It is to be noted, however, that a very high precision may be obtained with the simple Wheatstone bridge where the resistances of the arms are favorably chosen. With a galvanometer of sufficient sensitivity, measurements may be made to 1 part in 1,000,000 by shunting one or two arms.[1] The maximum sensitivity is

[1] "Simple Apparatus for Comparing Resistances," E. H. RAYNER, *London Electrician*, Sept. 10, 1915.

obtained when the four arms are equal; hence this condition should always be approached as nearly as possible by keeping r_1/r_2 near unity, and r_1 and r_2 as nearly equal to r as convenient.

A galvanometer with 100 to 500 ohms resistance will be satisfactory for nearly all classes of work. The resistance coils will dissipate about 1 watt without overheating, but care should be taken that the current does not become excessive when the ratio becomes large.

The slide-wire form of bridge is one of the earliest forms of the Wheatstone bridge. It is convenient and rapid where many similar measurements are to be made. It differs from the standard Wheatstone bridge in the respect that balance is obtained by varying the ratio r_1/r_2, instead of the resistance r (Fig. 80). This is accomplished by moving the contact b, Fig. 83, along a wire, ac, which forms the resistances r_1 and r_2. This wire should be uniform in cross-section and homogeneous so that the resistance per unit length will be constant. At exact balance (with n and n' short-circuited) the ratio of the lengths

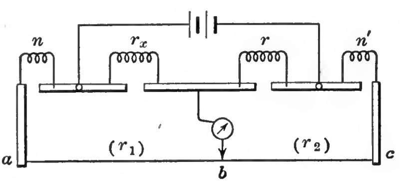


FIG. 83.

$$\frac{ab}{bc} = \frac{r_1}{r_2}$$

and

$$r_x = \frac{r_1}{r_2} \times r$$

as before.

The precision is a maximum when the sliding contact is at the center. When the slide wire is short, the precision decreases rapidly with settings toward either end. The length may be increased, in effect, by inserting equal resistances at n, n_1, which increases the sensitivity, but also decreases the permissible difference between r and r_x. In the actual instrument a long resistance wire is disposed of by winding it spirally on the circumference of a hard-rubber or marble disc, the contact being carried on a radial arm.

(*b*) *Carey-Foster Bridge.*—This type of bridge is particularly adapted to the comparison of low resistances. The distinctive feature is the elimination of the contact and other unknown

resistances which exist in the arrangement shown in Fig. 83, by taking two readings. The bridge is first balanced with the contact at x. The resistors r and r_x are next interchanged and a balance obtained at x_1. If the resistance of the slide wire per unit length is ρ, then,

$$r_x - r_1 = \rho(x_1 - x).$$

This method is especially suitable for comparison of standard resistors with each other and for temperature coefficient determinations.

(c) *Kelvin Double Bridge.*—The Kelvin double bridge[1] is also especially suitable for measurements of low resistances. It is the most generally used form, even in the most precise work, because of its accuracy and convenience. The principle is shown

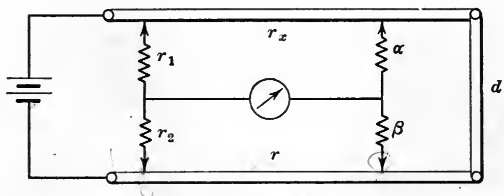


FIG. 84.

in Fig. 84. The bridge assumes the form of the Wheatstone bridge (Fig. 80) with the addition of an extra pair of ratio arms, α and β. When a balance is obtained:

$$\frac{r_x}{r} = \frac{r_1}{r_2} + \frac{d}{r}\frac{\beta}{\alpha + \beta + d}\left(\frac{r_1}{r_2} - \frac{\alpha}{\beta}\right).$$

If $r_1/r_2 = \alpha/\beta$, the expression following the addition sign (called the corrective term) becomes zero and $r_x = r_1/r_2$ as in the Wheatstone bridge. In practice, therefore, the ratio r_1/r_2 is kept as nearly equal to the ratio α/β as possible and the connecting resistance d is made as small as possible

If the connecting resistance, d, is removed, the net-work of conductors becomes a Wheatstone bridge in which, at balance

$$\frac{r_1}{r_2} = \frac{r_x + \alpha}{r + \beta}.$$

Therefore, a test for equality of the ratios r_1/r_2 and α/β may

[1] For discussion of the theory of the Kelvin double bridge see, "Methods of Measuring Electrical Resistance," E. F. NORTHRUP, p. 117.

be made as follows:[1] After a double-bridge balance has been obtained, remove the resistance d and if the bridge is still balanced, the ratio r_1/r_2 will be very nearly equal to the ratio α/β because r_1 and r will always be very low resistances compared with α and β and furthermore they are always in the same ratio. Consequently, it may generally be assumed without error that

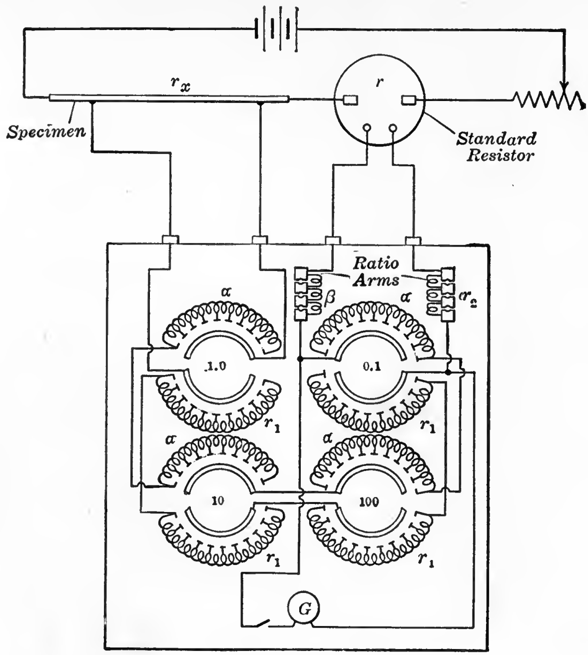


FIG. 85.

$r_1/r_2 = r_x/r$. If the bridge is not found balanced after removal of d, it may be balanced by shunting either α or β until exact balance is obtained.

Various other methods have been proposed for adjusting a double bridge in very precise measurements in order to eliminate the correction term. Wenner and Weibel[2] use a method which avoids removing the connecting resistance, d, a procedure which

[1] "Methods of Measuring Electrical Resistance," E. F. NORTHRUP, p. 120. Also Leeds and Northrup Co. catalogue No. 40, p. 22 (1911).

[2] "Adjustments of the Thomson Bridge in the Measurement of Very Low Resistances," F. WENNER and E. WEIBEL, *Bulletin*, Bureau of Standards, vol. 11, p. 65 (1914–1915). *Journal Wilmington Academy of Sciences*, Oct. 4, 1914.

is not always practicable especially when the resistances being compared and the resistance d are all very low.

There are two principal methods of balancing double bridges. In some forms, balance is obtained by adjusting the ratio r_1/r_2 (the ratio α/β being similarly adjusted at the same time) and in other forms both r and the ratio r_1/r_2 are adjusted. The first method is used in Wolff-Kelvin bridges (Fig. 85). The resistances r_2 and β are plugged in, in exactly equal amounts, while r_1 and α are adjusted by means of the four sliding-contact dials with 0.1-, 1-, 10- and 100-ohm coils respectively. Thus when the ratio r_1/r_2 is changed, the ratio α/β is automatically changed exactly the same amount. The standard, r, is a standard resistor external to the bridge proper as shown. The second method is used by the Leeds and Northrup Co. as shown in Fig. 86. In this case, a part of the standard, r, is a carefully cali-

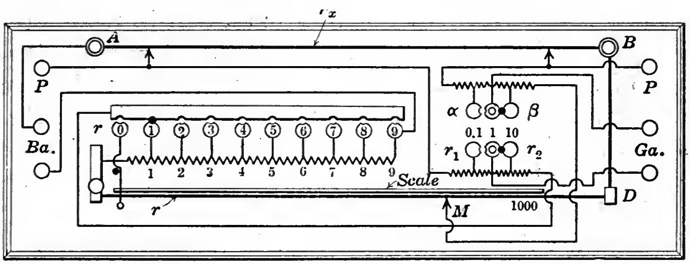


Fig. 86.

brated rod within the bridge and along which a contact, c, is moved to obtain the final adjustment, the preliminary adjustment being obtained by plugging in equal coils in r_1 and r_2. Holders A, B, are provided for wire specimens and special leads are connected to PP for irregularly shaped specimens.

146. Precautions To Be Observed when Using Bridges.—The contacts should be kept clean, especially where plugs are used. The use of unnecessary force on plugs wedges the contact blocks apart and results eventually in errors due to contact resistance. Where a "slide wire" is used, the contact should always be lifted before moving in order to avoid scraping the wire and thereby changing the resistance by reducing the cross-section. Where accurate results are required, the average of two readings with reversed currents should be used. The current in the bridge coils should not exceed the equivalent of 1 watt per coil.

In double-bridge measurements, the resistances of the four potential leads to r and r_x (Fig. 84) are included in r_1, r_2, α and β respectively. The resistance of these leads should, therefore, be as nearly equal as possible and as low as convenient in order to eliminate their effect.

Conductivity Measurements

147. General.—Conductance is the reciprocal of resistance, the property of a circuit which is obtained from the Ohm's law relation of current and potential. Conductivity is the reciprocal of resistivity or specific resistance, a property of the material composing the conductor as expressed by the relation

$$\rho = \frac{AR}{l}.$$

where ρ = resistivity or specific resistance, that is, resistance between opposite faces of a cube of the material, A = area of cross-section of the conductor (assumed uniform), R = resistance of conductor and l = length of conductor corresponding to the resistance R. The specific resistance will refer to an inch cube or a centimeter cube according as A and l are expressed in inches or centimeters.[1]

148. Relative Conductivity.—The relative conductivity of a conductor material is the ratio, expressed in per cent., of the conductivity (reciprocal of resistivity) of the material to that of a standard material at the same temperature. Stated in another way, the relative conductivity is the ratio of the resistivity of the standard to that of the material.

Relative conductivity may be expressed on two bases—mass conductivity and volume conductivity. Mass conductivity is the ratio of the resistances, at the same temperature, of specimens of the standard and of the material respectively having the same length and the same mass. Volume conductivity is the ratio of the resistances taken at the same temperature when the two specimens have the same length and the same uniform cross-section. Mass conductivity and volume resistivity are connected

[1] The official units as recognized by the A. I. E. E. are (*a*) the ohm-centimeter, the resistance in ohms between opposite faces of a centimeter cube and (*b*) the mho per centimeter, the conductance in mhos between opposite faces of a centimeter cube.

by density. Mass conductivity is, however, the more important and the one usually measured because conductor materials are usually bought and sold on a weight basis.

It is to be noted that the word "relative" is usually omitted, so that the term conductivity ordinarily refers to relative conductivity notwithstanding that conductivity is, strictly speaking, the reciprocal of resistivity.

149. Conductivity Standards.—Copper having a certain resistivity is practically universally used as the standard of reference when expressing the conductivity of a conductor.

The present[1] official standard of the A. I. E. E. is the "International Annealed Copper Standard" adopted by the International Electrotechnical Commission.[2] This standard is defined briefly as having at 20°C., a density of 8.89, a temperature coefficient of resistance at "constant mass" of 0.00393 and a resistance of 0.017241 ohm when in the form of a wire 1 meter long and 1 sq. mm. uniform cross-section.

From this definition the following resistivity values follow:

Mass Resistivity at 20°C.—

(*a*) Resistance of a wire of uniform cross-section, 1 meter long and weighing 1 gram = 0.15328 ohm.

(*b*) Resistance of a wire of uniform cross-section, 1 mile long and weighing 1 lb. = 875.20 ohms.

Volume Resistivity at 20°*C.*, 8.89 *Density.*—

(*a*) Resistance between opposite faces of a centimeter cube = 1.7241 microhms (1 microhm = 0.000001 ohm).

(*b*) Resistance between opposite faces of an inch cube = 0.67879 microhm.

(*c*) Resistance of a wire of uniform cross-section, 1 mil in diameter and 1 ft. long = 10.371 ohms.

The Matthiessen standard which is still used in many specifications is 0.14172 ohms at 0°C. for a wire of uniform cross-section 1 meter long and weighing 1 gram.

150. Conductivity by Calculation.—The conductivity of a specimen may be calculated from its dimensions, weight and resistance. This may be done either by comparing the resistance of

[1] For earlier conductivity standards including Matthiessen's Standard see "Copper Wire Tables," *Circular* No. 31, Bureau of Standards. Also "Standard Handbook for Electrical Engineers," 4th edition, p. 234, 1915.

[2] "International Standard of Resistance for Copper," International Electrotechnical Commission, *Publication* No. 28, March, 1914.

the specimen with that of a specimen of the standard copper of the same shape and volume or mass as prescribed in the definition of relative conductivity (par. 148). Or, which amounts to the same thing, the resistivity of the specimen may be calculated and compared with that of standard copper.

The conductivity or resistivity of irregular specimens cannot ordinarily be obtained directly. A part of the specimen at least must have a uniform cross-section of a shape whose area can be calculated. Usually the specimen can be machined to the form of a square or (preferably) round rod. Or, if an annular ring can be turned from the specimen, it can be cut radially at one point and the resistance measured between two points, the current being introduced at the two ends made by the slot. The resistivity of such a specimen is

$$K = \frac{Rt}{\theta} \log \left(1 + \frac{r_1 - r_2}{r_2}\right)$$

where R = measured resistance in ohms, t = thickness of the ring in the axial direction, r_1 = outside radius, r_2 = inside radius; t, r_1 and r_2 are in the same units. θ = angle in radians subtended by the path, the resistance of which was measured.

When $\frac{r_1 - r_2}{r_2}$ is less than 0.1, the computation may be made, with negligible error, as for a straight rod using the mean circumference as the length.

When the specimen is of a uniform cross-section but the shape is such that the area cannot be readily computed, such as that of a third rail for an electric railway, the area may be obtained from the weight of the specimen, its length and the density as determined from a small piece cut off of the specimen.

Where the resistance of the specimen is very low and the current being used is therefore relatively large, care should be taken to have the current distribution uniform throughout that portion of the specimen between the potential taps. Where the specimen is relatively short and straight, ends may be faced off perpendicular to the axis and the current introduced through copper plates clamped against these surfaces with a mercury amalgam between to insure complete contact. The potential taps should preferably be pieces of copper wire driven tightly into small holes drilled into the specimens, several such pairs of taps all exactly the same distance apart being set in the specimen if it is very

large. For example, in a specimen of a large third rail, there should be three (3) sets in the head, one set at the top and one set at the bottom of the web, and one set near each edge of the foot. A measurement is made between each pair of terminals and the average used in the calculation.

151. Conductivity by Direct Measurement.—In commercial measurements of electrical conductors, where speed combined with moderate accuracy is important, the conductivity is measured directly by comparison with carefully calibrated rod or wire standards. These so-called conductivity bridges all employ the principle of the Kelvin bridge.

In the Hoope's conductivity bridge[1] the conductivity of a specimen of wire is read directly from a scale graduated in per cent. The scheme is shown in Fig. 87 where r_x is the specimen to be measured, r_a standardized wire or rod and r_1, r_2, α and β are ratio coils. It will be seen that the arrangement is a regulation Kelvin double-bridge network of conductors. The four ratio coils are made exactly equal so that the ratio becomes unity and, at balance, the resistance between a and b on r_x is equal to the resistance between c and d on r. The ratio coils are about 300 ohms resistance in order to eliminate the effect of the resistance at the contacts a, b, c and d.

The test specimen, r_x, is cut exactly to a certain definite length (38 in.), accurately weighed and stretched taut between two clamps in the apparatus. The scales used with the particular standard corresponding to the diameter of this specimen are graduated in weights of 38-in. lengths in such a manner that with the contact d set at the weight of the test specimen, the resistance between c and d is that of a piece of wire of the same weight per inch as the test specimen but having 100 per cent. conductivity and a length equal to 100 parts on the scale I. The contact b (contact a being fixed at o on scale x) is shifted until balance is obtained when the conductivity is read directly from the scale x, 100 representing, of course, 100 per cent. conductivity.

One standard can be used with specimens ranging over three B. & S. gage diameters and with suitable standards the apparatus will measure wires from No. 0000 to No. 18 B. & S. gage.

In the method used by the Electrical Testing Laboratories for commercial conductivity measurements of electrical conductors,

[1] "A New Apparatus for Making Direct Measurement of Electrical Conductivity," Wm. A. Hoopes, *Electrical World and Engineer*, Nov. 14, 1903.

a standard Wolff form of Kelvin bridge is used, the carefully weighed specimen being compared with a standardized wire or rod as in a Hoope's bridge. The principle is as follows:

From the weight of a definite length of the test specimen (120 cm.), the length, l_1, is computed which would have a resistance just equal to that of a certain standard if the specimen had 100 per cent. conductivity. This length is computed as follows:

$$l_1 = \frac{W \times 100}{k}$$

where W = weight in grams of exactly 120 cm. of the test specimen, l = length in centimeters, k = constant $= \dfrac{0.15328 \times 1.2}{R_s}$; where 0.15328 = resistance in ohms of international standard

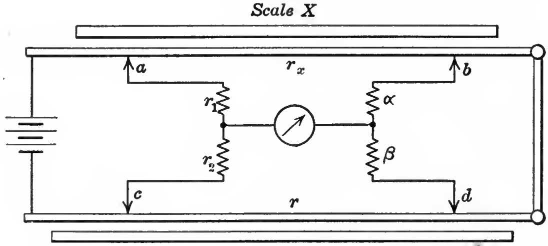


FIG. 87.

metergram, 1.2 = factor to correct from 120 cm. to 100 cm. and R_s = resistance of standard in ohms.

The standards are pieces of ordinary soft-drawn copper wire with tap wires silver-soldered to them about a meter apart. The resistance between the tap wires at 20°C. is carefully measured and the constant k computed once for all.

Having weighed 120 cm. of the specimen (120 cm. is used in order to allow sufficient wire for clamping), the length l is computed by the above formula and the contacts (corresponding to ab in Fig. 87) are set to include this length as measured on a scale alongside the specimen. The other contacts (corresponding to cd, Fig. 87) having been attached to the tap wires on the standard, the fixed ratio coils on the Kelvin bridge are set at 100 and the variable ratio arms adjusted until balance is obtained. With the standard and the unknown connected inverted to the bridge, the bridge will read conductivity directly in per cent.

One standard can be used with specimens from the same diameter to two B. & S. gages smaller. Theoretically, one standard can be used for all sizes of specimens but beyond the limits indicated, the length, *l*, becomes so short that the error in setting the contacts to the correct length becomes prohibitive.

In these conductivity bridges, care should be taken that the test specimen is in the cabinet inclosing the apparatus long enough to attain the temperature of the standard. The current should not be large enough to raise the temperature of the standard or the specimen, particularly if they are not the same size because then the heating would not be equal.

While the standard is usually of the same material as the specimen in order to eliminate the question of the temperature-resistance coefficient, it is obviously not necessary. Where they are not the same, proper correction must be made for the difference in density and in the temperature coefficient.

Earth Resistivity

152. In electrolysis investigations, the specific resistance of the earth in certain localities is sometimes important. Also, the specific resistance may be of value as giving some indication of the nature of the composition of the earth at a given locality whether it is moist, contains oil, or ores of relatively high conductivity, and so forth. Wenner[1] proposes a method which makes the determination of earth resistivity a comparatively simple matter, particularly as it does not involve disturbing the portion of the earth to be measured and measurements of large volumes at considerable depths can be readily made.

The method is briefly as follows: Four holes are made in the earth arranged in a straight line and uniformly spaced. All are the same depth, which is that at which information as to the resistivity is desired and the diameters of the holes are not more than 10 per cent. of the distance between them. In each hole an electrode is placed as indicated in Fig. 88 and so arranged that electrical contact is made only at the bottom as shown by the black areas in the figure. Such an arrangement constitutes a four-terminal conductor[2] in which the resistance depends upon

[1] "A Method of Measuring Earth Resistivity," Frank Wenner, *Bulletin*, Bureau of Standards, vol. 12, p. 469 (1915–16).

[2] For discussion of the four-terminal conductor see *Bulletin*, Bureau of Standards, vol. 8, p. 360 (1912).

the distance between the electrodes and the resistivity in the region having linear dimensions of the same order as the distance between the outside electrodes. By measuring the resistance between 2 and 3 the resistivity can be calculated from the depth

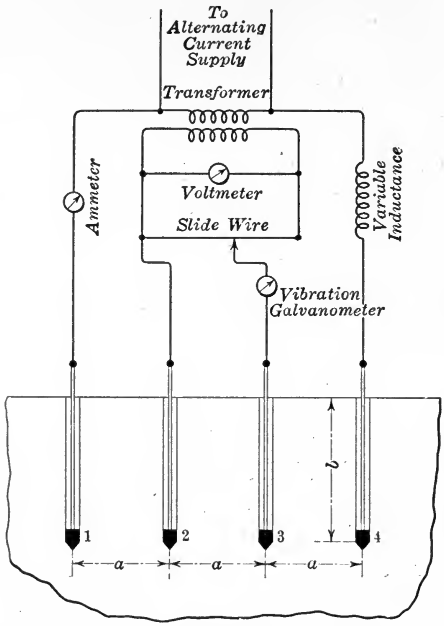


FIG. 88.

of the holes and the distance between them by means of the following formula:

$$\rho = \frac{4\pi aR}{1 + \dfrac{2a}{\sqrt{a^2 + 4b^2}} - \dfrac{2a}{\sqrt{4a^2 + b^2}}}$$

where ρ = volume resistivity (ohms per centimeter cube), R = measured resistance, a = distance between holes in centimeters, and b = depth of holes in centimeters.

The resistance, R, may, of course, be approximately determined (which is usually all that is required) by the fall of potential method with a continuous current flowing between terminals 1 and 4. Polarization and contact resistance difficulties may, however, be encountered and Wenner proposes the simple op-

position scheme using alternating current which is indicated in Fig. 88. The diagram is self-explanatory and from the ratio of the length of the slide, at which balance is obtained, to the whole, the resistance, *R*, can be calculated. The variable inductance (or a fixed inductance with a variable resistance in parallel) is used to bring the test current in phase with the potential drop. Obviously other detectors than a vibration galvanometer could be used (see Chapter 2.) With a high-frequency current (over 300 cycles) a telephone receiver could be used. Probably a telephone receiver would be sufficiently sensitive for an approximate determination at 60 cycles, particularly if the contact on the slide wire is rapidly made and broken.

Resistance of Rail Joints

153. General.—The testing of rail joints consists in determining, either, (*a*) the ratio of the resistance of a given length of rail, including a bonded joint, to that of the same length of continuous rail; or (*b*) the length of solid rail which has the same resistance as the joint. The resistance of rail bonds is usually expressed in the latter manner, whether measured in that way or as a ratio (*a*). The three principal methods employed are millivoltmeter, bridge and opposition.

154. Millivoltmeter Method.—In the millivoltmeter method, simultaneous readings are taken with 2 milliovoltmeters, one connected across the bond and the other across a definite length of rail. If the current fluctuations are not too rapid, only one instrument is necessary, provided there is a suitable arrangement of switches to change the connections in quick succession.

155. Bridge Method.—The Roller bond tester utilizes the principle of a slide-wire form of Wheatsone bridge (par. 144). The scheme is indicated in Fig. 89. Balance is obtained by moving the contact *b* back and forth. When balance is obtained

$$\frac{ab}{bc} = \frac{r_1 + m}{r_2 + n}$$

where ab = resistance of bond and bc = resistance of the standard length of rail. The resistances r_1 and r_2 have the effect of extending the slide wire and providing greater accuracy. In the actual instrument, the slide wire takes the form of a circle

and the scale is graduated to give the resistance directly in terms of the number of feet of the solid rail being tested.

156. Opposition Method.—The Conant bond tester is an example of the class in which the drop across the joint is opposed to that across a length of solid rail, the outer contact on the latter (corresponding to *c* in Fig. 89) being moved until the two potentials are just equal and opposite. The detector is a telephone receiver in series with a make- and break-device operated by a clock.

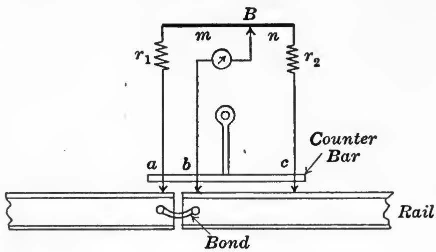


FIG. 89.

INSULATION RESISTANCE MEASUREMENTS

157. General.—The resistance of insulator and insulating materials is usually measured by either of two general methods, the direct-deflection method and the leakage or loss-of-charge method. In both methods the deflection of an indicating instrument is the measure of the resistance as distinguished from the "zero" methods which are usually employed to measure the resistance of conductors. For this reason, the accuracy usually attainable in the measurement of insulation resistance is much less than in the case of conductors. However, the true resistance of most insulating materials, especially organic insulations, is indefinite and the apparent resistance varies with so many conditions that only a more or less approximate value can be assigned. Where the resistance is high, the electrostatic capacitance also complicates the measurement.

158. Direct-deflection Method.—This method is simply an application of Ohm's law, the current being measured which a known potential will cause to flow through the specimen. When the resistance is of the order of one megohm, an ordinary voltmeter, used as an ammeter, will give results which are sufficiently accurate for most purposes. Two readings are taken,

one with the voltmeter directly across the battery or generator, and the other with the resistance to be measured connected in series with the voltmeter. The resistance is

$$R = r_v \times \frac{d - d_1}{d_1}$$

where $r_v =$ resistance of voltmeter (the greater the resistance per volt, the higher the precision), $d =$ deflection of voltmeter in first reading, $d_1 =$ deflection in second reading. Obviously, a portable galvanometer with series resistance may be used as a voltmeter.

When the resistance is moderately high, a high-resistance reflecting D'Arsonval galvanometer is employed. Fig. 94 shows the diagrammatic arrangement for measuring the insulation of a cable. The measurement is made as follows: after the galvanometer shunt, S, is set at the highest value and R_s is short-circuited, the main circuit is closed. The shunt is then decreased until the largest readable deflection is obtained. A reading is taken after 1 min. The galvanometer is then calibrated by repeating this procedure with the standard resistor r_s (usually 0.1 or 1 megohm) in circuit and the specimen short-circuited. The resistance of the specimen in megohms is

$$R = \frac{G}{d_1 S} \qquad \text{(megohms)}$$

where $d_1 =$ first reading, $S =$ multiplier corresponding to the shunt setting (that is 1, 10, 100, 1,000, or 10,000) and $G =$ galvanometer megohm-constant (par. 12) as obtained from the second measurement. The constant, G, $= dsr_s$; where $d =$ deflection, $s =$ shunt multiplier, $r_s =$ standard resistance in megohms.

159. Leakage or Loss-of-charge Method.—This method is the most suitable one for the measurement of very high resistances such as the resistance of porcelain, glass and similar materials or the surface resistance of such materials.

The leakage method is based on the fact that if the insulation resistance of a condenser is infinite, it will retain a charge indefinitely; whereas if the resistance between the condenser terminals (either the internal resistance or a resistance connected externally) is finite, the rate of loss of the charge (or leakage) will be a measure of that resistance. The principle of the method is

shown in Fig. 90 where the resistance to be measured, r, is connected in parallel with a condenser C. Key a is closed and immediately opened, thus charging the condenser. Key b is closed immediately after a is opened and the deflection, d_1, of the ballistic galvanometer noted. The process is repeated, a being left open a definite time, t sec., before b is closed and a second deflection d_2 observed. The resistance is then, assuming the condenser resistance is infinite.

$$r = \frac{t}{2.303C \log_{10}\left(\frac{d_1}{d_2}\right)} \quad \text{(megohms)}$$

where C is the capacitance of the condenser in microfarads.

The insulation resistance of the condenser is usually not infinite and a correction has to be made. The resistance of the condenser is measured in a similar manner with r disconnected. If r_2 is the resistance of the condenser and r_1 is the resistance obtained in the above measurement where r and r_2 are in parallel, the corrected value is, from the law of parallel circuits.

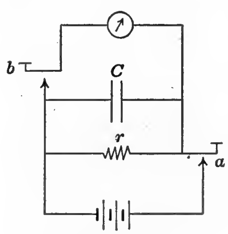


FIG. 90.

$$r = \frac{r_1 r_2}{r_2 - r_1} \quad \text{(megohms)}$$

For greatest precision, the time t should be such that the second deflection d_2 is about one-half to one-third of d_1.[1] In practice a special key is used in which a and b are combined so that they may be operated in rapid succession. The condenser should be of high grade, preferably with mica insulation, and as free as possible from "absorption" characteristics (see par. 316).

160. Specific Resistance of Insulating Materials.—The resistivity of solid materials is calculated from the resistance of a specimen of known and uniform thickness measured between two similar metallic electrodes of known area in intimate contact with the opposite and parallel faces of a specimen of the material as indicated in Fig. 91. Tin foil makes convenient and satisfactory electrodes if backed with blotting paper and sufficient weight to insure good contact. Sheet insulating materials can

[1] For discussion of the relation of capacitance and the time for greatest precision, see "Methods of Measuring Electrical Resistance," E. F. NORTHRUP, p. 172.

be measured in a similar manner, using brass discs or tin-foil electrodes.

The resistivity of liquid insulating materials may be determined with an accuracy sufficient for commerical purposes by pouring a specimen into a round glass cylinder or graduate of known internal area in which two circular, closely fitting disc electrodes are supported. One of the electrodes should be movable so that

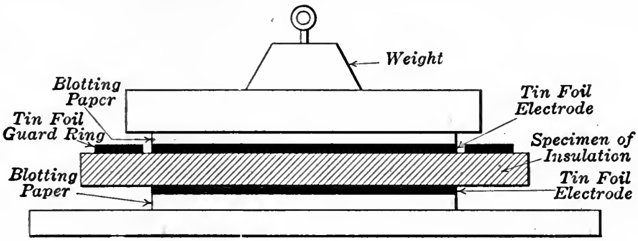


FIG. 91.

the resistance of columns of several different measured lengths can be measured. The first measurement should be taken as the zero or base reading and the results checked by calculation of the increase in resistance and the corresponding increase in the spacing of the electrodes at different settings. The apparatus devised by H. W. Fisher for capacitance measurements (par. 318) is convenient where many such measurements have to be made.

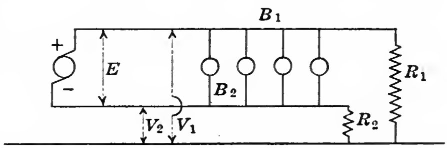


FIG. 92.

161. Insulation Resistance of Electrical Circuits.—The insulation resistance of a "dead" circuit can usually be most conveniently measured by the voltmeter method previously described or with one of the portable instruments described below.

When the circuit is "alive" the following method may be used.[1] Fig. 92 represents diagrammatically a system with lamps and motors connected. The resistances R_1 and R_2 which represent

[1] "Methods of Measuring Electrical Resistance," E. F. NORTHRUP, p. 210.

the insulation resistance from the positive and negative sides respectively, to ground are:

$$R_1 = \frac{R\,(D - d_1 - d_2)}{d_2} \text{ and } R_2 = \frac{R\,(D - d_1 - d_2)}{d_1} \quad \text{(ohms)}$$

where R = resistance of voltmeter in ohms; D = deflection (scale divisions) corresponding to circuit voltage, E; d_1 = deflection corresponding to V_1; d_2 = deflection corresponding to V_2. Incidentally the total leakage current to ground is

$$I = \frac{E}{(R_1 + R_2)} \quad \text{(amps)}$$

If the system is supplying continuous current, a D'Arsonval-type voltmeter is preferable if alternating current, an electrodynamometer-type instrument should be used. This method in general will measure 1 megohm with sufficient accuracy to check specifications. If the resistance is over 1 megohm, a galvanometer method will obviously be more accurate.

162. Precautions in Insulation Measurements.—Where the specimen has considerable capacitance and particularly where this capacitance absorbs a charge slowly (which is usually the case with long lengths of wire and cable) the time required for the deflection to become constant is more or less indefinite. Also, in such cases, the deflection obtained will often vary with the potential employed. In other words, the apparent resistance which is obtained will depend upon the time of application of the potential used and upon its value. The effect of time is shown by the curve in Fig. 93 which was plotted from galvanometer deflections and the time of application of a constant potential to a length of wire being measured for insulation resistance. It is therefore customary, when reporting insulation resistance measurements, to state the potential used and the time of application of the potential. The usual values are 100 to 500 volts and 1 min. respectively.

Where the conditions just described exist, it is apparent that care must be taken that the specimen is thoroughly free from electricity or is in a neutral state before testing. This is not always completely accomplished by merely short-circuiting the ends of the wire or cable or the two electrodes, for where absorption takes place to a considerable extent, a residual charge may be retained. In such cases it is desirable to reverse the

applied potential a number of times at as high rate as possible, beginning at a low rate of reversals. However, even after the above treatment, the deflections with direct and reversed current may not be equal. This does not necessarily mean that the specimen was not "neutral" because this condition will be encountered where the specimen is so small that the capacitance must be nil. However, it is usually only found in low-resistance insulators

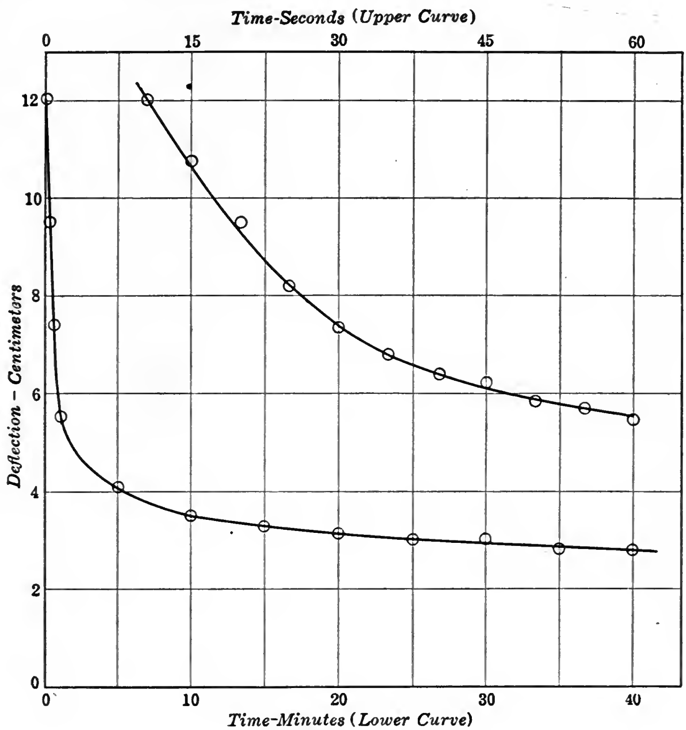


FIG. 93.

containing organic materials or traces of water or other liquid. In such cases the resistance corresponding to both deflections should be reported. The Standardization Rules of the A. I. E. E. (October, 1916) state that in insulation resistance tests of wire and cable, the conductor should be maintained negative with respect to the sheath or water in which it is immersed.

Leakage over the surface of specimens is a source of much trouble in damp weather. In wires and cables, the lead or braid

should be removed for 2 or 3 in. from the ends and the exposed insulation coated with hot, clean paraffine; or, just before measuring, these prepared ends may be carefully dried with an alcohol, Bunsen or other flame free from carbon. As a further precaution, a "guard" circuit may be arranged as shown by the dotted lines in Fig. 94. This consists of a few turns of fine copper wire twisted around the insulation close to the copper conductor and connected to the battery side of the galvanometer. In the case of solid specimens, the twisted wire is replaced with a ring of tin foil as shown in Fig. 91.

The side of the circuit which contains the galvanometer should be well insulated throughout. The battery should also be insulated as thoroughly as possible; this is a relatively easy matter when dry cells are used. The important point is to insure that all current passing through the specimen, and only that current,

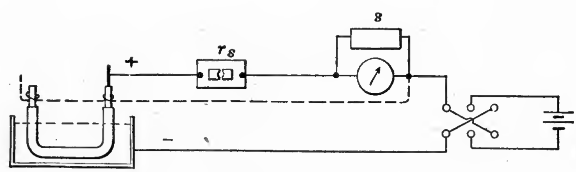


FIG. 94.

passes also through the galvanometer. The galvanometer, should, preferably, have a high resistance (order of 1,000 ohms) and a megohm sensitivity (par. 12) of several hundred megohms. The temperature should always be noted, because of the large coefficient which most insulating materials have.

RESISTANCE OF ELECTROLYTES

163. General.—The measurement of the resistance of electrolytes involves difficulties not encountered in metallic resistances due to the counter e.m.f. of polarization which develops when a continuous current is passed through an electrolyte. It is therefore necessary to use a rapidly reversing current and the several methods in use are all based on the scheme first proposed by Kohlrausch which employed a simple bridge arrangement with a rapidly reversed current.

A standard Wheatstone bridge may be used, but a slide-wire type is more convenient, especially when the slide wire is a long

one wound spirally on a marble cylinder. When such slide-wire resistance is a separate piece of apparatus, a bridge may be easily made up and used as indicated in Fig. 95 where R is a non-inductive box and r_x the electrolyte. If the slide-wire scale is divided into 1,000 parts, the resistance of the electrolyte is, at balance,

$$r_x = \left\{ \frac{a}{(1{,}000 - a)} \right\} R. \qquad \text{(ohms)}$$

When the source of energy is a battery or a continuous current generator, the necessary reversals may be obtained by using a secohmmeter with a D'Arsonval galvanometer as the detector. A secohmmeter is an apparatus consisting of two commutators on a common shaft which is driven by a small motor. One commutator is in the supply circuit and the other is in the galvanometer circuit, the brushes being so arranged and the circuits so connected, that the current to the bridge is rapidly reversed while current to the galvanometer is simultaneously rectified. A secohmmeter is more or less troublesome to use. A motor is required to operate it, the contact resistances at the commutators may be more or less variable and thermo e.m.fs. may be developed at the brushes due to heating.

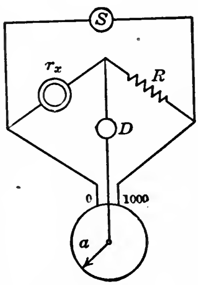


Fig. 95.

An induction coil with an interrupter is a more convenient method of providing the current. The secondary is connected to the bridge and a telephone receiver is used as a detector. However, due to the harmonics existing in an induction-coil current, it is not ordinarily possible to obtain absolute silence in the receiver.

The current from a Vreeland oscillator (par. 293) is admirably adapted to this class of measurements because, being a pure sine wave, it is free from harmonics and the frequency can be adjusted to the range of highest telephone sensitivity, viz., 800 to 1,000 cycles per second.

If the source is alternating-current energy of commercial frequencies, an alternating-current galvanometer (electrodynamometer instrument) may be used, the fixed coils being connected in series between the source, S, and the bridge and the moving coil in place of D (Fig. 95).

164. Specific and Relative Resistance of Electrolytes.—When the specific resistance or resistivity is required, a column of the liquid of known dimensions must be isolated. Fig. 96 shows a satisfactory method.[1] The glass tube is about 20 cm. long, 1 cm. internal diameter and open at both ends. The electrodes are of gold or platinum, the lower one being fixed in position and perforated, while the upper one is adjustable. The average cross-section must of course be carefully determined, preferably by a volumetric measurement with mercury. The temperature is

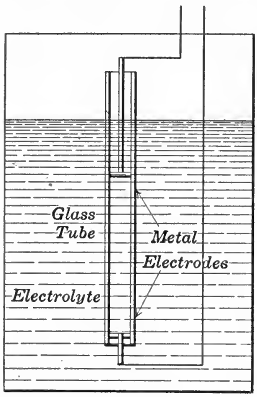


FIG. 96.

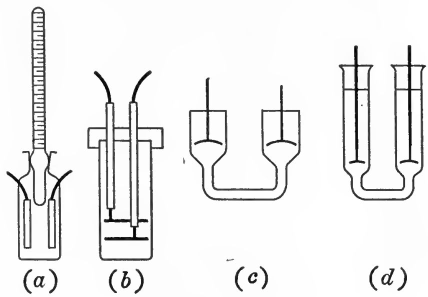


FIG. 97.

readily kept constant by stirring the liquid in the containing vessel.

In many cases only relative values are required, that is, the resistance in terms of another electrolyte as a standard. In such cases, each electrolyte is measured in the same container, the dimensions and the shape being unimportant. Fig. 97 shows a number of cells, *a* and *b* being adapted to poor conductors and *c* and *d* to good conductors.

INTERNAL RESISTANCE OF BATTERIES

165. General.—The resistance of a battery is, relatively, an uncertain quantity because it varies with so many conditions—current flowing, temperature, previous history and, if a storage battery, the state of charge or discharge. The resistance of a

[1] "Methods of Measuring Electrical Resistance," E. F. NORTHRUP, p. 241.

storage battery is abnormally high immediately after charging because of the gas in the plates and on the surfaces, hence such a battery should always be partially discharged, when measuring the resistance. The very rapid polarization which takes place also introduces a variable and measurements have to be made quickly.

Measurements may be made with either continuous current or alternating current, but, in general, continuous-current methods should be used only with primary batteries of very low resistance and with secondary or storage batteries. Alternating-current methods are more reliable.

166. With Continuous Current.—A simple continuous-current method is as follows: The e.m.f. of the cell or battery is first measured on open circuit. The circuit is then closed through a known resistance and the e.m.f. measured again, care being taken to do so quickly before polarization begins. The resistance is

$$R_x = \frac{R(E - E_1)}{E_1} \qquad \text{(ohms)}$$

where R = known resistance, E and E_1 = voltage before and after closing circuit, respectively. If R is unknown, the current may be measured and its reciprocal substituted for R/E_1 in the formula.

This method assumes that the internal resistance remains constant under all conditions, which is generally not the case, especially in dry cells. In a modification of this method, both readings are taken with the circuit closed, but with slightly different values of R. Then

$$R_x = \frac{(E_1 - E_2)R_1R_2}{E_2R_1 - E_1R_2} \qquad \text{(ohms).}$$

where R_1 and E_1 are the first resistance and e.m.f. respectively; and R_2 and E_2 are the corresponding values with the second resistance. The current may be measured instead of the e.m.f. and then

$$R_x = \frac{E_1 - E_2}{I_2 - I_1} \qquad \text{(ohms).}$$

167. With Alternating Current.—The Kohlrausch-bridge arrangement shown in Fig. 95 may be used with alternating current by inserting the cell or battery in place of the electrolyte cell.

The apparent resistance of a cell varies with the current flowing and therefore measurements should be made with a definite and known current. This is readily taken care of by connecting an ammeter in series with the cell and a variable resistance, R', in parallel with the cell and the ammeter. Thus the measurement can be made with the cell discharging at any rate.

Without the resistance R' connected, the resistance of the cell will be

$$r_x = \left(\frac{a}{1{,}000 - a}\right) R \qquad \text{(ohms).}$$

If the resistance R' is connected, the resistance of the cell will be

$$r_x = \frac{aR'R}{(1{,}000 - a)R - R_a} \qquad \text{(ohms).}$$

168. Direct-reading Instruments for Measuring Resistance.—These instruments, called ohmmeters, are very useful where many measurements have to be made rapidly and with only a moderate degree of accuracy. They are of two classes. In one class the principle of the slide-wire bridge is usually employed and in the other class the resistance is indicated by a pointer moving over a calibrated scale.

169. Bridge Instruments.—In the bridge type of instruments, which is suitable only for moderate resistances, the "rheostat" arm (r_2, Fig. 80) is a fixed resistance and the ratio of the "ratio" arms is adjusted until balance is obtained, instead of adjusting the resistance of the "rheostat" arm as in the ordinary bridge. A long wire forms the two ratio arms, the ratio depending upon the position of the movable contact. Obviously, with a fixed value for r_2 (Fig. 80) a scale at the side of the wire, $r + r_1$, can be so calibrated that the position of the contact indicates the resistance, r_x, directly in ohms. In some forms of this type of instrument, the wire is wound spirally on the edge of a disc around which the contact is rotated. In other forms, the wire is simply stretched between two posts and contact is made with a metal stylus held in the hand. This latter form permits the use of a telephone receiver as a detector, balance being indicated by the absence of the "click" which is heard upon touching the wire. Also, the range of the instrument is easily extended by having additional wires each with its appropriate scale. Obviously where the resistances to be measured are nearly alike, the slide

wire can be so selected that a short range and therefore relatively high precision is obtained.

170. Pointer Instruments.—It is obvious that any current-measuring instrument in series with the resistance to be measured and with a fixed potential may be calibrated in terms of resistance and thus become an ohmmeter. Thus an ordinary 100-volt voltmeter having 10,000 ohms resistance could be calibrated to indicate directly resistances ranging from a few hundred ohms to 200,000 or 300,000 ohms with a potential of 100 volts.

The Evershed "Megger"[1] is an instrument in which the measuring current is produced by a small generator and the resistance is indicated by means of an application of a differential-galvanometer principle. It is primarily applicable to high resistances only. The scheme employed is indicated in Fig. 98 where A is a coil in series with the resistance to be measured and B, B_1 are coils, which, with the resistance, R, are connected to a hand-driven generator D. All three coils are rigidly coupled together and are connected to a circuit by fine copper strips which exert no controlling force. When the generator is actuated, a current flows through coils, B, B_1 proportional to the e.m.f generated. If the external circuit is open, B and B_1 are deflected to the position where the least flux from the permanent magnets MM_1 will intersect them, that is, opposite the gap in the C-shaped iron piece about which the coils A and B_1 move. The pointer then stands at "infinity" on the scale. If now a finite resistance is connected across the terminals, the current flowing in A will produce a deflecting torque toward the position shown in the figure, and, as the system moves, the coils B and B_1 exert an opposing torque of constantly increasing magnitude. Hence the system comes to rest at a point where the two forces are balanced, the position depending upon the amount of the external resistance under measurement. The nominal speed is 100 r.p.m., but the indication is obviously independent of the speed. Where the

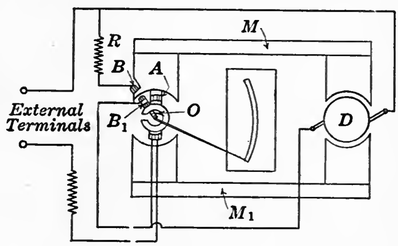


Fig. 98.

[1] *Circular* No. 740, J. G. Biddle, Philadelphia, Pa., 1910.

e.m.f. must be constant, a friction-clutch arrangement is provided which prevents reaching a speed over 100 r.p.m. The instrument is furnished with various voltages from 100 to 1,000, with corresponding resistance ranges of 10 to 2,000 megohms.

While as indicated, this instrument is normally designed for high-resistance measurements, a form is made which in conjunction with an auxiliary resistance box, can be used to measure moderate resistances up to several hundred thousand ohms as well as high resistances up to 40 megohms.

Resistances in Alternating-current Circuits

171. General.—The passage of alternating current through a circuit is opposed by the ohmic resistance, by the equivalent resistance of all the energy losses other than the loss due to ohmic resistance, and by the reactance.

172. Ohmic Resistance.—The ohmic resistance is the resistance to the passage of continuous current and is, therefore, measured with continuous current by the methods previously described.

173. Effective or Alternating-current Resistance.—This resistance is the value which is obtained when the power lost in a circuit is divided by the square of the current. It includes in addition to the ohmic resistance, the effect of any other source of lost energy, such as iron losses in a magnetic circuit, dielectric losses, and induced currents in a neighboring circuit. This effective resistance produces a potential drop which is in phase with the current.

The simplest method of determining the effective resistance is from the relation: $W = I^2R$, where W = watts measured with a wattmeter, I = current (amperes) and R = effective resistance (ohms).

When the power is small and the power-factor is low, a separately excited electrodynamometer may be used. The exciting current must be in phase with the current through the resistance to be measured in order to insure that the reading obtained when the moving coil is connected across the resistance is the component in phase with the current, that is, the effective resistance. This condition can be established in several ways: (*a*) By connecting the fixed coils across a non-inductive resistance which is in series with the resistance to be measured; (*b*) by connecting the fixed coils to a phase shifter (par. 240), and the moving coil

to a non-inductive resistance in series with the resistance to be measured. The phase shifter is then adjusted until a maximum deflection is obtained. A more sensitive method is to connect a condenser in series with the fixed coils and adjust the phase shifter for zero deflection. The dynamometer can be calibrated on a non-inductive resistance or on continuous current.

Wheatstone-bridge methods of measuring effective resistance may also be used by providing facilities for obtaining a balance for both resistance and inductance. The two "ratio" arms should be non-inductive, and the "rheostat" arm should contain both a variable resistance and a variable inductance so that complete balance may be obtained. The detector must indicate both states of balance and may be a separately excited electrodynamometer or a synchronously driven commutator or reversing key (par. 38). When using a dynamometer instrument, a resistance balance is obtained with the fixed coils excited from the same circuit, that is, in series with the bridge or across a shunt. Inductance balance is obtained with the fixed coils excited from a circuit 90° from the first, the moving coils being connected across the bridge in the usual manner. An Anderson-bridge (see par. 294) arrangement with a vibration galvanometer as the detector may also be used.[1]

174. Reactance and Impedance.—Where the constants of the circuit are not known, the reactance and impedance of an alternating-current circuit are most conveniently obtained by calculation from measurements made with an ammeter, a voltmeter and a wattmeter in accordance with the following relations:

$$(a)\ X = \frac{W}{I^2}\tan\theta$$

$$(b)\ X = Z\sin\theta$$

$$(c)\ Z = \frac{E}{I}$$

$$(d)\ Z = \frac{W}{I^2\cos\theta}$$

where X = reactance in ohms, Z = impedance in ohms, W = power in watts, I = current in amperes, E = total potential drop in circuit in volts, and $\cos\theta$ = power factor = W/EI.

The separately excited electrodynamometer with a phase

[1] "Effective Resistance and Inductance of Iron and Bimetallic Wires," JOHN M. MILLER, *Bulletin*, Bureau of Standards, vol. 12, p. 207 (1915–1916).

shifter provides a convenient means of making accurate measurements of any component in an alternating-current voltage. The fixed coils are excited from the phase shifter and the moving coils are connected to the circuit to be measured. Adjusting the phase shifter until maximum deflection is obtained gives a deflection proportional to the impedance drop from which the impedance can be calculated if the current in the circuit is known and the dynamometer has been calibrated. The dynamometer may be calibrated on continuous current or by connecting the movable coil across a non-inductive shunt in the circuit being measured. In the latter case, if the current is the same as in the measurement, the calibration is obtained directly in ohms. The excitation must, of course, be kept at the same value.

The reactance can be similarly calculated from the reactance or "90°" drop; this is obtained by exciting with a current 90° in phase from the current in the circuit. The latter relation can be established by connecting a condenser in series with the moving coil and across the non-inductive shunt referred to above, and then adjusting the phase shifter until zero deflection is obtained. If the moving coil is then transferred to the circuit to be measured, the deflection will be proportional to the reactance drop.

CHAPTER VIII

POWER MEASUREMENTS

175. General.—Power is the rate of expending energy. When a quantity of electricity, q, is passed through a circuit against a constant difference of potential, e, the work done, that is the amount of electrical energy expended, is qe.

The instantaneous power, or power at any instant during the transfer of this quantity, is

$$e\frac{dq}{dt} = ei$$

where i and e are the instantaneous values of current and potential differences respectively.

The unit of average power, or simply power as it is usually expressed, is the watt which is a joule of energy expended in 1 sec. Power expressed in watts is, therefore,

$$W = \frac{QE}{t}$$

where Q = quantity of electricity in coulombs, E = difference of potential in volts and t = time in seconds.

175a. Continuous-current Power.—In a circuit supplied by a battery or a continuous-current generator, energy is expended at a uniform rate; hence,

$$e\frac{dq}{dt} = \frac{QE}{t} = IE$$

where Q = quantity in coulombs, E = e.m.f. in volts, t = time in seconds and I = current in amperes.

176. Alternating-current Power.—In an alternating-current circuit, the power at any instant, is, as stated above, the product of the current and the potential at that instant. When the load consists only of resistance, the current wave, I, and the potential wave, E, are in phase as shown in Fig. 99. If the products of the instantaneous values of current and potential are plotted, the curve P is obtained. The average value of this curve is the power equivalent of a continuous current producing the same effect. Furthermore, this power, W, is equal to the product

EI, where E = mean effective volts and I = mean effective amperes. These values of potential and current are indicated by instruments in which the deflections are proportional to the square of the current.

When the circuit contains inductance or capacitance (or the equivalent), the current and the potential will not be in phase. In the case of an inductive load, the current will lag behind the potential, as shown in Fig. 100. The power curve, P, then will not be all on one side of the axis, but a part will be negative as indicated. If the current lags sufficiently, Fig. 101, the power curve will be positive half of the time and negative the other half; the average power will then be zero. This difference in phase, or time relation between the current and the potential, is called the phase angle and is usually expressed in degrees, an entire cycle being 360°. If the current and the voltage wave forms are sinusoidal, the average value of the power is

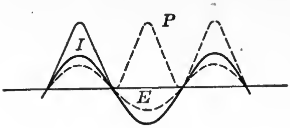


FIG. 99.

$$W = EI \cos \theta$$

where θ is the phase angle.

The power-factor of an alternating-current circuit is the ratio

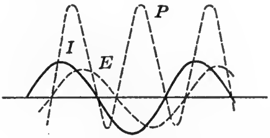


FIG. 100.

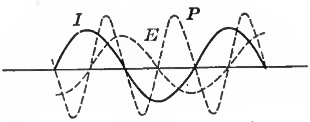


FIG. 101.

of watts to "volt-amperes," or W/EI. When the current and potential wave forms are sinusoidal, the power-factor is also obviously equal to $\cos \theta$. But when either or both current and potential wave forms are not sinusoidal the phase angle, θ, as obtained from the relation $\cos \theta = W/EI$, has no significance.

177. Measurement of Power.—Direct measurements of power are made with instruments (wattmeters) which indicate directly the average of the products of the instantaneous values.

In continuous-current circuits, the power is usually determined from the product of the current and potential measured simultaneously. Wattmeters may be used but they are less accurate

than the permanent-magnet type of instruments that would be used for current and potential measurements.

Alternating-current power is most directly and accurately measured with wattmeters. Where the power is unidirectional but pulsating, as in a rectifier circuit, the power consumption of a storage battery or a motor may be approximately measured with a voltmeter and an ammeter of the permanent-magnet type. Such instruments would give a more nearly correct result than dynamometer instruments. On the other hand, the reverse will be the case with a load of incandescent lamps or heating devices. In both cases, the error will depend upon the wave shape and the character of the load. The true power will, however, always be shown by a wattmeter.

178. Precision Measurements.—The unit of power is the watt which is equal to the product of 1 ampere and 1 volt. The measurement of power means, therefore, fundamentally a comparison with the standard ohm and the standard cell, consequently precision measurements must be made with an instrument which is equally accurate on continuous and alternating currents in order that it may be calibrated on continuous current. Such measurements are most accurately made with reflecting two-circuit electrodynamometers. They are similar to those used in current and e.m.f. measurements except that the fixed coils are wound with relatively heavy wire and are connected in series with the circuit being measured, while the moving coil is of fine wire and connected across the circuit. Great care is used in the construction. Eddy currents in the windings or framework would produce distorting fields; they are eliminated by using braided strands of fine wire in the fixed coils and nonmetallic materials in the framework. The inductance of the moving coil is made as small as possible. The effect of external magnetic fields is eliminated by connecting the moving coils, or the fixed coils, or both, astatically. The fixed coils are often divided into several sections which may be connected in various series and parallel combinations to give large deflections over a wide range of power intensities. Instruments of this type made by the General Electric Co. have current capacities from 5 to 125 amp. and above, with corresponding sensitivities as high as 10 and 200 watts respectively at full scale deflection, with a 50-cm. (19.7 in.) scale at 100 cm. (39.4 in.) distance.

Reflecting instruments of this type are used for the most pre-

cise measurements. For secondary standards, there are available a number of semiportable indicating instruments employing this principle and although having a lower order of precision and sensitivity, they are quite sufficient for most commercial requirements. Typical of this class of instruments are the Westinghouse and Duddell-Mather standard wattmeters.

The Westinghouse "precision" wattmeter is an electrodynamometer instrument and is similar to the corresponding ammeter (par. 122), except that the moving coils are wound with fine wire and are connected across the line instead of in series with the fixed coil. There are usually two or three current ranges, and the series resistance for the moving coil is mounted in a separate box. It can, therefore, be used with a wide range of voltages from 10 volts up. Although considerable metal is used in the construction of this instrument, it is so arranged that the eddy-current error is not appreciable at commercial frequencies.

The Duddell-Mather type of wattmeter as made by R. W. Paul is also an electrodynamometer instrument. It is a carefully constructed semiportable, secondary standard in which the current element comprises four fixed coils connected in series, and the potential or moving element consists of four coils connected in series, all astatically arranged. The moving element is suspended by a silk fiber and a spiral spring furnishes the controlling force. It is a torsion-head instrument like the Westinghouse wattmeter. The fixed winding is stranded and braided, and is subdivided into ten sections which may be connected in various series and parallel combinations by means of a plug box. The series resistance for the moving coil is external to the instrument. The ranges may be varied from 10 volts and 1 amp. to any voltage and 100 amp. No metal whatever, other than that in the winding and the spring, is used in the instrument.

Wattmeters

179. General.—Practically all wattmeters used in all ordinary engineering measurements, both portable and switchboard forms may be classified as one of two types—electrodynamometer or induction.

180. Electrodynamometer Type Instruments.—Weston, model 16, and the General Electric, model P_3, instruments are well-known examples of this class. Fig. 102 shows the general arrangement of the circuits. The current or series element consists of

two fixed coils wound with heavy wire or strip, which are connected in series with each other and with the main circuit. The potential or shunt element is a moving coil mounted on a shaft supported between jewel bearings and placed between the two fixed coils. This coil consists of a large number of turns of fine wire, as in voltmeters; it is connected in series with a relatively large amount of non-inductive resistance, across the main circuit. The controlling force comprises one or more spiral springs.

The General Electric Co. inclined-coil instrument is similar in principle, but the center lines of the fixed coils and the moving coil make an angle of about 45° with each other, instead of 90°, the object being to make the scale more open, that is, more uniformly graduated.

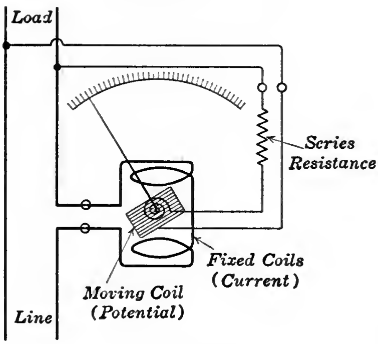


FIG. 102.

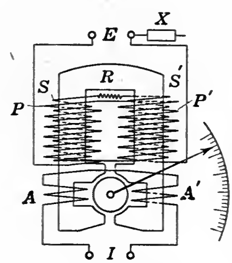


FIG. 103.

The Whitney wattmeter also operates on the electrodynamometer principle, but is a torsion-head instrument, the moving element being kept in a fixed position by twisting the torsion head to which the control spring is attached. The pointer attached to this head moves over the scale. This arrangement permits using a very long scale, extending practically around a full circumference.

181. Induction-type Instruments.—The Westinghouse portable wattmeter is the most important example of induction-type wattmeters. The principle is exactly the same as that of the induction watt-hour meter (par. 222), but instead of allowing the moving element to rotate, the torque is opposed by a spiral spring and hence the deflection is proportional to the power. Fig. 103 shows the schematic arrangement, where AA are the current or series coils, PP the potential or shunt coils,

and *SS* the compensation coils by means of which, together with the adjustable resistance, *R*, the exact quadrature relation is obtained as in watt-hour meters. This type is also made in polyphase form by having two sets of current and potential elements acting on a common moving disc, or drum, as in the polyphase watt-hour meter. It is obvious that instruments of this type are limited to the frequency for which they are designed.

182. Calibration.—Wattmeters of the electrodynamometer type are calibrated with continuous current, taking the average of direct and reversed readings. It is customary to make such tests at a fixed potential, usually 100 or 200 volts, and vary the current to give the required watts. The potential is held constant at some convenient value by means of one standard (standard voltmeter or potentiometer) and the current is read on another standard (standard ammeter, or potentiometer with standard resistance). It is convenient to obtain the potential and the current from separate sources, because then the process of adjusting one circuit will not affect the other. Also, in the case of instruments of large capacity, this scheme economizes energy as only 3 or 4 volts are necessary for the current circuit.

Induction-type wattmeters must be calibrated on alternating current of the frequency for which they are designed. This calibration is made by comparison with a secondary standard, which in turn is calibrated on continuous current (par. 178). Polyphase instruments may be calibrated as single-phase instruments by connecting the current circuits in series and the potential circuits in parallel. In the case of certain forms of induction-type instruments stray magnetic flux from one element may affect the other, in which case the calibration should be made on a polyphase circuit.

183. Temperature Errors.—Changes in temperature alter the tension of the control spring and the resistance of the moving coil. The latter may be reduced to a negligible value, when the potential of the circuit is sufficiently high, by the use of series resistance wire having a low temperature coefficient. Spring changes may be considerable; hence it is always advisable to leave wattmeters in circuit only long enough to get a reading, the instrument being cut out of circuit by a disconnecting switch in the potential (shunt) circuit and a short-circuiting switch in the current (series) circuit. The makers of some of the higher-grade wattmeters, that is, secondary standards, furnish a temperature coefficient which should always be used in accurate measure-

ments, particularly when the series resistance in the potential circuit is made small in order to increase the sensitivity where the voltage is low.

184. Inductance Errors.—The inductance errors in wattmeters may, under certain conditions, become very important. While the theory of the electrodynamometer type of wattmeter assumes that the potential circuit is non-inductive, this is not strictly true in the actual instrument because of the inherent inductance of the movable coil. Ordinarily, however, the non-inductive series resistance is sufficiently large to make the effect of this inductance negligible at ordinary frequencies and power-factors. But with low power-factors, the lag angle in the potential circuit may have to be considered (see par. 197).

The power in an alternating-current circuit is $W = EI \cos \theta$, where W = power, I = current, E = e.m.f., and $\cos \theta$ = power-factor of the circuit. When the power-factor is unity, I and E are in phase, but the potential-circuit current usually lags slightly behind E, because of the inductance in the movable coil, thus producing the effect of a small power-factor. If the angle between the current and potential in the potential circuit is, for example, 2°, $\cos \theta = 0.9994$ and the error is ordinarily negligible. If the main circuit power-factor is 50 per cent., the lag angle in the wattmeter is $(60 - 2) = 58°$. The cosine of 60° is 0.50 while the cosine of 58° is 0.53, thus introducing an error of 6 per cent. The error corresponding to various instrument lag angles and at any power-factor can be found in the tables in par. 202 by simply taking the "displacement" angle therein as the wattmeter lag angle.

185. Stray Magnetic-field Errors.—The stray-field error in unshielded, non-astatic, electrodynamometer wattmeters may be anything from zero to 25 per cent. with an alternating magnetic field of 5 lines per square centimeter, and from zero to 75 per cent. at 10 lines per square centimeter, depending upon the direction of the field and the coil deflection. A shield, properly made and placed, is extremely efficient, reducing the effect of a field of 20 lines per square centimeter to practically zero without introducing eddy current or other errors. The latest types of wattmeters are provided with such shields in the form of a well, built up of rings of laminated steel in which the wattmeter elements are mounted.

A stray field is indicated when, with the potential current alone connected to the main circuit, a deflection is obtained. Its

effect may be eliminated from a measurement by rotating the instrument until this deflection becomes zero and noting the direction of the pointer. Then, with the current circuit connected, the instrument is rotated until the deflected pointer is in the same position as before. The stray field, then, has no effect on the moving coil and there will be no error.

Wattmeters of the Kelvin balance type, in which the coils are astatically arranged, are practically immune from errors due to stray fields, except in an intense field which is not uniform throughout the space occupied by the moving system. Such a condition may arise, for example, when the wattmeter is close to a conductor carrying a very large current.

Induction-type instruments employ much stronger field strengths and are not appreciably affected except by very intense fields.

186. Electrostatic-field Errors.—As in all movable-coil instruments, an error may be caused by electrostatic attraction between the movable coil of the wattmeter and an adjacent body at a different potential. This adjacent body is usually within the instrument itself. A charge may be induced on the glass over the scale or on the metal cover (where the case is non-metallic) by accidental rubbing and this will attract the pointer causing a false deflection. Such a condition is readily removed by touching the glass or case with the hand.

An error from this cause is frequently encountered when the wattmeter is being used with its potential and current circuits connected to separate sources between which there is a difference of potential, thus causing an electrostatic force between the fixed and movable coils. The remedy in this case is of course to connect the two coils together at one point. To avoid accident in case the circuits are accidentally connected at some other point, this connection at the wattmeter may be made with a voltmeter which should always indicate zero.

Wattmeter Connections

187. General.—Care should be taken so to connect a wattmeter into the main circuit that the moving-coil end of the potential circuit and the current coil are on the same side of the circuit being measured. Otherwise there may be sufficient electrostatic attraction between the two windings to produce an error, as pointed out in the preceding paragraph, or, if the potential is

sufficiently high, the insulation between the windings may be broken down. The latter may be guarded against by connecting the binding post at the moving-coil end of the potential circuit to the proper current post with fine fuse wire.

188. Single-phase Circuit.—One wattmeter connected as shown in Fig. 104 will read true watts. The power may also be measured with three voltmeters or three ammeters.

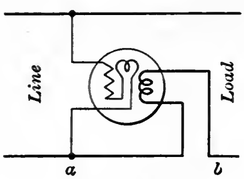


FIG. 104.

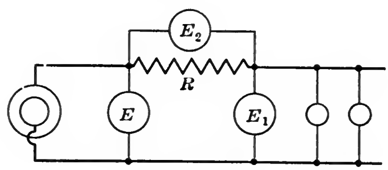


FIG. 105.

In the three-voltmeter method, a known non-inductive resistance, R, is connected in series with the load as shown in Fig. 105, where E, E_1 and E_2 are points where voltmeter readings are to be taken. The power in watts is

$$W = \frac{E^2 - E_1^2 - E_2^2}{2R} \qquad \text{(watts)}$$

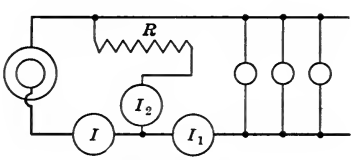


FIG. 106.

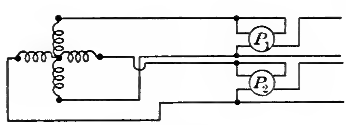


FIG. 107.

Similarly, in the three-ammeter method, Fig. 106, the power in watts is

$$W = R\left\{\frac{I^2 - I_1^2 I_2^2}{2}\right\}. \qquad \text{(watts)}$$

189. Two-phase, Four-wire Circuit.—Two wattmeters, connected as shown in Fig. 107 are sufficient, these conditions being equivalent to two single-phase circuits. The total power is obviously the arithmetical sum of the readings of the two instruments.

190. Two-phase, Three-wire Circuit.—Two wattmeters should be connected as shown in Fig. 108, the total power being the *algebraic* sum of the two readings. This connection is correct for all conditions of load, balance and power-factor. One watt-

meter may be used as in Fig. 109, provided there is no load across the outer conductors and the phases are *balanced* as to load and power-factor.

191. Two-phase, Four-wire, Interconnected Circuit.—Three wattmeters can be used, connected as in Fig. 110, the total power being the *algebraic* sum of the three readings. This connection

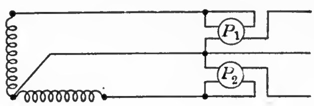


FIG. 108.

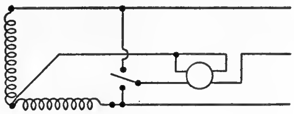


FIG. 109.

is correct for all conditions of load, balance and power-factor. Two wattmeters, one in each phase, will give the true power only when the load is balanced.

192. Three-phase, Three-wire Circuits.—Two wattmeters may be used, connected as indicated in Fig. 111, the total power being

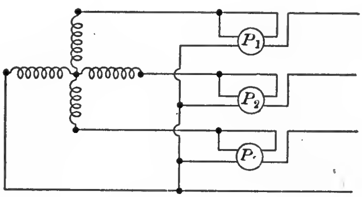


FIG. 110.

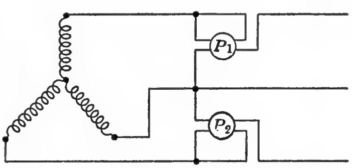


FIG. 111.

the *algebraic* sum of the two readings (see polyphase watt-hour meters, pars. 227 and 228). At unity power-factor, each instrument will indicate half the total power and at 50 per cent. power-factor one instrument will indicate the total power, the other instrument reading zero. At less than 50 per cent. power-factor, one instrument will read negative (see par. 232 for method of verifying power-factor).

193. Three-phase, Three-wire Circuits, Balanced Load.—When the load is *balanced*, the power may be measured with one wattmeter by the following methods:

(*a*) With "*star*" *box* or artificial neutral as shown in Fig. 112. The total power is three times the reading of the wattmeter. The resistance in each leg of the star box should be non-inductive and small compared with that of the potential circuit of the watt-

meter, so that the current taken by the latter will not disturb the potential at the neutral point.

(*b*) With "*Y*" *box* as shown in Fig. 113. The total power is three times the wattmeter reading. This arrangement is similar

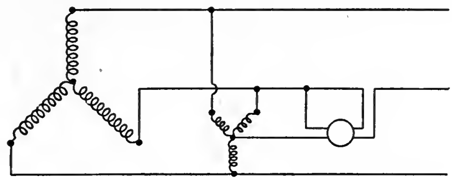


FIG. 112.

to (*a*), one leg of the star box being replaced with the potential circuit of the wattmeter itself. The other two legs have the same resistance as the potential circuit of the wattmeter.

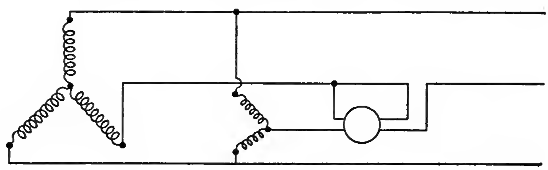


FIG. 113.

(*c*) With a "*T*" *reactance coil* as shown in Fig. 114. The total power is twice the wattmeter reading. The impedance of the reactance coil must be small compared with that of the potential

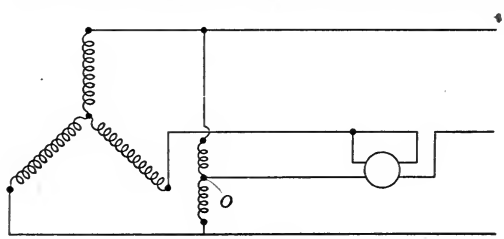


FIG. 114.

circuit of the wattmeter, so that the current taken by the potential circuit will not disturb the potential at O.

194. Three-phase, Four-wire Circuits.—Three wattmeters are used as shown in Fig. 115. The total power is the *algebraic* sum of the three readings. This method is correct for all conditions of load, balance and power-factor. A three-phase "star"

system with a *grounded neutral* is virtually a four-wire system and the power should be measured with three wattmeters. Obviously, if the load is balanced, one wattmeter can be used, the total power being the indication of the wattmeter multiplied by three. In that case, the current coil should be connected in series with one conductor or phase wire and the potential coil between that conductor and the neutral.

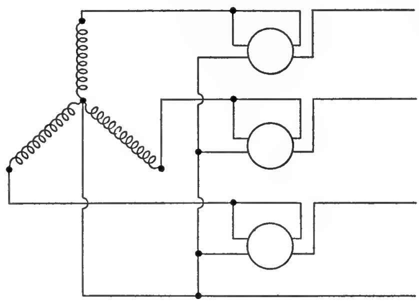


FIG. 115.

195. "N"-phase Circuit.—In any system whatsoever, of "*n*" phases the true power may be measured by connecting a wattmeter in each phase, the current coil being in series with the line and the potential coil connected between that line and any common point, *P*, of the system, which may or may not be the neutral. The total power is the *algebraic* sum of the readings of all of the wattmeters, so connected.[1]

MEASUREMENT OF SMALL POWERS

196. General.—In addition to the problem of measuring a very small power *per se*, there is often present the additional complication of either high potential and small current, low potential and large current, or a low power-factor combined with either of the preceding conditions.

197. With Dynamometers.—When the power is very small, a watt or less, a high sensitivity reflecting electrodynamometer with separate circuits is necessary, particularly with a low power-factor and extreme values of current and potential. Such conditions are encountered when measuring the losses in dielectrics, instrument circuits and in magnetic circuits. The fixed coils may

[1] "Direct and Alternating-current Testing," F. BEDELL, p. 228.

be connected directly in series with the main circuit or to a non-inductive shunt in the circuit. At low potentials or very small power-factors, the series resistance in the movable-coil circuit may not be sufficient to eliminate the effect of inductance but this can be accomplished by shunting the resistance with a condenser as shown in Fig. 116.[1] With the movable-coil circuit short-circuited at *ab*, the capacitance C or resistance R_2 is adjusted until there is no deflection with maximum current in the fixed coil (that is, $L_2 = CR_2^2$). Calibration is of course made with continuous current.

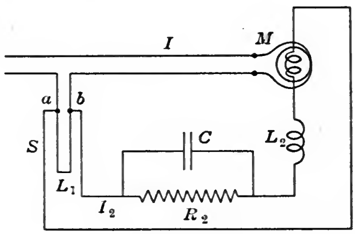


Fig. 116.

198. With Portable Wattmeters.—Portable wattmeters of the electrodynamometer type (par. 180) are available in standard makes with ratings from 25 watts, 0.5 amp. and 75 volts full scale to 10 kw., 1,000 amp. and 100 volts full scale (10 per cent. power-factor). In these large-current, low-power-factor instruments, special care is used in the design and construction in order to avoid eddy-current errors.

The necessary sensitivity is obtained in small-capacity instruments by the use of a large number of turns in the fixed coil. The instrument losses, current as well as potential, are therefore relatively large and have to be taken into account. It will be noted in Fig. 114 that the instrument is measuring its own current circuit loss. If the instrument loss cannot be neglected, it is better to connect the potential circuit to the load side (*b* instead of *a*) and include the potential circuit loss in the measurement instead of the current circuit loss, because the former not only remains constant but is more easily calculated.

199. Wattmeters Compensated for Losses.—Wattmeters are often arranged to correct or compensate automatically for this loss by means of a few turns on the fixed coil, connected in series with the potential circuit and in opposition to the fixed coil. This arrangement cannot be used with instrument potential

[1] "The Compensated Two-circuit Electrodynamometer," E. B. Rosa, *Bulletin*, Bureau of Standards, vol. 3, p. 43 (1907).

"Compensated Dynamometer Wattmeter Method of Measuring Dielectric Energy Loss," G. B. Shanklin, *General Electric Review*, October, 1916.

transformers nor when checking the wattmeter with separate sources of e.m.f. and current. A separate connection is usually provided, however, for this purpose. In general, it is safer always to use this "independent" connection, making allowance for the potential loss, when necessary, by calculation, because the compensation may not be correct for all positions of the moving coil. However, where the correction for instrument loss is so large that the quantity being measured is the difference between two larger quantities, routine measurements of this character may best be made with a compensated instrument if the compensating field coil has been properly installed.[1] In any case, the amount of under- or over-compensation at different coil positions can be determined by test and proper correction made.

200. High-voltage Power.—When the power to be measured is of the order of 50 watts or more and the potential is as high as 15,000 volts, a portable wattmeter of small capacity in conjunction with a potential transformer may be used. Due account must be taken of the ratio and phase-angle errors particularly where the power-factor is low, which is usually the case in measurements of this class. If the power is only a few watts or the potential is much over 15,000 volts, recourse must be had to more elaborate methods. Probably the most sensitive scheme devised for measurements of this class is that first proposed and developed by Prof. Ryan,[2] viz., the cathode-tube power indicator. Later, Minton[3] developed the "cyclograph," a cathode-tube wattmeter employing the same principle. The principle of this method is as follows: A stream of electrons from the cathode, R (Fig. 117), is concentrated by a focusing coil, F, and then intercepted by a metallic diaphragm, D, in the center of which is a very small hole, thus allowing a pencil of rays to pass to the other end of the tube where it falls on a fluorescent screen, S. The beam passes between two pairs of electrodes or quadrants, $a\ a'$ and bb' which are connected to air condensers, c and c', so arranged that the pairs of electrodes are excited by potentials respectively proportional to the current through and the poten-

[1] "Compensating Wattmeters," A. L. Ellis, *Transactions,* A. I. E. E., vol. 31, p. 1579 (1912).

[2] "A Power Diagram Indicator for High-tension Circuits," H. J. Ryan, *Transactions,* A. I. E. E., vol. 30, p. 1089 (1911).

[3] "An Investigation of Dielectric Losses with the Cathode Ray Tube," John P. Minton, *Transactions,* A. I. E. E., vol. 34, p. 1627 (1915).

tial across the specimen of insulation, X, being tested. The electrostatic field produced by each pair of electrodes deflects the beam in proportion to the excitation and a resultant curve similar to that shown at the left in Fig. 117 is produced on the screen. The area of this curve is proportional to the power.

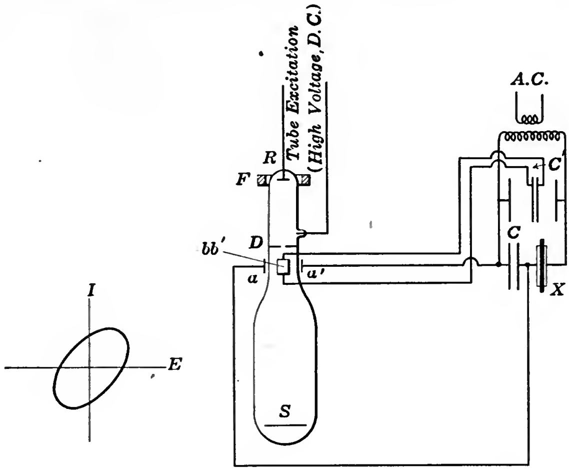


FIG. 117.

Fig. 118 indicates the scheme in a simple modification of the three-voltmeter method for measuring power proposed by Irwin[1] which is applicable in certain cases where the power-factor is

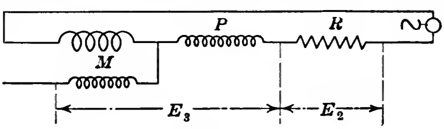


FIG. 118.

low, as for example, the losses in a Pupin loading coil. P is the coil, the losses in which are to be measured, R is a known non-inductive resistance and M a variable mutual inductance. The secondary of the latter is adjusted until the indication of an electrostatic voltmeter connected to read $E_2 + E_3$ is a minimum. Then E_2 and E are each measured. The power in watts is

$$W = \frac{E_2 E_3}{R} \qquad \text{(watts)}$$

where E_2 and E_3 are in volts and R is in ohms.

[1] J. T. IRWIN, *London Electrician*, Feb. 7, 1913.

Measurement of Large Powers

201. General.—Switchboard wattmeters are usually limited to 200 amp. and 440 volts. Above these values, instrument transformers with 5-amp. and 110-volt wattmeters are employed. In fact, current transformers are often used with less than 200 amp. in order to simplify the switchboard wiring.

Portable wattmeters are available with a current capacity as high as 1,000 amp., and, with suitable series resistance (multipliers) in the potential circuit, may be used directly on potentials of the order of 2,000 or 3,000 volts. In such cases, however, particular care must be exercised to observe the precautions indicated in par. 186.

Fortunately, it is seldom necessary to connect wattmeters directly in the circuit. Modern instrument transformers are so well designed and such accurate methods for determining their characteristics have been developed (pars. 111 and 130) that power measurements may be made with ample accuracy by using them in conjunction with the more accurate, low-capacity wattmeters.

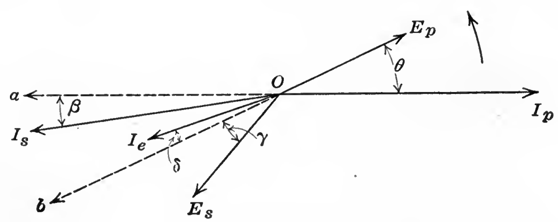


Fig. 119.

202. Errors Due to Phase Displacement.—The phase angle between the current and the potential in a wattmeter being used with instrument transformers may differ from that in the circuit being measured due to three causes: (*a*) the characteristics of the potential winding, causing the current in the potential circuit to be out of phase with the impressed potential; (*b*) the phase angle of the current transformer; (*c*) the phase angle of the potential transformer.

Let Fig. 119 represent vectorially in simple form, but greatly exaggerated, the fundamental relations between the current and the potential in the primary circuit and between the current and the potential in the winding of a wattmeter connected to the secondaries of instrument transformers (sine wave assumed).

The primary current I_p and the primary potential E_p are out of phase by the angle θ, that is, the power-factor of the circuit is lagging and equal to cos θ. If the transformers had the characteristics of theoretically perfect transformers, the secondary current I_s would be in the phase position a, 180° from I_p and the secondary potential E_s would be in the phase position b, 180° from E_p. However, the current transformer has a phase angle β which usually tends to decrease the angle between current and potential in the wattmeter (angle aob) making the power-factor higher, and hence the indication higher, than it ought to be. On the other hand, the potential transformer has a phase angle γ which usually tends to increase the phase angle in the wattmeter and make the indications lower. Furthermore, if the potential circuit of the wattmeter has appreciable inductance, the potential circuit current I_e will lag behind the impressed potential by the angle α which tends to reduce the phase angle in the wattmeter and make it indicate high. The net error is, therefore, the resultant of these three angles, the sign of each of which may be positive or negative according as it tends to increase or decrease the wattmeter indication.

The error in any instance is

$$\text{Error (in per cent.)} = \frac{\cos\theta' - \cos\theta}{\cos\theta'} \times 100$$

where $\cos\theta'$ = apparent power-factor in wattmeter circuits as obtained by measurement with a voltmeter, the ammeter and the wattmeter; $\cos\theta$ = power-factor of the primary circuit = $\cos(\theta' \pm \alpha \pm \beta \pm \gamma)$ where α = equivalent phase angle of the potential circuit of the wattmeter, β = phase angle of the current transformer and γ = phase angle of the potential transformer. The sign of each angle is made positive when its effect is such as to tend to reduce the phase angle in the wattmeter, that is, decrease the apparent power-factor and make the wattmeter read high. Hence, if the resultant of $\pm\alpha \pm\beta \pm\gamma$ is positive, the error is positive, that is, the wattmeter indication is too high by the percentage obtained by calculation from the formula.

In general θ is lagging, α is very small and positive, β is relatively large and positive and γ is small and negative. The net error is, therefore, usually positive, making the wattmeter indications high.

Positive Errors

Errors in per cent. when the observed watts are greater than the true watts, *i.e.*, (*a*) where $(\pm\alpha \pm\beta \pm\gamma)$ is positive and the power-factor of the primary circuit is lagging, or (*b*) where $(\pm\alpha \pm\beta \pm\gamma)$ is negative and the power-factor of the circuit is leading. The true watts are obtained by multiplying the observed watts by the per cent. error shown and subtracting from the observed watts.

Total displacement angle $(\pm\alpha \pm\beta \pm\gamma)$	Power-factor in wattmeter circuit, $\cos\theta'$										
	0.10	0.15	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90
10′	2.90	1.90	1.45	1.15	0.95	0.65	0.50	0.40	0.30	0.20	0.15
20′	5.80	3.85	2.85	2.25	1.85	1.35	1.00	0.80	0.60	0.45	0.30
30′	8.70	5.75	4.30	3.40	2.80	2.00	1.50	1.15	0.90	0.65	0.45
40′	11.60	7.70	5.70	4.50	3.70	2.70	2.05	1.55	1.20	0.90	0.60
50′	14.50	9.60	7.15	5.65	4.65	3.35	2.55	1.95	1.50	1.10	0.70
1° 0′	17.40	11.50	8.55	6.75	5.55	4.00	3.05	2.35	1.80	1.30	0.85
10′	20.30	13.45	10.00	7.90	6.50	4.70	3.55	2.75	2.10	1.55	1.00
20′	23.20	15.35	11.45	9.05	7.45	5.35	4.05	3.15	2.40	1.80	1.15
30′	26.10	17.30	12.85	10.15	8.35	6.05	4.55	3.55	2.70	2.00	1.30
40′	29.00	19.20	14.30	11.30	9.30	6.70	5.10	3.90	3.00	2.25	1.45
50′	31.90	21.15	15.75	12.40	10.20	7.40	5.60	4.30	3.30	2.45	1.60
2° 0′	34.80	23.05	17.15	13.55	11.15	8.05	6.10	4.70	3.60	2.70	1.75
10′	37.70	25.00	18.60	14.70	12.10	8.75	6.65	5.10	3.90	2.90	1.90
20′	40.60	26.90	20.05	15.85	13.05	9.40	7.15	5.50	4.25	3.15	2.05
30′	43.50	28.85	21.45	16.95	13.95	10.10	7.65	5.90	4.55	3.35	2.20
40′	46.40	30.75	22.90	18.10	14.90	10.80	8.15	6.30	4.85	3.60	2.35
50′	49.30	32.70	24.35	19.25	15.85	11.45	8.70	6.70	5.15	3.85	2.50
3° 0′	52.20	34.60	25.80	20.40	16.75	12.15	9.20	7.10	5.50	4.05	2.65
10′	55.10	36.55	27.25	21.55	17.70	12.80	9.70	7.55	5.80	4.30	2.85
20′	58.00	38.50	28.65	22.70	18.65	13.50	10.25	7.90	6.10	4.55	3.00
30′	60.95	40.45	30.10	23.85	19.60	14.20	10.75	8.35	6.40	4.75	3.15
40′	63.85	42.40	31.55	24.95	20.55	14.85	11.30	8.75	6.75	5.00	3.30
50′	66.75	44.35	33.00	26.10	21.50	15.55	11.80	9.15	7.05	5.25	3.45
4° 0′	69.65	46.30	34.45	27.25	22.40	16.25	12.35	9.55	7.35	5.45	3.60
10′	72.55	48.25	35.90	28.40	23.35	16.90	12.85	9.95	7.70	5.70	3.80
20′	75.50	50.15	37.30	29.55	24.30	17.60	13.40	10.35	8.00	5.95	3.95
30′	78.40	52.05	38.75	30.70	25.25	18.30	13.90	10.80	8.30	6.20	4.10
40′	81.30	54.00	40.20	31.85	26.20	19.00	14.45	11.20	8.65	6.45	4.25
50′	84.20	55.90	41.65	33.00	27.15	19.65	14.95	11.60	8.95	6.70	4.45
5° 0′	87.10	57.80	43.10	34.15	28.10	20.35	15.50	12.00	9.30	6.90	4.60

If the equivalent total displacement angle $(\pm\alpha \pm\beta \pm\gamma)$ is computed, the error can be obtained directly from the accompanying tables[1] which have been calculated according to the above formula for various values of total displacement angle and for various apparent power-factors in the *wattmeter* circuits. It is to be noted that these errors have been computed to the nearest 0.05

[1] From tables in the paper, "Electrical Measurements in Circuits Requiring Current and Potential Transformers," L. T. Robinson, *Transactions*, A. I. E. E., vol. 28, p. 1005 (1909).

per cent. only. The precision with which power, power-factor, energy and phase angle can be measured does not justify computing the errors more accurately. Obviously, errors for power-factors and phase angles between those listed in the tables can be obtained by interpolation.

Negative Errors

Errors in per cent. when the observed watts are less than the true watts *i.e.*, (*a*) where ($\pm\alpha \pm\beta \pm\gamma$) is negative and the power-factor of the primary circuit is lagging, (*b*) where ($\pm\alpha \pm\beta \pm\gamma$) is positive and the power-factor of the primary circuit is leading. The true watts are obtained by multiplying the observed watts by the per cent. error shown in the table and adding to the observed watts.

Total displacement angle ($\pm\alpha \pm\beta \pm\gamma$)	Power-factor in wattmeter circuit, $\cos\theta'$										
	0.10	0.15	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90
10′	2.90	1.90	1.40	1.15	0.90	0.65	0.50	0.40	0.30	0.20	0.15
20′	5.80	3.85	2.85	2.25	1.85	1.35	1.00	0.75	0.60	0.45	0.30
30′	8.65	5.75	4.25	3.40	2.75	2.00	1.50	1.15	0.90	0.65	0.40
40′	11.55	7.65	5.70	4.50	3.70	2.65	2.00	1.55	1.15	0.85	0.55
50′	14.45	9.60	7.10	5.65	4.60	3.30	2.50	1.95	1.45	1.10	0.70
1° 0′	17.35	11.50	8.55	6.75	5.55	4.00	3.00	2.30	1.75	1.30	0.85
10′	20.25	13.40	9.95	7.85	6.45	4.65	3.50	2.70	2.05	1.50	0.95
20′	23.10	15.35	11.35	9.00	7.35	5.30	4.00	3.05	2.35	1.70	1.10
30′	26.00	17.25	12.80	10.10	8.30	5.95	4.50	3.45	2.60	1.90	1.25
40′	28.90	19.15	14.20	11.20	9.20	6.60	5.00	3.85	2.90	2.15	1.35
50′	31.75	21.10	15.60	12.35	10.10	7.25	5.50	4.20	3.20	2.35	1.50
2° 0′	34.65	23.00	17.05	13.45	11.05	7.95	6.00	4.60	3.50	2.55	1.65
10′	37.55	24.90	18.45	14.60	11.95	8.60	6.45	4.95	3.75	2.75	1.75
20′	40.40	26.85	19.85	15.80	12.85	9.25	6.95	5.35	4.05	2.95	1.90
30′	43.30	28.75	21.25	16.95	13.75	9.90	7.45	5.70	4.35	3.20	2.00
40′	46.15	30.65	22.65	18.10	14.70	10.55	7.95	6.10	4.65	3.40	2.15
50′	49.05	32.60	24.10	19.25	15.60	11.20	8.45	6.45	4.90	3.60	2.25
3° 0′	51.95	34.50	25.50	20.40	16.50	11.85	8.95	6.85	5.20	3.80	2.40
10′	54.80	36.35	26.90	21.45	17.40	12.50	9.40	7.20	5.50	4.00	2.50
20′	57.70	38.25	28.30	22.55	18.30	13.15	9.90	7.60	5.75	4.20	2.65
30′	60.55	40.10	29.70	23.60	19.25	13.80	10.40	7.95	6.05	4.40	2.75
40′	63.45	42.00	31.10	24.65	20.15	14.45	10.85	8.30	6.30	4.60	2.90
50′	66.30	43.85	32.50	25.70	21.05	15.10	11.35	8.70	6.60	4.80	3.00
4° 0′	69.20	45.75	33.90	26.80	21.95	15.75	11.85	9.05	6.85	5.00	3.15
10′	72.05	47.65	35.30	27.90	22.85	16.40	12.30	9.40	7.15	5.20	3.25
20′	74.90	49.50	36.70	29.00	23.75	17.05	12.80	9.80	7.40	5.35	3.35
30′	77.75	51.40	38.10	30.05	24.65	17.65	13.30	10.15	7.70	5.55	3.50
40′	80.60	53.30	39.50	31.15	25.55	18.30	13.75	10.50	7.95	5.75	3.60
50′	83.50	55.20	40.90	32.25	26.45	18.95	14.25	10.90	8.25	5.95	3.70
5° 0′	86.35	57.05	42.30	33.35	27.35	19.60	14.70	11.25	8.50	6.15	3.85

Where a polyphase circuit is being measured with a polyphase wattmeter and the characteristics of the transformers used with the two elements are similar, the error can be computed on a

single-phase basis from the power-factor of the primary circuit as measured with a polyphase power-factor meter or otherwise. The secondary power-factor can then be computed from the average resultant phase angle ($\pm \beta \pm \gamma$) of the transformers and the error determined as previously explained. For, while the error may be very different in the two elements because of the difference in power-factor, it will be inversely proportional to the power in each element, hence a single power-factor may be used. When, however, the transformers have different phase angles and high accuracy is desired, the per cent. error should be calculated for each element separately—the net or combined single error for the wattmeter being calculated on the basis of the power in each element. Thus, if a polyphase meter indicates 100 watts and the power-factor is such that one element is measuring 30 watts with a phase-angle error of 5 per cent. and the other element 70 watts with a phase angle error of 2 per cent., the net error would be 2.9 per cent.

Power-factor

203. General.—The power-factor of a circuit is the ratio of the true power in watts as measured with a wattmeter to the "apparent power" obtained from the product of the current and potential measured in amperes and volts respectively. When the current is unidirectional and non-pulsating, that is, ordinary continuous current, the power-factor is obviously always unity. When the current is unidirectional and pulsating, such as the current delivered by a rectifier, the power-factor may be slightly less than unity under certain conditions. If the current is alternating, the power-factor will only be unity when the current and potential are in phase (see par. 176). This is an exceptional condition in ordinary circuits because there is ordinarily more or less inductance or capacitance or both in the circuit.

When the current and potential wave forms are both sinusoidal, the ratio of watts to volt-amperes is also equal to the cosine of the phase angle between the current and potential, that is, power-factor $= \cos \theta = W/EI$. If, however, either the current or potential wave form is distorted from a sine curve, the phase angle is indeterminate and, therefore, $\cos \theta$ has no meaning. Therefore, all measurements of power-factor which are based on $\cos \theta$ assume sine-wave current and potential.

204. Single-phase Circuits.—The power-factor in single-phase circuits is the ratio of watts to volt-amperes.

205. Polyphase Circuits.—The power-factor of polyphase circuits which are balanced is the same as that of the individual phases. When the phases are not balanced the true power-factor is indeterminate. For all practical purposes, however, it is sufficiently correct to assume the power-factor to be that obtained by methods which give the average of the power-factors of the separate phases.

In the wattmeter-voltmeter-ammeter method, the power-factor is, for a two-phase, three-wire circuit,

$$\frac{W}{EI\sqrt{2}}$$

I being the current in the middle wire and E the potential between the outer wires. For a three-phase, three-wire circuit, the power-factor is

$$\frac{W}{EI\sqrt{3}}$$

In both cases, W is in watts, E in volts and I in amperes.

In the two-wattmeter method, the power-factor of a two-phase, three-wire circuit is obtained from the relation $W_2/W_1 = \tan\theta$, where W_1 is the reading of a wattmeter connected in one phase (outside wire) in the same manner as in a single-phase circuit, and W_2 is the reading of a wattmeter connected with its current coil in the first phase, in series with the first wattmeter, and the potential coil is connected across the second phase. Obviously, if the load is steady, one wattmeter can be used for both readings. If the phases are not balanced, the readings should be repeated with the instruments connected in the second phase, the true power-factor being taken as the average of the two results. On a three-phase, three-wire circuit the two wattmeters are connected as for measuring power. The power-factor is calculated from the relation of the two values of power indicated by the two instruments by either of the following formulas or other formulas which may be derived therefrom:

$$\cos\theta = \frac{1}{\sqrt{1 + 3\left(\dfrac{W_1 - W_2}{W_1 + W_2}\right)^2}}$$

$$\tan\theta = \frac{\sqrt{3}\,(W_1 - W_2)}{W_1 + W_2}$$

where W_1 = watts indicated by the instrument measuring the larger amount of power and W_2 = watts indicated by the instrument measuring the smaller amount of power. In the case of the second formula, the power-factor, $\cos \theta$, is of course obtained from a table of natural functions. If the phases are not balanced, the power-factor is taken as the average of the three values obtained for the three possible positions of the wattmeters taken in turn, that is, (*a*) in phases 1 and 2, (*b*) in phases 2 and 3, (*c*) in phases 1 and 3.

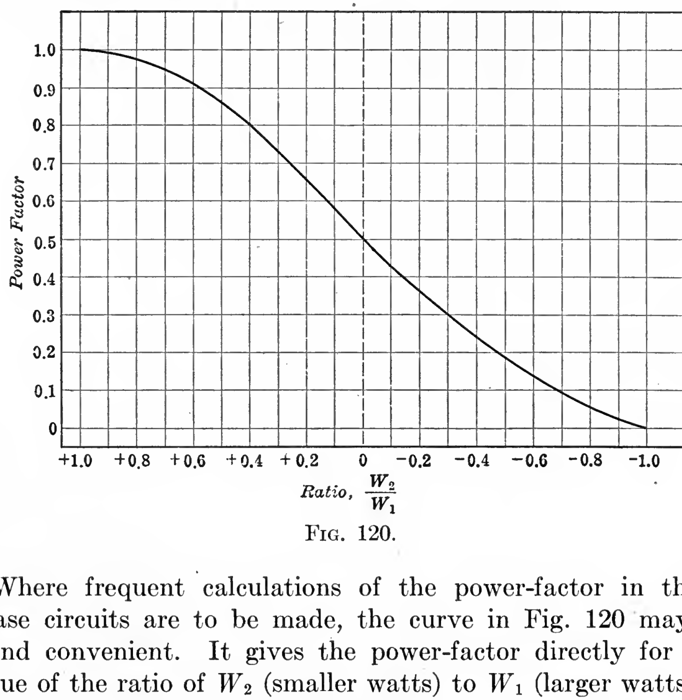


FIG. 120.

Where frequent calculations of the power-factor in three-phase circuits are to be made, the curve in Fig. 120 may be found convenient. It gives the power-factor directly for any value of the ratio of W_2 (smaller watts) to W_1 (larger watts).

Attention is called to the necessity for noting the sign of W_2. For power-factors over 0.5, W_2 will be positive and the ratio, W_2/W_1, will be negative. When it is not certain whether the power-factor is greater or less than 0.5, a test can be made as described in par. 232 in connection with watt-hour meters.

POWER-FACTOR METERS

206. General.—There are two general classes of power-factor meters, those involving the principle of electrodynamometer

wattmeters and those based on the principle of induction wattmeters.

207. Weston Meters.—Weston Electrical Instrument Co. power-factor meters employ the electrodynamometer principle. The essential features of a single-phase power-factor meter are shown in Fig. 121. It will be noted that the arrangement is

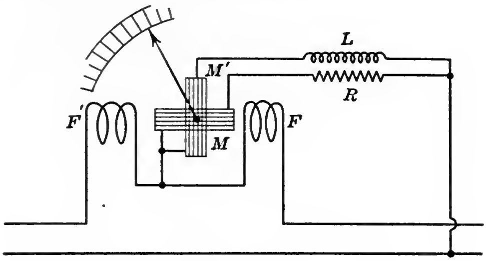


Fig. 121.

similar to that in wattmeters, except that there are two coils, M, M', in the moving system instead of one. One coil, M, is connected across the line and in series with a resistance, R, while the other coil, M' is connected in series with an inductance, L. The current in the coil, M', will therefore be about 90°

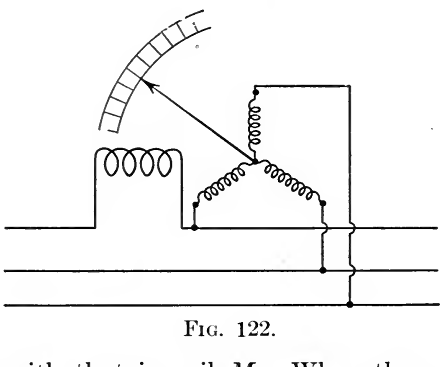


Fig. 122.

out of phase with that in coil M. When the power-factor is unity, the reaction between the fixed coils, F', F, and the moving, M, will be a maximum, while that between F', F and M' will be a minimum. The torque exerted on M will cause the moving system to take the position of minimum torque, that is, where the plane of M will be parallel to that of F' and F; the corresponding mark on the scale will therefore be 100. Similarly, at zero power-factor, coil M' will exert all of the torque and cause

the moving system to take a position where the plane of M' will be parallel to that of F' and F; the corresponding indication is therefore zero. Theoretically, the indications will be affected by the frequency because the current in L depends upon the frequency, but by proper design of the reactor, L, the effect of moderate variations in frequency can be eliminated.

In the polyphase meter, Fig. 122, the inductance, L, is not required and the instrument is, therefore, entirely independent of frequency. There are three coils in the moving system, one conducted across each phase. The principle of operation is exactly the same as in the single-phase instrument, the moving system taking up a position where the resultant of the three torques will be a minimum, which position will vary with the average power-factor of the circuit.

208. Westinghouse Meters.—In Westinghouse power-factor meters, the dynamometer principle is used in some types as in the Weston instruments. In others the induction principle is employed in the same manner that is applied in synchroscopes (par. 334).

209. General Electric Meters.—General Electric power-factor meters employ the electrodynamometer principle in polyphase instruments, in the same manner as that outlined above. No single-phase instruments are made by this company.

210. Calibration of Power-factor Meters.—In order to check a power-factor meter, a circuit in which the power-factor can be readily changed is necessary. Such a circuit is usually arranged with a phantom load, that is, the potential and current are obtained from separate sources or separate phases of the same source. The variation of the power-factor is obtained by providing means for shifting the phase of the current with respect to the potential or *vice versa* (see par. 240). With the power-factor meter connected to such a circuit, it can be checked by comparison with the ratio of the "artificial" watts, as shown by a wattmeter, to the product of volts and amperes. If the circuit is three-phase, comparison is made with the ratio of the watts to $\sqrt{3} \times$ voltamperes.

This method of comparison is, however, open to the objection that it is not accurate at high power-factors because the two quantities, watts and volt-amperes, are too nearly equal. Methods in which the "wattless" or reactive power component is measured directly are, therefore, preferable.

In the case of a single-phase meter, the latter method is applied by connecting the power-factor meter and a single-phase wattmeter to one phase of a two-phase source, using a phantom load and means for varying the power-factor as in the first method. The current coil of a second single-phase wattmeter is connected to the second phase of the source. It is apparent that the second wattmeter indicates the "wattless" component of the apparent power or volt-amperes in the circuit on the first phase while the first wattmeter indicates the power component. The ratio of the watts indicated by the second wattmeter (W_2) to that indicated by the first wattmeter (W_1) is equal to tan θ.

The "wattless-component" method may be applied to polyphase power-factor meters in several ways. If a phantom three-phase circuit with facilities for varying the power-factor is available (such as two revolving field alternators with a common shaft and bed plate, with provision for shifting the phase of one armature with respect to the other, current being taken from one machine and potential from the other), the power component, W_1, may be measured with a polyphase meter connected in the proper manner and the "wattless component," W_2, with another wattmeter connected similarly except that the potential coils are interchanged with respect to the first instrument. Then,

$$\frac{W_2}{W_1} = \frac{2EI \sin \theta}{\sqrt{3}EI \cos \theta} = \frac{2}{\sqrt{3}} \tan \theta.$$

Similarly, for a two-phase meter tested on a phantom two-phase circuit,

$$\frac{W_2}{W_1} = \frac{2EI \sin \theta}{2EI \cos \theta} = \tan \theta.$$

Two single-phase wattmeters may, of course, be used and the power-factor obtained as indicated in par. 205.

Probably the simplest method of checking polyphase power-factor meters is that indicated in Fig. 123 where PS is a phase shifter (described in par. 240), PF is the power-factor meter, W_1 a single-phase wattmeter with a "star box" connection from the three-phase balanced supply to the potential circuit (that is, $R_1 = R_2 =$ resistance of the potential circuit of the instrument, see par. 193), and W_2 is a second single-phase wattmeter. Wattmeter W_1 serves to measure a quantity proportional to the power component and wattmeter W_2 measures a quantity proportional to the "wattless component." Since the phase shifter is usually

of small current capacity, being normally used to "shift" the potential rather than the current when changing the power-factor, a step-down transformer, T, with a suitable rheostat, R, is employed to supply the current to the current circuits of the three instruments. With this arrangement, the power-factor meter is quickly checked by simply adjusting PS until the desired indication is obtained and then noting the indications of W_1 and W_2. The true power-factor is obtained from the relation

$$\tan \theta = \frac{W_2}{\sqrt{3}W_1}$$

where W_1 and W_2 are the watts corresponding to the indications

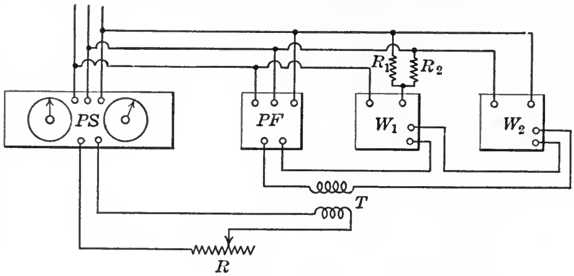


FIG. 123.

of wattmeters W_1 and W_2, respectively—corrected, of course, for scale errors, multipliers, constants and so forth. Obviously, if the instruments are alike and have negligible scale corrections, the indications may be used directly.

211. Wattless-factor and Wattless-component Meters.—A wattless-factor meter is a power-factor meter connected to indicate $\sin \theta$ instead of $\cos \theta$, the scale being marked to correspond. Similarly, a wattless-component meter is a wattmeter connected to read the wattless component instead of the power component. The change in the connections of both instruments obviously consists in shifting the potential circuit excitation 90°. In a two-phase circuit, this means simply interchanging potential connections but in three-phase circuits potential transformers with special taps are required in order to get the necessary shift of 90°.

Where these instruments are used, it is largely for psychological reasons.[1] For example, the wattless factor corresponding to 97

[1] "Measuring Idle Volt-amperes," H. B. TAYLOR, *The Electric Journal*, 1914.

per cent. power-factor is 25 per cent. Suppose that in a particular case, the power-factor was very important and was to be kept as near 100 per cent. as possible. It is reasonable to expect that the operator would give the matter closer attention if the instrument indication changed from 0 to 25 for a change in power-factor from 100 to 97, than if the indication changed only from 100 to 97.

CHAPTER IX

ENERGY MEASUREMENTS

212. General.—It is estimated that the revenue from the electrical energy produced in the United States in 1916 by central stations alone was over $400,000,000. The greater portion of this energy was sold on a basis which involved measuring the energy. It is evident, therefore, that, from a commercial standpoint, the importance of the measurement of energy exceeds that of all other electrical measurements.

The electrical energy expended in a circuit is the product of the quantity of electricity delivered to the circuit and the electrical pressure applied. That is

$$W = QE = IEt$$

where W = energy, Q = quantity, E = potential, I = current and t = time.

The units of energy are the joule, which is the energy represented by 1 watt of power being expended for 1 sec. (watt-second), and the watt-hour, which is the energy represented by 1 watt of power being expended for 1 hr. The kilowatt-hour, which is the thousand multiple of watt-hour, is the unit in more general use.

An instrument which measures electrical energy is called a watt-hour meter (often incorrectly called an integrating wattmeter, recording wattmeter or simply wattmeter). All watt-hour meters are practically small motors in which one revolution represents a certain amount of energy and the speed is proportional to the power. The revolving element operates a registering mechanism on which the energy consumption corresponding to the total number of revolutions is recorded. Meters for continuous current are usually of the type which utilize the electrodynamic principle of continuous-current motors, while those for alternating current utilize the principle of induction motors.

Continuous-current Energy

213. Types of Meters.—Continuous-current energy is measured with two types of watt-hour meters, commutator meters and mercury-motor meters.

214. Commutator-type meters are similar in principle to shunt motors. The essential features are shown in Fig. 124. The moving element consists of an armature, *a*, a commutator, *c*, and a light metal disc, *d*, all mounted on a steel shaft which rotates in a jewel bearing. The armature is connected to the external circuit by means of very light silver-tipped brushes. In series with the armature is a light-load compensation coil, *s*, and a resistance, *r*. In some meters the resistance, *r*, is included in the compensating coil, *s*.

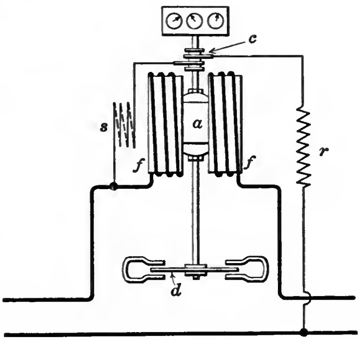


Fig. 124.

The principle of operation is as follows: The torque is proportional to the current in the armature coils and to the field strength. Since there is no iron in the magnetic circuit, the field strength is always proportional to the field current, hence the torque is proportional to the two currents as in a dynamometer-type wattmeter. The current in the armature being proportional to the line potential, and the field current equal or proportional to the line current, the torque is proportional to the power. In order to make the speed proportional to the power, a mechanical load must be provided, in which the counter torque will be proportional to the speed. This load usually takes the form of a circular disc, *d* (Fig. 124), of thin copper or aluminium which revolves between the poles of one or more permanent horseshoe magnets, with poles very close together. The eddy currents induced in the disc react with the permanent-magnet field, producing a counter torque which will always be proportional to the speed. As the load current increases, the torque of the motor element increases and the speed increases, because there is practically no counter e.m.f. in the armature, *s*. But as the speed increases, the counter torque of the disc or generator element also increases and a speed is finally reached where the two torques balance each other and the speed remains constant.

Thus, theoretically, the speed will always be proportional to the power in the circuit. Each revolution represents a definite amount of energy, and by connecting the shaft to a suitable recording mechanism similar to that of gas and water meters, the total energy consumed is automatically registered.

The essential differences between this type of watt-hour meter and a two-pole shunt motor are as follows: (*a*) entire absence of iron in the magnetic circuits; (*b*) the armature element is connected across the circuit and carries a very small current, while the field element is in series with the circuit and carries the main current; (*c*) the speed increases as the field strength increases which is opposite to the effect in a shunt motor because of the practical absence of counter e.m.f. in the meter armature.

In practice, certain conditions prevent the speed from being always proportional to the load, the principal factors being friction and temperature. Bearing friction is reduced to a minimum by using polished sapphire or diamond jewels, with either a polished, cone-shape shaft end or a steel ball. Thus the contact surface is reduced practically to a point. The weight is reduced by using hollow shafts and very light aluminium or non-metallic frames for the armature windings. Commutator friction is reduced to a minimum by making the commutator diameter as small as possible (about $\frac{1}{10}$ in.), and using round brushes so that contact is made practically at a point. The gears in the registering mechanism are made as light as possible. The effect of friction is smaller as the torque is increased; hence the ratio of torque to weight is made as large as possible by making the armature spherical, which gives the maximum torque for the minimum amount of weight of wire.

Friction in commutator meters cannot be entirely eliminated and its effect is marked at light load, necessitating the use of a compensating device (*s*, Fig. 124). This usually consists of a few turns of fine wire on the field coil, which are connected in series with the potential circuit, thereby adding a small torque to that produced by the main field. These turns are wound in a separate coil and the amount of compensation can be adjusted for each meter individually; either by altering the position of the coil with respect to the field coils, as in General Electric and Westinghouse meters, or by changing the number of active turns by means of a convenient switching arrangement, as in Columbia and Duncan meters.

Temperature changes affect the performance of commutator meters by: (*a*) changing the resistance of potential circuit; (*b*) changing the resistance of the drag disc; and (*c*) changing the strength of the permanent magnets. These changes produce a combined or resultant effect, which is compensated for by using, for the series resistance in the potential circuit, one or more materials so combined that the final resistance-temperature coefficient of the series resistance counteracts all of the other effects combined.

215. Mercury-motor Meters.—This type is most prominently represented by the continuous-current meter made by the Sangamo Electric and Manufacturing Co. Fig. 125 shows diagrammatically the circuits and scheme of operation. D is a solid copper disc floating in mercury; F is a float which supports the shaft and eliminates a jewel bearing; H is a laminated iron core and C is a chamber filled with mercury. The flux produced in the core, H, by the shunt coil, traverses the disc at two points which are diametrically opposite. The line current passes from L to L_1, through the mercury and diametrically through the disc. This disc being cut by the flux, a torque is produced which is proportional to the current and the potential.

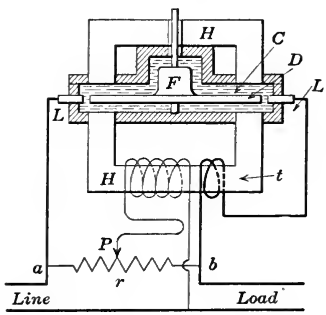


FIG. 125.

The friction due to the disc rotating in the mercury is compensated in two ways. One method is shown in Fig. 125. A high resistance, r, is connected in shunt with the armature, D, and the potential circuit is completed by the sliding contact, P. When the slider is at b, practically all of the shunt current will pass through D because of the resistance, r, thus adding to the torque. When the slider is at a, practically none of the potential-coil current will traverse the armature. In the second method, a circuit consisting of two wires of dissimilar metals which form a thermocouple, is connected in parallel with the mercury-chamber terminals, L, L_1. This couple is surrounded with a heating coil connected in series with the potential circuit. The e.m.f. produced by the couple causes current to flow through the disc, thus producing the necessary additional torque to overcome

the effect of friction at light load. The increased mercury friction at high speed is compensated by a series turn (or half a turn), *t*, on the core *H* (Fig. 25). The drag or counter torque is obtained with a separate disc and permanent magnets (not shown) in the manner described for commutator meters.

216. Meter Speed Adjustments.—The speed of both commutator and mercury-motor meters is adjusted principally by shifting the drag magnets diametrically with respect to the meter shaft, thus altering the retarding torque. The torque is a minimum when the magnets are close to the shaft and a maximum when they are near the edge of the disc. In some meters, a micrometer adjustment is provided for making small changes in speed. It consists of a movable piece of soft steel or a steel screw which bridges the gap between the magnets, and shunts more or less of the flux from the drag disc. Any change in the magnets produces the same percentage effect at all speeds so that the accuracy curve is merely shifted parallel to itself.

In addition to the magnet adjustments which affect all loads equally, the speed at light load (10 per cent. and less) is adjusted independently by the methods previously described. The effect of these compensating devices is substantially inversely proportional to the load, that is, the effect at 5 per cent. load is twice that at 10 per cent. load.

217. Metering Large Currents.—The construction of meters of the commutator type for use on heavy-current circuits becomes very difficult because of the large conductors required in the field. While such meters have been built for direct connection in 20,000-amp. buses, the cost is excessive and the operation is not entirely satisfactory. The Columbia Meter Co. has developed a commutator meter for switchboard use which is operated from a shunt. The mercury-motor meter is particularly adapted to operation with shunts and the Sangamo mercury meter is used with shunts for all capacities above 10 amp., all meters being rated at 10 amp. capacity.

Large currents may be metered by connecting several small meters in parallel. Care should be taken to make the resistances of the several branches equal if the meters are of the same capacity; or, if the meters are of different capacities, inversely proportional to the capacities of the meters. This will insure that none of the meters is overloaded.

218. Three-wire circuits are metered with two, standard two-wire meters (one meter on each side of the circuit) or a three-wire meter. In the commutator type, the three-wire meter is the same as the corresponding two-wire meter except that the two field coils (which should be exactly alike) are separated electrically and one is connected in each outer wire in such a manner that their fields are cumulative as before. When the load is exactly balanced, the conditions are obviously the same as in a two-wire meter and when the load is unbalanced the two fields add together so that the speed is proportional to the total current.

It is to be noted that this type of meter will not register correctly if the potentials are not balanced, a condition which may occur with an unbalanced load if the resistance of the wiring is too high. With both current and potential unbalanced, the total load is

$$W = E_1 I_1 + E_2 I_2$$

where E_1, and I_1 are the potential and current respectively of one side of the system at the meter and E_2, I_2 are the potential and current for the other side. If the potential coil of the meter is connected across the side E_1, the load registered by the meter is

$$W_m = E_1(I_1 + I_2)$$

and the error will be

$$W - W_m = I_2(E_1 - E_2),$$

that is, the product of the difference between the potentials and the current in the side to which the potential circuit of the meter is *not* connected. The error will be positive or negative according as E_1 is greater or less than E_2.

If the potential circuit is connected across the outer wires, the error will be smaller, equal to

$$W - W_m = \tfrac{1}{2}(I_1 - I_2)(E_1 - E_2)$$

and always the same sign, that is, the meter registration will always be too large.

The error due to unbalancing is not of practical importance in installations which have been properly wired. When it does become significant, the best solution of the difficulty where commutator meters are used is to employ two two-wire meters.

Sangamo three-wire mercury meters consist of two two-wire meter elements, one above the other, with a common shaft. Thus the energy consumption of the two sides of the circuit are automatically added together. Obviously in this type of meter the accuracy of registration is unaffected by unbalancing of the circuit.

219. Typical Meter Data.—The data in the following tables are given for the purpose of showing some of the design and operating characteristics of modern continuous-current watt-hour meters and the order of magnitude of some of the quantities involved. They may vary more or less between individual meters. Conclusions as to performance should not be drawn from these figures because many conditions enter into practical operation which may quite outweigh apparent advantages in design.

The following data in Table I apply to modern two-wire meters and are taken from the "Electrical Meterman's Handbook" issued by the National Electric Light Association, 1912.

Table I

Make	G. E. CO.	Westinghouse	Sangamo	Duncan	Columbia
Type	C-6	CW-6	D	E	D
Rating	5 amp., 110 volts	5-amp., 110 volts	10 amp., 110 volts	5 amp., 110 volts	10 amp., 110 volts
Speed, full load, r.p.m.	45.8	41.7	25	36.7	30
Torque, full load, gram-mm.	170	140	55	180	90
Weight, moving element, grams	97	80	3[1]	130	90
Ratio, torque to weight	1.75	1.75	18	1.4	1
Drop, current circuit, volts at rated current	1.15	1	0.03	......	0.55
Loss, current circuit, watts at rated current	5.75	5	0.3	......	5.5
Loss, potential circuit, watts at 110 volts	5.1	4.5	5	5	2
Resistance, armature, ohms	825	1,185	1,850[2]	......	2,660
Resistance, compensating coil, ohms	65	315	....	......	450
Resistance, series resistance, ohms	1,540	800	450	......	3,000
Resistance, potential circuit, total ohms	2,430	2,300	2,300	......	6,110
Ampere-turns, field	300	600	....	910	700
Ampere-turns, armature	800	1,500	....	......	2,100

[1] In mercury. The weight in air is 32 grams.
[2] Potential field coils.

The following data in Table II, for three-wire, 220-volt meters, are taken from a report of an investigation made by the Bureau

TABLE II

Make Type Capacity	G. E. CO. C-6 5 amp.	Westinghouse CW-6 5 amp.	Sangamo D 10 amp.	Duncan E 5 amp.	Columbia D 10 amp.
Speed, full load, r.p.m.	45.8	45.8	27.5	36.7	33
Torque, full load, gram-mm.	167	149	39	143	75
Weight, moving element, grams	102	96	7	156	98
Ratio, torque to weight	1.64	1.55	5.45	0.92	0.76
Loss, current circuit, at rated current, watts	5.7	5.2	0.2	5.7	6.5
Loss, potential circuit, at 110 volts, watts	4.4	4.5	9.2[1]	4.6	2.2
Resistance, armature, ohms	930	980	....	1,290	2,300
Resistance, compensating coil, ohms	1,790	1,710	....	12	230
Resistance, series resistance, ohms	....	....	....	1,340	3,080
Resistance, total potential circuit, ohms	2,720	2,690	5,290	2,640	5,610
Range of full-load adjustment, per cent.	68	92	35	155	68
Range of light-load adjustment, per cent.	13.8	22	4.6	5.6	13.8
Change of full-load rate by maximum change of light-load adjustment, per cent.	0.7	1.5	1.5	0.3	0.7
Change of light-load rate by maximum change of full-load adjustment, per cent.	66	104	38	154	66
Temperature coefficient, per cent. change per degree C.	+0.10	+0.07	+0.12	+0.10	+0.26
Flux density, full load, gausses	75	70	....	85	70
Starting current, amperes[2]	0.03–0.05	0.07–0.09	0.02	0.02–0.06	0.06–0.07
Creeping potential, volts[2]	130–130	130–130	160–260	100–130	130–130
Difference in balance of current elements, per cent[2]	0.4–1.6	0.0–0.8	....	1.5–5.1	1.1–4.2
Effect of reversing polarity, per cent. change at:					
100 per cent. load	0.2	0.4	1.9[3]	0.3	0.5
50 per cent. load	0.5	0.9	2.9	0.6	0.8
Effect of short-circuit, per cent. change on:					
120 volts (240 to 500 amp.)	1.1	1.9	2.3	2.9	2.5
240 volts (400 to 730 amp.)	5.6	4.7	0.0	11.6	34
Effect of voltage variation, per cent. change at:					
110 per cent. normal voltage	−1.2	−1.2	−2.2	−0.8	−1.2
90 per cent. normal voltage	+1.0	+1.2	+2.2	+0.8	0.0
Drop across armature at 100 volts, volts	37.6	40.1	....	53.8	45.1
Counter e.m.f., full load, volts	0.19	0.17	0.0001	0.13	0.13
Friction torque, per cent. of full-load torque:					
Brushes	0.76	0.60	....	0.40	0.50
Gears	0.06	0.03	....	0.04	0.13
Bearings	0.08	0.08	....	0.16	0.15
Air	0.11	0.13	....	0.11	0.10
Total	1.01	0.84	5.5	0.71	0.88
Drag magnet torque, per cent. of full-load torque	98.99	99.16	94.5	99.29	99.12
Diameter of commutator, centimeters	0.24	0.24	....	0.465	0.265
Number of commutator segments	8	8	....	8	3
Thickness of drag discs, centimeters	0.065	0.065	0.090	0.150	0.115
Diameter of drag discs, centimeters	12.66	12.70	10.18	13.35	11.40
Brush pressure, grams	1.5	1.8	....	0.57	0.24
Number drag magnets	4	4	2	2	2
Total drag flux, gausses	7,300	7,300	8,100	11,400	9,700

[1] 220-volt meter.
[2] Where two values are given, they are maximum and minimum of three meters.
[3] Thermocouple form of light-load compensation.

of Standards.[1] The figures given, are, in most cases, averages for three meters.

Alternating-current Watt-hour Meters

220. Types of Meters.—Alternating-current energy may be measured with two types of meters, the standard commutator type as used on continuous current and the induction type. The former are now seldom used, however, because the latter are more accurate and are much less expensive in first cost and in maintenance.

221. Commutator Meters.—When this type of meter is used on alternating current there will be an error due to the inductance of the potential circuit, which makes the current in that circuit lag behind the applied potential. Where the power-factor is unity, this angle is not sufficient to be serious, but on low power-factors, the error cannot be neglected and has to be compensated. One method of accomplishing this is to shunt the series coils with a non-inductive resistor, thereby producing a lag between the main current and that in the series coils. This resistance is made of such a value that the angle of lag is the same as that in the potential circuit. Its value is

$$R_2 = \frac{RX_1 - XR_1}{X}$$

where R, R_1 and R_2 are the resistances of potential, series and shunt circuits, respectively, and X and X_1 are the reactances of the potential and the series circuits, respectively.[2]

222. Induction-type watt-hour meters operate on the principle of the rotating magnetic field of the induction motor. The essential features of the principal makes of watt-hour meters are shown in the diagrammatic sketch, Fig. 126. P is the potential coil; S is the series coil, and C is a compensating coil. A metallic disc is free to revolve between the poles. The alternating magnetic fluxes from these poles will establish currents in the disc about as indicated by the arrows in the sketch at the right, which shows a portion of the disc and the poles. The potential winding, P, has many turns and is, therefore, highly inductive,

[1] "A Comparative Study of American Direct-current Watt-hour Meters," T. T. Fitch and C. J. Huber, *Bulletin*, Bureau of Standards, vol. 10, p. 161 (1914).

[2] "Electrical Meters," C. M. Jansky, p. 187.

while the series winding, S, is practically non-inductive; thus the fluxes produced by these circuits are practically 90 times degrees apart. Each flux is in phase with the current which produces it; and the e.m.f. generated in the disc, which is cut by the flux, is in time quadrature with the generating flux. Therefore, if it is assumed that the fluxes due to P and S, respectively, are in quadrature, the eddy currents produced by P will be a maximum at the same instant that the flux from S is a maximum, and *vice versa*. Thus, a torque will be produced which is proportional to the instantaneous product of the eddy currents in the disc and the flux from the pole under which the current is flowing. This torque is proportional to the power used in the load circuit, providing the time-phase difference of the currents in coils P and S is exactly 90° at unity power-factor. The necessary retarding action or counter torque is obtained with permanent

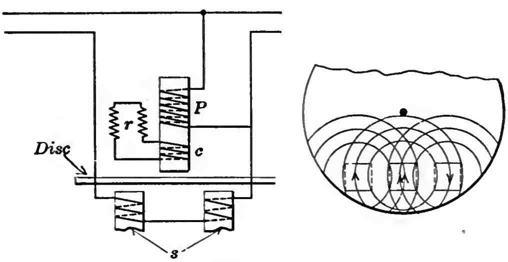


FIG. 126.

magnets on this same disc as described for continuous-current meters.

If the phase quadrature is not exact, the meter will obviously not register correctly under any condition. In consequence of the ohmic resistance of the potential circuit, the current is never exactly 90° behind the impressed e.m.f. At any instant, the flux E_f (Fig. 127) from the potential pole, instead of being in phase with the eddy currents, I_e, due to the line current, is slightly behind, as indicated in Fig. 127. The torque is therefore proportional to the product I_e and oa, instead of I_e and E_{ff}. The meter will therefore run slow, but as a practical matter the error is so small that at unity power-factor it is insignificant. The error rapidly becomes large, however, as the power-factor decreases. As practically all alternating-current circuits have a power-factor less than unity, a compensating coil is used to eliminate the error. This coil is a short-circuited winding placed

on the potential pole (see *c*, Fig. 126) and in which a current is induced 90° behind the generating (potential) flux. Its flux, S_f, will be in phase with this short-circuit current and therefore 90° from E_f, with which it will combine. By adjusting the value of the resistance (lag adjustment), *r* in Fig. 126, the resultant flux can be brought into exact phase with I_e and the meter will then register correctly on all power-factors. It is evident that with a lagging power-factor in the circuit, a meter will be slow if "underlagged" and fast if "overlagged." The opposite results will occur with a leading power-factor.

The friction in an induction-type meter is much less than in a commutator-type meter because of the absence of a commutator and an armature. On the other hand, the torque is less (compare tables of characteristic data, par. 219 and par. 229) so that the effect of friction at light load still has to be compensated. The principle employed in practically all meters is that in which a flux is produced at the potential pole face, slightly out of phase with the main flux. Then eddy currents will be produced in the disc which will be in phase with a small component of the main flux, giving rise to a slight torque which can be made sufficient to overcome the friction torque. This "out-of-phase" flux is produced in various ways in different meters. A common method is to place a short-circuited copper circuit or thin copper punching ("shading strip") in the potential-pole air gap, in an unsymmetrical position, so that the desired unblanced flux will be obtained. In the Columbia meter, the effect is accomplished by unbalancing the flux of the two potential poles by means of magnetic shunts.

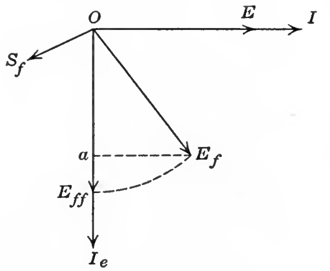


FIG. 127.

223. Meter Adjustments.—Convenient means are usually provided for adjusting the meter speed at light load and full load. The position of the light-load compensation coil can be changed by means of conveniently located screws and the light-load speed thus altered. Speed adjustment at all loads is obtained by shifting the drag magnets with respect to the axis, as in continuous-current meters, or by shunting the flux by means of a movable soft-iron keeper bridging the air gap. The power-factor or lag

adjustment is made at the factory and if properly done should never require readjustment. Consequently, in most modern meters no special provision is made for readjusting the lag.

224. Instrument Transformers.—When the capacity of the circuit is over 200 amp., current transformers are generally used to step down the current to 5 amp. If the potential is over 440 volts, current transformers are almost invariably used, irrespective of the magnitude of current, in order to insulate the meter from the line; in such cases potential transformers are also used to reduce the potential to 110 volts.

225. Errors Due to Transformers.—The register dial of a standard watt-hour meter is always marked to indicate directly the energy passed through the circuit to which it is connected. If transformers are used, the dial marking and the test constant marked on the disc (par. 236) are based on the theoretical performance of the transformers, that is, the nominal ratio and a zero phase angle. Actually, however, the true ratio always differs from the nominal or theoretical ratio and there is always a phase angle of greater or less magnitude (see checking current and potential transformers, par. 130 and par. 111 respectively). Therefore, two sources of error exist where transformers are used, one due to the ratio and one due to the phase angle. These errors, combined, may easily be as much as 2 per cent. and should therefore be taken into account where high accuracy is important, which is usually the case in large installations.

The ratio error is obviously equal to the difference between the actual and the nominal ratio expressed in per cent. of the latter. If the actual ratio is higher than the nominal ratio, the meter will under-register and if it is lower than the nominal value, the meter will over-register.

The error due to the phase angle of the transformer is discussed in connection with power measurements. The amount of the error corresponding to various phase angles and power-factors is given in the tables on pages 170 and 171. It should be noted when applying these corrections to polyphase meters that the value of the power-factor to be used is that of the meter circuits and not that of the line.

In the case of potential transformers, the potential and the volt-ampere load connected to the secondary are constant. Consequently the errors are constant and can be compensated for by means of the regular speed adjustments on the watt-hour meter.

In current transformers, the connected volt-ampere load is also constant but both the ratio and the phase-angle errors vary with the current. Therefore, complete compensation cannot be made. The best that can be done is to compensate for the error corresponding to average load conditions by means of the speed adjustments in the meter.

Where the meter is a polyphase one with two similar current and two similar potential transformers, the error may be calculated on a single-phase basis from the average ratio and phase angles of the current and potential transformers, respectively, and the average power-factor of the circuit. Theoretically, the error should be computed for each element of the meter separately because the power-factor is different in the two elements. However, the torque of each element is inversely proportional to the power-factor, so that with similar sets of transformers, the percentage error in registering the load will be substantially the same.

It is to be noted that where high accuracy is desired and transformer errors are to be compensated, transformers should not be interconnected in any way because of the impossibility of determining the load and therefore the errors. Each meter element should be connected directly and independently to its own potential and current transformers. Under these conditions, the load and consequently the errors, are perfectly definite.

226. Three-wire, single-phase circuits may be metered with two single-phase meters or with a three-wire meter, a meter with two current coils, one of which is connected in series with each outer conductor of the circuit. The potential coil is preferably connected across the outer conductors. When the load is unbalanced or the power-factor less than unity, there will be a small error as in the case of a three-wire meter on a continuous-current circuit (par. 218). Where these conditions are serious, two meters should be used.

227. Polyphase Circuits.—The energy consumed in a polyphase circuit is measured in the same manner that the power in similar circuits is measured (pars. 189 to 194 incl.). The total energy in a two-phase or three-phase, three-wire circuit is the algebraic sum of the readings of two single-phase meters just as the total power is the algebraic sum of the readings of two wattmeters. A three-phase four-wire circuit requires three meters (unless balanced) that is, one meter less than the number

of wires. A three-phase circuit with a grounded neutral should be considered a four-wire circuit.

A polyphase meter may be used instead of two single-phase meters. Such a meter consists simply of two single-phase meter elements mounted on one frame with a common case and with a common shaft. Thus the algebraic summation is automatically performed and when one element tends to run backward (power-factor less than 50 per cent.) it simply counteracts the torque of the other element, so that the speed is still proportional to the net power in the circuit.

228. Phase Relations in Polyphase Meters.—Fig. 128 shows the phase relations between the current and the potential in a

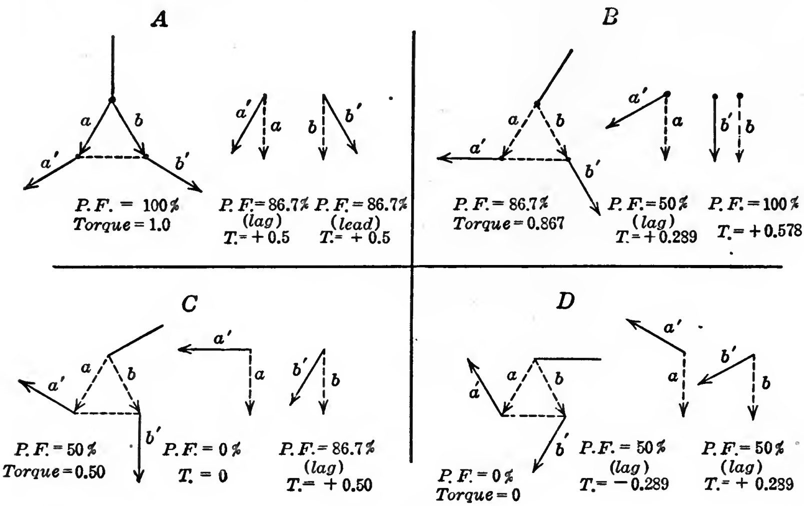


FIG. 128.

three-phase circuit with a constant volt-ampere load at various power-factors together with corresponding phase relations in the current and potential circuits of a polyphase meter, or two single-phase meters. The dotted lines represent potential vectors and the solid lines current vectors, the letters *a* and *b* identifying the corresponding currents and potentials in the line and in the meter circuits, respectively. The conditions at 100 per cent. power-factor are shown in Fig. 128 (*a*). Since there is an angle of 30° between the potential and the current in the meter element in each phase, each element produces half the total torque and operates at 86.7 per cent. power-factor, one lagging and the other

TABLE III

	Make and type					
	G. E. CO. I-10	Westinghouse AO	Fort Wayne K-4	Sangamo H	Duncan M	Columbia C
Speed, full load, r.p.m.	36.0	25.0	36.7	40.0	36.7	30.0
Torque, full load, gram-mm.	46.6	36.0	45.0	40.0	115.0	80.0
Weight, moving element, grams.	26.3	15.0	21.0	15.6	46.0	30.0
Ratio, torque to weight	1.77	2.32	2.14	2.5	2.5	2.66
Drop, current circuit, volts at 5 amp.	.....	0.3	0.12	0.1	.....	0.1
Loss, current circuit, watts at 5 amp.	0.98	0.75	0.59	0.5	.....	0.5
Loss, potential circuit, watts at 110 volts	2.5	1.6	1.75	1.85	1.25	1.5
Power-factor, potential circuit, per cent.	.....	18.0	17.0	35.0	.....	70.0

TABLE IV

Make	A	B	C
Type	I-10	I-14	K-5
Year	1913	1914	1914
Rating	5-amp., 110-volt, 60-cycle	5-amp., 220-volt, 60-cycle (3-wire)	5-amp., 220-volt, 60-cycle (3-wire)
Torque, full load, gram-mm.	50.4	50.7	47.5
Weight, moving element, grams	25.6	11.0	11.3
Ratio, torque to weight	2.0	4.6	4.2
Drop, current circuit, volts at 5 amp.	0.15	0.1	0.1
Loss, current circuit, watts at 5 amp.	0.6	0.4	0.4
Loss, potential circuit, watts at 110 volts	1.45	1.0	0.9
Power-factor, potential circuit, per cent.	10.0	18.0	18.0
Effect of power-factor variation, per cent. change:			
100 to 50 per cent. power-factor, 10 per cent. load.	1.2	2.4	2.3
100 to 50 per cent. power-factor, 100 per cent. load.	1.6	1.3	1.1
Effect of voltage variation, per cent. change:			
90 to 110 per cent. normal volts, 10 per cent. load.	0.1	0.6	0.7
90 to 110 per cent. normal volts, 100 per cent. load.	0.7	0.2	0.1
Effect of frequency variation, per cent. change:			
95 to 105 per cent. normal frequency, 10 per cent. load.	0.7	0.6	0.5
95 to 105 per cent. normal frequency, 100 per cent. load.	1.0	0.3	0.5
Range of light-load adjustment (at 10 per cent. load)	15.0	30.0	37.0
Range of full-load adjustment (at 100 per cent. load)	45.0	30.0	30.0
Starting load, watts	2.7	4.0	4.5
Temperature coefficient, per cent. change, per degree C.	0.10	0.08	0.07
Total friction, per cent. full-load torque (approx.)	0.2	.....	.....

leading. At 86.7 per cent. power-factor in the circuit (Fig. 128, *b*), one element is producing one-third of the total torque at 50 per cent. power-factor and the other element produces two thirds of the total torque at 100 per cent. power-factor. At 50 per cent. power-factor in the circuit (Fig. 128, *c*), the torque and power-factor in element *a* is zero; while element *b* produces the total torque at a power-factor of 86.7 per cent., lagging. At zero power-factor in the line, the power factor is 50 per cent. lagging in each element but the torques are in opposite directions and therefore produce no rotation (Fig. 128, *d*).

229. Typical Meter Data.—The following data in Table III apply to modern single-phase, 60-cycle, 5-amp., 110-volt meters. They are taken from the "Electrical Meterman's Handbook" issued by the National Electric Light Association, 1912.

The data in Table IV show the performance of three prominent makes of meters manufactured and tested in 1913 and 1914.

Installation and Maintenance of Watt-hour Meters

230. Selection of Suitable Meter Capacity.—The rating of a watt-hour meter should be so chosen that it will operate at as nearly full load as possible without danger of overloading. The average load should come within the limits of best accuracy, that is, between 25 per cent. and 125 per cent. of the rated capacity of the meter. In large installations, the individual conditions determine the proper rating to use. The "Code for Electricity Meters" (A. E. I. C. and N. E. L. A., 1912) recommends for small installations, the following capacity of watt-hour meters in per cent. of connected load.

	Per cent.
Residences and apartments	25
Retail stores, offices	60
Wholesale stores, lights used intermittently	25
Restaurants, cafés, non-flashing signs, etc., where all of the lamps are used at one time	100
One motor	100

In order to maintain the average accuracy as high as possible it is often advantageous to divide the load, and to meter separately small loads of long duration and large loads of short duration; or to combine two or more loads which are about equal but whose maximum values occur at different times.

Where current transformers are to be used, the ratio should if

possible, be such that the current will not fall below 25 per cent. of the normal rating of the transformer and thus insure operation at the straight portion of the ratio and phase-angle curves (par. 130). With this condition the ratio and phase-angle errors can be approximately compensated by adjusting the speed of the meter. The phase-angle and ratio errors increase with the volt-ampere load on the secondary of a transformer and therefore these errors should be kept a minimum by not connecting other apparatus to transformers from which watt-hour meters are being operated. The practice of connecting meters to transformers used for relays and tripping coils on high-tension switches is particularly objectionable because such devices function only at abnormally high currents and the current transformers are rated accordingly. Therefore, under normal load conditions, the transformer is frequently operated at 5 to 10 per cent. of its rating where the ratio and phase-angle errors are not only relatively large because of the small current and the large reactance of such devices, but the accuracy of the meter itself is lower.[1]

231. Location.—Watt-hour meters should be located as near the service entrance to the building as practicable, at a point subject to a minimum of vibration and free from moisture. They should be easily accessible for reading, inspecting, testing and so forth. The matter of accessibility for testing is very often overlooked when installing a meter, especially on switchboards, with the result that an abnormal amount of time is required every time the meter is tested and inspected. This wasted time would repay many times over the cost of a little extra care in locating the meter more conveniently.

Meters should be securely attached to a solid wall and carefully leveled. If several meters are installed together, the minimum distance between centers should not be less than 15 in. in order to eliminate the possible effect of the field of one meter on the other. Similarly, care should be taken that conductors leading to one meter do not affect another meter.

232. Connections.—Watt-hour meters are connected to the circuit in exactly the same manner as wattmeters (pars. 189 to 194 incl.), care being taken that the potential circuit is so connected that its current does not pass through the meter current circuit and thus cause the meter to measure its own potential

[1] "Measurement of Energy with Instrument Transformers," ALEXANDER MAXWELL, *Transactions*, A. I. E. E., vol. 31, p. 1545 (1912).

loss. In the case of three-wire meters one current coil is connected in each outer conductor, care being taken that rotation is in the same direction for both coils. Polyphase meters are connected just as two single-phase wattmeters are connected for power measurements.

It is obviously extremely important that the various circuits of a polyphase meter be properly connected. If, for example, the current-coil connections are interchanged, and the line power-factor is 50 per cent., the meter will run at a speed corresponding to 100 per cent. power-factor, thus giving an error of 100 per cent.

The test for correct connections for a polyphase meter is as follows: If the line power-factor is over 50 per cent. rotation will always be forward when the potential or the current circuit of either element is disconnected, but in one case the speed will be less than in the other. If the power-factor is less than 50 per cent., the rotation in one case will be backward.

When it is not known whether the power-factor is less or greater than 50 per cent., this may be determined by disconnecting one element and noting the speed. Then change the potential connection from the middle wire to the other outside wire and again note the speed. If the power-factor is over 50 per cent., the speed will be different in the two cases, but in the same direction. If the power-factor is less than 50 per cent., the rotation will be in opposite directions in the two cases.

233. Effect of Stray Fields on Watt-hour Meters.—Certain types of watt-hour meters are readily affected by external magnetic fields, particularly those which contain no iron, such as the usual types of continuous-current commutator meters. This external field may weaken or strengthen the drag magnets, increase or decrease the retarding torque, or, and more generally, increase or decrease the strength of the field in which the armature element is rotated. The effect on the magnets can be eliminated by using two or more magnets astatically arranged, a practice which is now common. The effect on the drag disc and on the meter field can only be eliminated either by removing the source of the field or by so locating the meter that there will be no component of the stray field in a direction which will give trouble. The usual sources of such fields are electric conductors and often these can be relocated so that there will be no influence on the meter.

The field intensity in modern, standard types of commutator watt-hour meters is about 75 to 150 lines per square centimeter. If the field strength of the meter is known, the effect of an adjacent conductor carrying current can be readily computed. The following table shows what error might be expected in a meter, having a field strength of 150 lines per square centimeter, when a conductor is placed about 2.5 in. directly behind and parallel to the meter shaft, in which position the plane of the conductor field coincides with the meter field and produces the maximum effect. Obviously in meters with other field strengths, the errors will be in proportion.

STRAY FIELD ERRORS, CONTINUOUS-CURRENT METERS, COMMUTATOR TYPE
Approximate error in registration, per cent.

Current in conductor, amperes	Per cent. load on meter					
	5	10	20	50	100	150
400	55	30	15	6	3	2
800	...	60	30	12	6	4
1,200	...	90	45	18	9	6
1,600	...	...	60	25	12	8
2,000	...	...	75	31	15	10

Continuous-current mercury-motor-type meters and alternating-current induction-type meters are much less liable to stray-field errors because they contain iron, and their field intensities are therefore very much greater than in the commutator-type meter. The error produced in modern induction meters by the field from a conductor carrying 50 amp. and placed in any position 15 in. behind the meter will not exceed 0.5 per cent. Reasonable care should be exercised, however, and meters should not be placed too close together nor close to conductors carrying very heavy currents.

Where stray magnetic fields are unavoidable, as in large central stations and substations, commutator-type meters are made more or less astatic by using two armatures, one above the other (each measuring half the total torque), or by making use of a four-pole meter field. The drag element, magnets and disc, are frequently enclosed in a separate shielding case of cast iron.

A test for the presence of stray fields where a commutator

meter is located, may be made as follows: A long thread is passed around the shaft of the meter three or four times so that by pulling the ends back and forth, the armature can be revolved rapidly. A portable galvanometer is connected to the brushes and with no current in the field coils, the armature is given a few rapid turns. If there is any appreciable field, an e.m.f. will be induced in the armature and produce an indication in the galvanometer.

234. Maintenance of Watt-hour Meters.—Watt-hour meters require careful maintenance if the average accuracy is to be maintained reasonably close to 100 per cent. The natural tendency is to run slow on account of increased friction due to bearing wear, commutator corrosion, brush wear, and so forth. On the other hand, the drag magnets may weaken with age or become partially demagnetized due to an excessive overload or short-circuits, thus tending to make the meter run too fast. The best interests of both the service company and the consumer are served by periodic inspections and tests of all meters. Commutator-type meters obviously require more attention than induction meters.

Modern practice in regard to the frequency of testing watt-hour meters is indicated in the following tabulation.[1]

Commutator-type meters	Minimum period between tests
25 amp. and less, 110 and 220 volts	12–15 months
50 to 150 amp., 110 and 220 volts	9–12 months
150 to 600 amp., 110 and 220 volts	6 months
Over 600 amp., 110 and 220 volts	3 months
All over 300 volts	6 months
Induction-type meters	
25 amp., and less, single-phase	24 months
25 amp., and over, single-phase	12 months
150 amp. and less, polyphase	12 months
150 amp. and over, polyphase	6 months

These periods are naturally subject to wide variation. When conditions are particularly favorable, such as where the load is very light for long periods, the frequency of testing may be decreased. On the other hand, where the moving element is heavy and the meter is loaded, more frequent tests are justified. Systematic records will give in a short time data from which the

[1] "Electrical Meterman's Handbook," National Electric Light Association, p. 356, 1912.

most economical test period for important meters can be determined.

Laboratory Tests of Watt-hour Meters

235. General.—Watt-hour meter testing may be divided into two kinds, laboratory tests and service tests. The former are made in the laboratory or test room for the purpose of adjusting the meter or to determine its performance characteristics under various conditions. Service tests are made on the meters where they are installed, primarily for the purpose of maintaining the accuracy at 100 per cent. as nearly as practicable.

Routine test-room tests usually consist in adjusting the meters to register correctly at light load (10 per cent.) and at full load with the adjustments in the middle position. It is noted that the meter does not creep at normal voltage, that it rotates at the proper minimum load and that the general operation is normal. In the case of alternating-current meters a test is also often made at a low power factor.

Tests which may be called special tests, as distinguished from those of a routine nature, obviously cover a wide range. The more usual tests for characteristic data and performance under various conditions are as follows:

Accuracy curve from 2 per cent. load to 150 per cent. load.
Effect of variation in power-factor.
Effect of variation in voltage.
Effect of variation in frequency.
Effect of variation in temperature.
Torque and weight of moving element.
Effect of external magnetic fields.
Effect of temporary excessive overloads.
Losses in windings.
Potential drop in current circuit, current and power-factor in potential circuit.
Starting load and load at continuous rotation.
Voltage at which creeping is produced.
Range of adjustments.

Other tests which are sometimes made include the following:

Friction, per cent. of full-load torque.
Field strength.
Amount of light-load compensation.
Effect of distorted wave form.

Those tests which present more or less difficulty in execution are discussed further in later paragraphs.

236. Meter Constants.—Various constants are used in connection with watt-hour meters. The principal ones are defined as follows:[1]

Register constant is the number by which the register readings must be multiplied to obtain the registration. They are ordinarily used only on large-capacity meters and are marked on the register.

Gear ratio is the number of revolutions of the rotating element per revolution of the first dial hand.

Watt-hour constant is the registration reduced to watt-hours per revolution of the rotating element. It has a definite value for each type and rated capacity of meter.

Watt-second constant is the registration reduced to watt-seconds per revolution of the rotating element. It is equal to the watt-hour constant multiplied by 3,600.

Test constant is the constant assigned by the manufacturer for use in the test formula for his meter.

237. Accuracy of Watt-hour Meters.—The accuracy or precision of a watt-hour meter is the percentage of the total energy passed through the meter which is registered on the register dials. The watt-hours registered in a given time are noted while the actual watts are simultaneously measured during the same period with standard instruments. On account of the time required to get an accurate reading from the register, it is customary to use the revolutions of the rotating element in a given short period of time instead of the register indications. Since the energy represented by one revolution, or the watt-hour constant, has been assigned by the manufacturer and marked on the meter disc, the watt-hours registered by the meter in a given period will be $K_h \times R$, where K_h is the watt-hour constant, and R is the number of revolutions. The accuracy of the gear ratio between the rotating element and the first dial of the register can be determined by count.

In the actual determination of the accuracy of registration of a watt-hour meter it is customary to convert the meter watt-hours to watts rather than the true watts to true watt-hours. Thus:

[1] "Code for Electricity Meters," pp. 95, 96.

$$\text{Per cent. accuracy} = \frac{\text{meter watt-hours}}{\text{true watt-hours}} \times 100$$
$$= \frac{\text{meter watts}}{\text{true watts}} \times 100$$
$$= \frac{\dfrac{K_h \times 3{,}600 \times R}{S}}{W} \times 100$$
$$= \frac{K_h \times 3{,}600 \times R}{S \times W} \times 100$$

where K_h = watt-hour constant, 3,600 = number of seconds in 1 hr., R = revolutions in the test period of S sec., W = true average power in watts during the test period as measured with indicating instruments. The last formula is the standard formula used in testing watt-hour meters.

When the test constant, K, as assigned by the manufacturer and marked (usually) on the disc, differs from the watt-hour constant K_h, the formula is changed accordingly as follows:

Make	K in term of K_h	Formula for meter watts
Columbia	$K = 3{,}600K_h$	$P = R \times K/S$
Duncan	$K = 66K_h$	$P = 60 \times R \times K/S$
Fort Wayne	$K = 36K_h$	$P = 100 \times R \times K/S$
General Electric	$K = K_h$	$P = 3{,}600 \times R \times K/S$
Sangamo	$K = 3{,}600K_h$	$P = R \times K/S$
Westinghouse	$K = 3{,}600K_h$	$P = R \times K/S$

K_h = watt-hour constant, K = test constant, R = number of revolutions in S sec., P = meter watts.

238. Source of Power.—Obviously the source of energy for meter testing should be as steady as possible. Storage batteries are largely used for continuous-current meters. Special alternators, whose speed can be controlled, are used for alternating-current meters. The testing load may be banks of lamps or rheostats in series with which the meter and the standard instruments are connected. A preferable method is to separate the current and the potential circuits and connect them to independent sources, the former being a relatively large current, low-voltage source and the latter a high-voltage, low-current source. Conditions are more easily adjusted by this method and with large meters, a saving of energy is effected.

Where separate sources are used for the current and the poten-

tial, carbon rheostats are convenient for adjusting the current, and high-resistance rheostats connected "potentiometer" style are convenient for controlling the voltage, as indicated in Fig. 129. It is to be noted, however, that resistance in the potential circuit of alternating-current meters will alter the quadrature phase relation (par. 222) and, therefore, the voltage regulation for alternating-current meters should be obtained with a variable ratio auto-transformer, an induction regulator, or by varying the field of the alternator.

239. Standards.—The correct measurement of the energy which passes through the meter involves the accurate measurement of power in watts (see Chapter VIII) and of time in seconds.

In testing continuous-current meters, the power is obtained by measuring the potential and current. The precision desired determines whether ordinary portable or special laboratory indi-

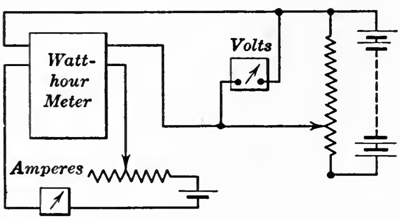


FIG. 129.

cating instruments should be used. Where reliability of results is important, potentiometers may be used with advantage if the source of the power is sufficiently steady. This is not because of their greater accuracy, which is much higher than the inherent variable performance of watt-hour meters justifies employing, but because the errors in indicating instruments, or those introduced in the application of their corrections, is eliminated. Incidentally, if potentiometers of the deflection type (par. 57) are used, a saving in time may be effected if the time required to check indicating instruments and apply their corrections is considered.

In testing alternating-current meters, indicating wattmeters are universally used to measure the power. While the power could be obtained from the potential and current as measured with a voltmeter and ammeter respectively, this method could be used only at unity power-factor and is not as accurate. Con-

tinuous-current, permanent-magnet type instruments are considerably more accurate on the whole than alternating-current instruments, so that while, with continuous current, power is more accurately measured with an ammeter and voltmeter than with a wattmeter, the reverse is true with alternating current.

In the measurement of time, stop watches are very generally used in laboratory as well as service tests. While stop watches usually indicate to $\frac{1}{10}$ sec., the error is considerably greater than this amount. The hand is pushed forward by steps of $\frac{1}{10}$ sec. (in high-grade watches) so that a portion of a step is lost when the second hand is thrown in or taken out of mesh. Furthermore, the observational errors in starting and stopping are not negligible although, being accidental errors, they tend to cancel themselves when several observations are made.

If readings are taken over a period of 100 sec., the errors in measuring time with a high-grade stop watch may be reduced to the order of ± 0.2 per cent., if correction for the inherent fixed error of the watch, as determined by comparison with a standard clock beating seconds is made. The greatest reliability of the result and the highest precision will be obtained by taking the average of five observations, each 100 sec. long, and made with five different watches. This procedure will require only a little more than the time required for a single observation if the first watch is started at the beginning of a certain revolution, the second watch at the beginning of the second revolution, etc., stopping them in the same order after the lapse of the required number of revolutions corresponding to about 100 sec.

High accuracy in timing watt-hour meters can be obtained, by employing a recording chronograph and a high-tension jump spark.[1] The scheme is shown diagrammatically in Fig. 130, A very light rider, P', made of a loop of fine wire, is placed on the edge of the drag disc, D, of the meter thus making a slight projection. A well-insulated needle point, P, is supported close to the edge of the disc and connected to the end of the high-tension winding of a small-capacity transformer, T, having a ratio of about 110 volts to 3,000 or 4,000 volts. The primary circuit includes an alternating-current relay, M, which in turn actuates the magnet that operates the chronograph pen, K.

[1] "A New Method of Timing Watt-hour Meters," Gordon Thompson, *Electrical World*, Feb. 1, 1913. See also *Bulletin*, Bureau of Standards, vol. 10, p. 175 (1914).

When the rider on the disc comes opposite the needle point, a spark passes and the sudden increase in the primary current increases the drop across the resistance, R, in series with the primary winding, thus causing the relay M to close the chronograph circuit. A contact-making clock, C, gives a record of seconds. The current is limited by suitable resistance, R', in the primary circuit. A precision of at least 0.1 per cent. is very easily obtained. In addition to high precision, the method has the advantages of being automatic and of leaving a permanent record of the speed.

Where meters are being tested or adjusted in quantities it is customary to use master or standard watt-hour meters and thus

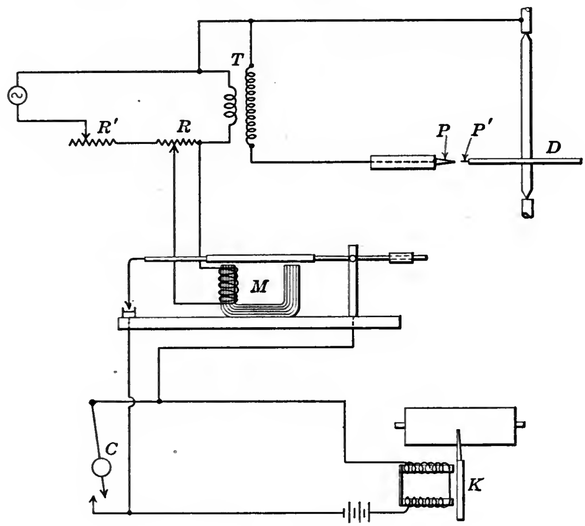


FIG. 130.

eliminate the separate measurement of time and watts. The accuracy is then obtained directly from the ratio of the revolutions in the same interval of time. The load does not have to be steady and by employing suitable mercury cup or other contact devices on the standard to operate a sounder, the revolutions of the standard can be noted by ear while the observer's entire attention is given to the meters being tested.

240. Power-factor variation for testing alternating-current meters can be obtained by several methods. In the *two-alternator* method, two generators are mounted on a common shaft.

The stationary members (either the armatures or the fields) are arranged to be movable about the shaft with respect to the base and to each other. Thus with the potential coil of the meter connected to one machine, and the current coil to the other, any phase relation can be obtained by shifting the movable member of one machine with respect to the corresponding member of the other machine.

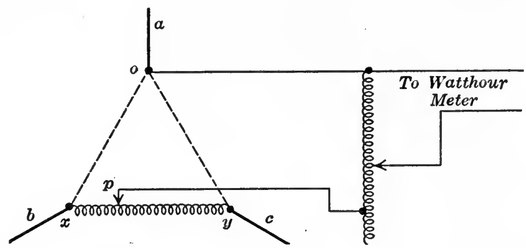


FIG. 131.

In the *transformer* method, a transformer with a large number of steps, or a variable-ratio auto-transformer, is connected across one phase of a polyphase circuit and the potential coil of the meter is connected in such a manner that any phase relation can be obtained. Thus, referring to Fig. 131, the current coil of the meter is connected in series with conductor *a* of a three-phase circuit, and the potential coil is connected to *o* and to

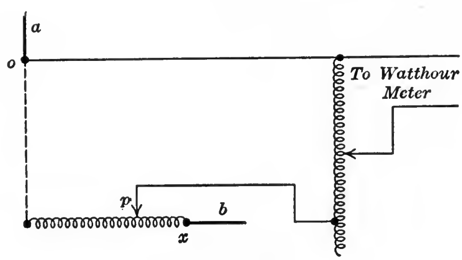


FIG. 132.

p, the latter being a tap on a transformer connected across phase *bc*. It is apparent that any phase angle between the current and the potential can be obtained in the range from 0 to 60° by moving the connection point *c* along suitable taps connected to the transformer winding. Angles from 60° to 90°, lead or lag, can be obtained by changing the transformer to either of the other two phases and the meter connection from *o* to the corresponding

point, x or y. These changes can be instantly made by providing suitable switching arrangements. A similar arrangement can be used on a two-phase circuit, Fig. 132.

It is convenient to introduce another similar transformer between the taps o, p and the meter, for the purpose of compensating for the variations in the voltage between o and p, when p is shifted, thus keeping the voltage constant at the meter. The connections are shown at the right of Figs. 131 and 132.

The movable contact, p, may be shifted from one fixed tap to another by a suitable rheostat type of switch or by means of a rotating arm making direct contact with the winding. A phase-shifting device employing this scheme is shown in Fig. 133. One auto-transformer changes the phase relation and the other

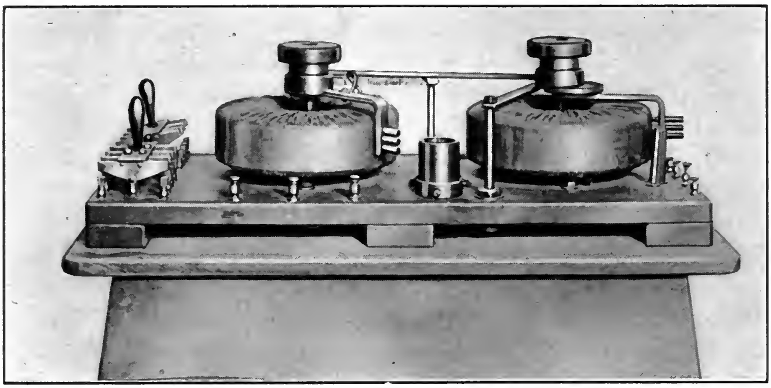


Fig. 133.

adjusts the voltage to a constant value as indicated in the preceding paragraphs. They are annular rings made up of about 35 lb. of ring punchings of sheet steel about 4 in. inside diameter and 9 in. outside diameter, the face of the ring being about 2.5 in. wide. A layer of fiber is placed on the face and a single layer of No. 16 B. & S. magnet wire is wound thereon. After being thoroughly shellaced and dried, the insulation is scraped off that part of the winding on the face of the rings. An arm pivoted at the center of the ring carries small carbon brushes which make contact with this exposed surface of the winding so that steps of one turn each are obtained. This particular design can be used with potentials up to 220 volts on either 25 or 60 cycles. While it will carry 5 amp., it is not desirable to employ

this type of compensator where much current is required because of the wave distortion due to magnetic leakage.

In the apparatus shown, there are two rotating arms on the potential coil. One arm is interlocked with the arm on the phase-shifting coil by means of a cam device in such a manner that when altering the phase angle by turning the left-hand handle, the right-hand arm on the potential coil, is automatically shifted the correct amount to maintain the voltage constant at the exact value which is fixed by the original setting of the second arm on the potential coil. By means of the gang switches at the right, any phase angle from 0 to 90°, and either lead or lag, can be obtained.

In the *reactance-coil* method, a reactance coil is introduced in the current circuit, the reactance being varied by moving an iron core in and out of the coil. It is difficult to obtain low power-factors with this method unless a separate low-potential current circuit is used, and in that case there is a possibility of wave-form distortion.

241. Torque.—The torque of a meter is usually measured under normal conditions and at rated load by noting the force in grams which is necessary to prevent rotation, when exerted at the edge of the drag disc or at the end of an arm attached to the shaft. The torque arm, that is, the perpendicular distance between the line in which the force is measured and the shaft, is usually measured in millimeters so that the torque is obtained in gram-millimeters.

The force may be measured in a variety of ways. The principle of a torque balance is shown in Fig. 134. A very light wire, *a*, is attached to the meter shaft, *s*. A light thread, *b*, is connected to this wire at right angles and attached to a light rod, *c*, which swings in a vertical plane on a knife edge, *k*, a little above its center. To this rod is attached a balanced crossarm, *mn*, on one side of which is a movable weight, *w*. When a torque is exerted on the disc, the force is counter-balanced by shifting the weight, *w*, until the vertical rod assumes its initial zero position. Then the meter torque is

$$T = \frac{ly}{x} \times w \qquad \text{(gram-mm.)}$$

T is in gram-millimeters, if l, y and x are in millimeters and w is in grams. Care should be taken that *a*, *b* and *c* are at right angles to each other.

In another scheme, the thread b is attached to the end of a light wire fastened to a shaft mounted between jewel bearings. Movement of this shaft is opposed by a spiral watch spring. When the torque is applied the spring shaft is brought back to its initial position (the two arms and the thread being kept perpendicular to each other) by a torsion head, to the outer end of which the spring is attached. A pointer, attached to the torsion head, moves over a circular scale and indicates the movement in degrees which was necessary. The corresponding torque is obtained directly in gram-millimeters by attaching weights to the arm. In order to eliminate the effect of the slight friction in

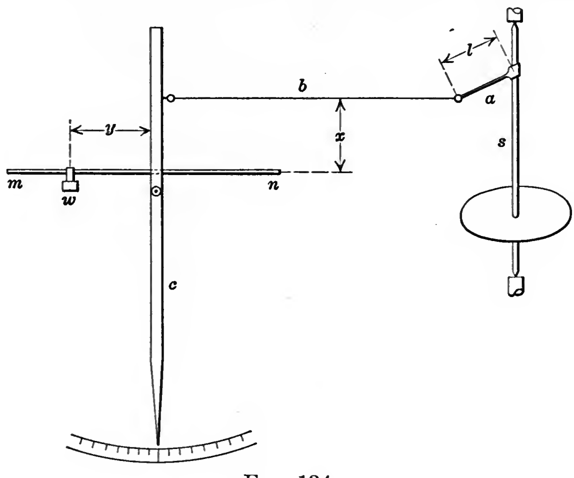


FIG. 134.

the bearings as much as possible, the average of the two readings should be taken. One should be taken when increasing the spring tension until the shaft is brought to the zero position from the low side and the other when decreasing to the zero position from the high side.

The Agnew form of torque balance[1] is a very simple arrangement in which the force component of the torque deflects the bob of a simple pendulum. The deflection produced is a measure of the torque.

242. Stray-field Tests.—A convenient method of producing a uniform field to test the effect of external fields on the per-

[1] "A Device for Measuring the Torque of Electrical Instruments," P. G. AGNEW, *Bulletin*, Bureau of Standards, vol. 7, p. 45 (1911).

formance of watt-hour meters is to use a Helmholtz arrangement of coils[1] constructed as follows: Two coils with rectangular cross-section are wound on the edges of two circular non-metallic discs which are supported in two vertical and parallel planes with their axes common. The mean distance between the coils is equal to the mean radius of the coils. The coils are connected in series in such a manner that their fields are cumulative. The field intensity throughout the central portion of the space between the planes of the two coils will be practically uniform and directly proportional to the current in the coils. The field intensity in lines per square centimeter will be

$$H = \frac{32\pi IN}{50r\sqrt{5}}\left(1 - \frac{d^2}{15r^2}\right) \quad \text{(gilberts per cm.)}$$

where I = current in amperes, N = number of turns in each coil, r = mean radius in centimeters, and d = depth of each winding in centimeters. By placing the meter at the center of the space between the coils and turning the latter to various positions, the effect of superposed fields in any relative direction can be determined.

With a pair of coils constructed according to the following specifications, the field intensity in the central portion will be almost exactly 2 gausses (lines per square centimeter) per ampere.

Size of wire (d.c.c. magnet), B. & S.	16.00
Number of turns per coil (N)	56.00
Number of layers	8.00
Number of turns per layer	7.00
Depth of winding, cm. (d)	1.12
Width of winding, cm	0.98
Mean radius, cm. (r)	25.18

The inductance of each coil will be about 5.1 millihenrys.

243. Field Strength.—The field strength of two-pole, continuous-current meters may be measured with Helmholtz coils as follows: The meter is placed so that the armature is at the center of the apparatus and with the axis of the field coils coinciding with that of the two coils. A portable galvanometer is connected to the brushes and a cord is passed around the shaft a few times so that by pulling alternately on one end and then on the other, the armature will be given a few rapid turns in one direction and then

[1] "Absolute Measurements in Electricity and Magnetism," ANDREW GRAY, vol. 2, p. 254.

the other. Normal current is passed through the meter field coils and current is passed through the coils of the apparatus in a direction to produce a field which opposes that in the meter. This latter current is adjusted until the meter field is just balanced as indicated by zero deflection of the galvanometer. The meter field is then obviously equal to that produced by the Helmholtz coils.

244. Losses.—The losses in continuous-current meters are best obtained by calculation from the resistances of the circuits as measured by standard methods (Chapter VII).

The loss in the potential circuit of alternating-current meters is measured with a wattmeter. As it amounts to only 1 to 3 watts, the measurements should be made on several meters together by connecting the potential circuits in parallel. The power-factor is very low, about 10 to 20 per cent., and, therefore, the wattmeter should be calibrated under that condition. Obviously, care should be taken that the circuits have reached normal working temperature and that the losses in the circuits of the measuring instrument are eliminated.

The iron loss in the current circuit is negligible for commercial purposes so that the total loss can be calculated with sufficient accuracy from the resistance as measured with continuous current.

245. Friction.—The driving torque of a commutator-type continuous-current meter is at all times strictly proportional to the field current, if the compensation coil is disconnected and the armature current kept constant. The total friction torque (bearing, brushes, windage and register gearing) can be conveniently measured with ample accuracy by removing the drag magnets, disconnecting the conpensation coil and noting the field current at which the speed remains constant at the full-load value; the armature current being kept constant in the meantime, at the normal value corresponding to normal potential when the compensation coil is connected. Under these conditions, the driving torque is equal to the friction torque. It is, therefore, the same proportion of the full-load torque as the corresponding current is of the full-load current.

This method is not applicable to alternating-current meters, however, because there are other losses beside those due to friction. Furthermore, the loss is not the same at such an extremely light load as at full load even though the speed is kept the same. In other words, the efficiency of the meter as an induction motor

is not the same under the two conditions. The friction torque can, however, be determined by the deceleration method and accurate results will be obtained if the work is carefully done. In this method the drag magnets are removed and the meter (now considered as an induction motor) is brought up to a high speed, that is, a speed considerably higher than the full-load speed. It is then disconnected from the circuit, and the total time elapsed (in seconds) from the beginning of the first revolution observed, is noted at the end of each successive revolution. Since, at any speed, the power required to balance the friction torque is supplied by the kinetic energy at that moment,

$$P = K\omega \frac{d\omega}{dt} \times 10^{-7} \qquad \text{(watts)}$$

where P = power in watts, K = moment of inertia of the revolving element, ω = speed in radians per second.

The value of $d\omega/dt$ is obtained from the time-revolution data as follows: The speed in radians per second (1 revolution = 6.28 radians) is calculated at the end of each revolution from the time elapsed between the end of the revolution just before and the end of the revolution just after the one being calculated. These speeds are then plotted against time and the tangent, ω/dt, is computed from this curve for that speed, in radians per second, which corresponds to the speed in revolutions per minute at which the friction torque is desired.

The moment of inertia of the rotor of an alternating-current meter can be calculated from the formula for a cylinder, that is,

$$K = \frac{mr^2}{2}$$

where K = moment of inertia in c.g.s. units, m = mass in grams and r = radius in centimeters. The moment need be calculated for the disc only, as that for the shaft is usually negligible.[1]

The friction torque corresponding to the power, P, is calculated from the relation

$$T = \frac{P \times 10{,}192 \times 60}{2\pi \times R} = 973{,}758 \frac{P}{R} \qquad \text{(gram-cm.)}$$

[1] For deceleration methods of measuring meter friction see "A Comparative Study of American Direct-current Watt-hour Meters," T. T. Fitch and C. J. Huber, *Bulletin* Bureau of Standards, vol. 10, p. 174 (Reprint No. 207). Also "Electric Meters," K. Schmiedel, *London Electrical Review*, Dec. 22, 1911.

where T = torque in gram-centimeters and R = speed in revolutions per minute.

246. Three-wire meters are usually tested as two-wire meters by connecting the two current circuits in series. This procedure assumes that the two field coils are equal, an assumption which can be checked by determining the accuracy with one coil in circuit at a time. If the coils are alike, the speed and hence the accuracy, will be the same in the two cases and equal to that with the coils in series. With the same current, the torque with only one coil in circuit will of course be only half that with both coils in circuit, but the accuracy curve is practically a straight line from 50 per cent. to 100 per cent. load, so that any difference of importance between coils will be apparent. However, at 10 per cent. load, the accuracy curve is not a straight line and therefore the torque must be kept the same with one coil as with two coils. That is, for a comparison at 10 per cent. load or less, the current with one coil should be double that with both coils.

247. Polyphase meters are usually tested as single-phase meters by connecting the current circuits in series and the potential circuits in parallel. It should be first determined, however, that the elements are equal in accuracy by testing each separately and also that there is no interference between the two elements, that is, no stray field from one element which affects the accuracy of the other element.

A test for independence of elements can be made by connecting one element to phase I of a two-phase circuit. With first 20 per cent. and then 100 per cent. rated current in this element, the other element is successively connected to phase I, as follows: (*a*) current circuit not connected, voltage circuit connected direct and then reversed; (*b*) voltage circuit not connected, current circuit connected direct and then reversed; (*c*) repeat, connecting to phase II instead of phase I. It will be observed that under these various conditions the maximum effect of any stray field from the current or voltage circuit of one element will be obtained in the second element because the stray field will be in phase with one or other of the two fields in the second element.

Service Tests of Watt-hour Meters

248. General.—There are two general methods of making tests on meters where they are installed: (*a*) with indicating

instruments and a stop-watch; and (*b*) with standard watt-hour meters. The general procedure is the same in both cases.

249. Procedure.—The usual practice in testing watt-hour meters at the place of installation is to cut the meter out of service by shunting the current circuit, the potential circuit being left connected to the service. The load for testing small meters (50 amp. and less) is usually obtained with a portable adjustable lamp bank or rheostat, connected across the line and in series with the current circuit of the meter. For large meters, portable water rheostats are often used but a more economical procedure is the employment of a portable storage battery of two or three cells, the current being controlled with a carbon rheostat in series. Step-down transformers with low-voltage secondaries are used for testing alternating-current meters; regulation of the current being obtained by varying the number of turns of the winding employed or by means of a carbon rheostat.

250. Indicating-instrument Method.—In this method, the time of a given number of revolutions at a known load is observed. With continuous-current meters, the load is measured with an ammeter and a voltmeter. The ammeter may be omitted by using as a load an accurately standardized resistance, the power in watts then being

$$W = E^2/R$$

where W = watts, E = volts and R = resistance of load in ohms. In the case of alternating-current meters, the load is preferably measured with a wattmeter.

251. Rotating-standard Method.—This method is the most used, because only one observer is required and it is more accurate with fluctuating loads. Rotating standards are watt-hour meters similar to standard house-type, service meters, except that they are made with extra care, and are usually provided with more than one current and one potential range, and are more portable. A pointer, attached directly to the shaft, moves over a dial divided into 100 parts, so that fractions of a revolution are easily read. This standard meter is used by connecting it in series with the meter to be tested; the accuracy of the latter is determined by the "switch" method or the "eye-and-ear" method.

In the "switch" method, the register only (in continuous-current standards), or the entire moving element (in alternating-

current standards) is started at the beginning of a revolution of the meter under test by means of a suitable switch, and stopped at the end of a given number of revolutions of the meter under test. The accuracy is determined by direct comparison of the number of whole revolutions of the meter under test with the corresponding whole number of revolutions and a fraction, of the standard.

In the "eye-and-ear" method, the number of whole revolutions of the standard is compared with the corresponding whole number of revolutions and a fraction, of the meter under test. The revolutions of the standard are counted by ear by means of a telephone receiver and an electrical contact on the shaft, while those of the test meter are observed by eye.

252. Large-capacity Continuous-current Meters.—When the meter is of the shunted type and certain data are known, the meter itself can be disconnected from the shunt, checked as a 5- or 10-amp. meter as the case may be, and the overall accuracy computed. The required data are the resistance of the shunt between the points where the meter is connected, together with the potential which is necessary at the shunt end of the leads from the meter to the shunt to force various values of current through the meter. Furthermore, these data must be known for working conditions: it must be known that the resistance of the shunt and the instrument circuit is constant for all temperatures, that any thermo e.m.fs. present are negligible and that there were no loose contacts before the circuits were disturbed. Usually, positive information on all of these points is lacking, so that a test of this kind is generally not very reliable.

The only entirely reliable method of testing large-capacity meters is to connect a standard in the main circuit in the regulation manner. Rotating standards cannot be used in the ordinary way because it is not practicable to construct them above about 100 amp. capacity. Indicating instruments are, therefore, the usual standards and where the load on the circuit is sufficiently steady, or it is practicable to provide a separate rheostat load, or use a storage battery, the indicating instrument method is satisfactory. When, however, the load is unsteady and the meter is so large that a separate load cannot be provided, an accurate test is difficult to make. This is particularly the case with meters on railway circuits. The best that can be done is to take a large number of observations of short duration perhaps only 15 sec. or even 10 sec. long.

A method has been proposed[1] which meets this last condition by employing a Wheatstone-bridge arrangement of shunts together with a rotating standard. The meter does not have to be cut out of service. The connections are shown in Fig. 135, where *a* and *b* are two fixed standard resistances or shunts, forming two arms of the bridge. The third arm is shown at *c*. A rotating standard with an adjustable fixed resistance *d* and a carbon rheostat *e* constitute the fourth arm. When *e* is adjusted until the portable galvanometer shows zero deflection, the ratio of watts passing through the two meters is $a/(a + b)$.

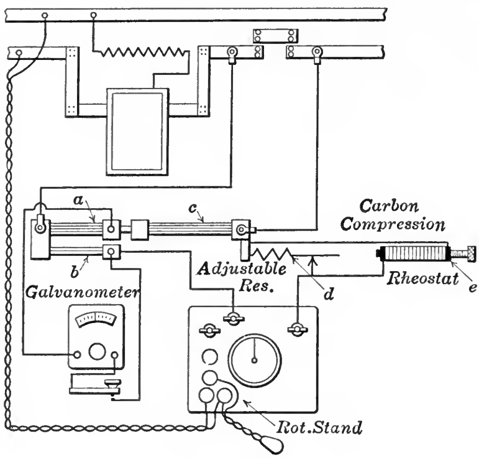


Fig. 135.

253. Average Accuracy of Watt-hour Meters.—The accuracy of a meter varies with the load so that it is impossible to represent the accuracy at all loads by a single figure. However, it is often desirable to assign a value for the average accuracy. Such a value obviously must be based on an arbitrary rule. The "Code for Electricity Meters"[2] recommends the following: When feasible, test the meter at 10 per cent. and 100 per cent. of capacity and at the "normal load." Multiply the accuracy at normal load by three, add to this the accuracies found at each of the other two loads and divide the sum by five. This result is

[1] "Wheatstone-bridge Rotating-standard Method of Testing Large-capacity Watt-hour Meters," C. H. Ingalls and J. W. Cowles, *Transactions*, A. I. E. E., vol. 31, p. 1551 (1912).

[2] *Q.v.*, p. 99, 1912 edition.

to be taken as the average accuracy. The "normal load" shall be taken as the percentage of the total rating of the connected load indicated in the following table.

	Per cent.
Residence and apartment lighting	25
Elevator service	40
Factories (individual drive), churches and offices	45
Factories (shaft drive), theatres, clubs, entrances, hallways, and general store lighting	60
Saloons, restaurants, pumps, air compressors, ice machines and moving picture theatres	70
Sign and window lighting, and blowers	100

254. Precautions in Watt-hour Meter Testing.—There are numerous precautions which should be observed in testing watt-hour meters. The following are the most important:

The test period should always be sufficiently long and a sufficiently large number of independent readings should be taken to insure the desired accuracy. In service tests, the period should preferably be not less than 30 sec. and the number of readings not less than three. In laboratory tests, five 100-sec. readings are preferable.

The capacity of the standards should be so chosen that readings will be taken at reasonably high percentages of their capacity, in order to make observational or scale errors as small as possible.

Where indicating instruments are used on a fluctuating load, their average deflections should be estimated in such a manner as to include the time of duration of each deflection, as well as the magnitude.

Instruments should be so connected that neither the standards, nor the meter being tested, are measuring the potential-circuit loss of the other; that the same potential is impressed on both, and that the same load current passes through both.

When the meter under test has not been previously in circuit, sufficient time should be allowed for the temperature of the potential circuit to become constant, preferably not less than 10 min.; this is important with continuous-current meters, especially in the case of rotating standards. In some types of the latter, special provision is made for rapid heating.

The effect of stray fields must be guarded against by locating the standards and arranging the temporary test wiring in a judicious manner (see par. 233). The effect on a continuous-

current rotating standard may be reduced to a minimum by placing the plane of the field coils in the direction taken by a compass needle.

Specifications for Watt-hour Meters

255. General.—Probably the most complete specifications for the performance, installation, testing and maintenance of watt-hour meters in the United States are those included in the "Code for Electricity Meters"[1] prepared by the Electrical Testing Laboratories under the joint direction of the Meter Committees of the A. E. I. Co.'s and the N. E. L. A. The Code is generally recognized as authoritative. It is the basis of the rules issued by many of those State public utility commissions which have adopted regulations covering the subject of electricity meters. While scientific accuracy and correct technical principles are the basis of the Code, the commercial phase of meter practice is not neglected. It covers the entire field of representative American meter practice, including definitions, standards, specifications for meters and auxiliary apparatus, installation tests and maintenance. The essential features of the performance specifications for acceptable meters are given herewith.

256. Accuracy under Normal Conditions.—

Load, per cent. of rated current	Permissible maximum deviation from 100 per cent. accuracy	
	C.c. meters	A.c. meters
2.0	(Rotation shall be continuous)	
5.0	±7.5	±3.0
10.0	±3.0	±1.5
20.0	±2.0	±2.0
50.0	±2.0	±2.0
100.0	±2.0	±1.5
150.0	±2.0	±3.0

The difference between the accuracy at 10 per cent. and that at 100 per cent. of rated current shall not exceed, in continuous-current meters, 4 per cent.; and in alternating-current meters, 2 per cent. No meter shall rotate (creep) at zero load, even with 110 per cent. of normal voltage.

[1] Copies of the "Code" may be obtained from the National Electric Light Association, 39 West 39th Street, New York City.

257. Permissible Effect of Voltage Variation.—

Load, per cent. of rated current	Permissible maximum deviations from normal accuracy			
	C.c. meters		A.c. meters	
	Voltage		Voltage	
	10 per cent. low	10 per cent. high	10 per cent. low	10 per cent. high
11.1	±5.0	±5.0	±2.0	±2.0
100.0	±3.0	±3.0	±1.5	±1.5

258. Permissible Effect of Frequency Variation.—A change of 5 per cent. either way from the rated frequency shall not cause a change of more than 1.5 per cent. in accuracy at either 10 per cent. or 100 per cent. of rated current.

259. Permissible Effect of Power-factor Variation.—

Load, per cent. of rated current	Power-factor, per cent.	Permissible maximum change from normal accuracy
13.3	75	±2.0
20.0	50	±4.0
100.0	75	±2.0
100.0	50	±4.0

260. Permissible Effect of Temperature Variation.—The accuracy of both continuous-current and alternating-current meters shall not change more than 0.2 per cent. per degree C. from 20°C. to 40°C. at either 10 per cent. or 100 per cent. of full load. This applies to meters operating on shunts, whether the shunt and the meter are at the same temperature or at different temperatures.

261. Permissible Effect of Temporary Overloads.—The application of an overload of 300 per cent. (400 per cent. of meter rating) for 2 sec., three times in succession, shall not change the accuracy more than indicated in the following table:

Load, per cent. of rated current	Permissible maximum change from normal accuracy	
	C.c. meters	A.c. meters
10	±5.0	±1.0
100	±3.0	±1.0

At least 15 min. should elapse after the application of the overload, to permit the meter to reach normal temperature.

262. Permissible Effect of External Magnetic Fields.—(*a*) *Continuous-current Meters.*—An approximately uniform field of 0.1 line per square centimeter at the center of the armature and in the direction to produce the greatest effect, shall not produce a change of more than 2.5 per cent. in the accuracy at 10 per cent. load.

(*b*) *Alternating-current Meters.*—The external field is to be produced by a current of 50 amp. in a straight conductor 6 ft. long, with return leads arranged to form a rectangle 6 ft. square, lying in the plane of the board on which the meter is mounted. The accuracy of the meter shall not be changed more than 2.5 per cent. at 10 per cent. load with the conductor in any of the following positions: (1) 15 in. (38.1 cm.) behind the shaft and horizontal; (2) 15 in. behind the shaft and vertical; (3) 15 in. to the right or left of the shaft.

263. Permissible Effect of 100 Per Cent. Unbalancing in 3-wire Meters.—When the entire load is carried by either coil, the change in accuracy at 10 per cent. and 50 per cent. loads (3-wire basis), in both continuous-current and alternating-current meters, from that under normal conditions, shall not exceed 2.0 per cent.

264. Permissible Voltage-drop in Current Windings.—At 100 per cent. rated current, the drop shall not be more than 1.5 per cent. of the rated voltage for meters of less than 10 amp. capacity in either continuous-current or alternating-current meters. For meters of 10 amp. capacity and over, the drop shall not exceed 0.75 per cent. of the rated voltage.

265. Polyphase Meters.—Before polyphase meters may be tested as single-phase meters (current coils in series and voltage coils in parallel), a test for independence of elements shall not show a deviation from the normal accuracy of more than 0.1 per cent. under any of the conditions indicated. This test is made by connecting one element in the usual manner and noting the accuracy at 20 per cent. and 100 per cent. loads with direct and reversed connections, respectively, on the voltage and current windings in turn, of the other element. If the deviation is greater than 1 per cent. the meter must comply with certain performance specifications which are prescribed for polyphase meters and it must always be tested in service on a polyphase

circuit. If the deviation is less than 1 per cent. the meter may be tested as a single-phase meter.

Ampere-hour Meters

266. General.—Commercial measurements of quantities of electricity are usually made with ampere-hour or coulomb meters. Their principal application is in connection with the operation of storage batteries but they are used extensively in Europe—and to a very limited extent in this country—as the basis of charging for electrical energy consumed by small customers of central-station companies. Where so used, the potential is assumed to remain constant at a "declared" value and the scale is graduated in the corresponding kilowatt-hours instead of coulombs or ampere-hours. This "declared" value of potential is that potential which, if multiplied by the ampere-hour consumption, would give the kilowatt-hours which would have been shown by a watt-hour meter if such a meter had been in circuit. Obviously, this potential cannot ordinarily be stated in advance with exactness and, therefore, this method of determining the energy consumption is subject to more or less error. However, where the charge is low and the consumption small, this disadvantage may be more than counter-balanced by the lower maintenance and investment costs, particularly where the chemical type of meter is used. Some engineers believe that this method of metering is the only one that will meet the growing demand for a low-cost method for use with very small consumers.

Ampere-hour meters may be divided into two classes: electrochemical meters and electromotor meters.

267. Electrochemical Meters.—Meters of this type are essentially voltameters. The quantity of electricity is measured by the effect of electrolyzing a suitable substance by the passage of all or of a part of the load current. A zinc-sulphate solution was used in the Edison ampere-hour meters, but water or mercury are employed in the principal types of modern meters.

In the *Edison chemical meter,* which is now obsolete, two zinc plates were immersed in a sealed jar containing a zinc sulphate solution and connected in parallel with a shunt which was in series with the load. The energy consumption was obtained by measuring the decrease in weight of the anode (positive) plate, 1.224 grams representing 1 amp.-hr.

The *Bastian* ampere-hour meter is a modern meter in which the electrolysis of water is employed. The change in the length of a column of water as it is decomposed by the passage of the current, is used to measure the consumption. A scale at the side of the tube is calibrated to read in kilowatt-hours at the declared voltage. A little caustic soda is added to the water to decrease the resistance and a layer of oil is floated on top of the water to prevent evaporation and to facilitate reading.

The *Wright* meter is the principal example of the mercury type. Metallic mercury is carried into a mercurous nitrate solution from a platinum-cup anode and deposited in a platinum-cup cathode, from which it flows into a fine tube. The height of this column is a measure of the energy consumption, a graduated scale being placed at the side of the tube. The electrodes are connected in parallel with a shunt, so that only a small portion of the load current passes through the solution. Provision is made for easily emptying the mercury from the tube into the anode receptacle when the former becomes filled. This type of meter has been highly developed and inherent errors due to variation in temperature, concentration of the solution, level of mercury, effect of vibration, etc., are largely eliminated in the latest forms.

268. Electromotor Meters.—Electromotor ampere-hour meters are similar to watt-hour meters, except that the field is produced by permanent magnets instead of electromagnets. The rotating element is geared to a register which is calibrated in watt-hours for a given assumed voltage. There are two general types, the electromagnetic and the mercury flotation. The former is not made or used very much in this country.

The *Chamberlain and Hookum* meter is an example of the electromagnetic type. It employs a flat (pan-cake) armature winding mounted on an aluminium disc which also serves as the drag element. Connection is made to the circuit through a commutator and brushes in the usual manner. The armature is connected to a low-resistance shunt which is in series with the load.

The *Sangamo ampere-hour meter* is a well-known example of the mercury-flotation type. It is practically the same as the Sangamo continuous-current watt-hour meter, the essential difference being the substitution of permanent magnets for electromagnets.

CHAPTER X

MAXIMUM DEMAND MEASUREMENTS

269. General.—The various costs which make up the total cost of supplying electrical energy from a central station to a consumer may be classified as follows:

(*a*) Costs which vary with the number of consumers such as installation of the service, testing and maintenance of the meter, meter reading, billing, and so forth. These costs are independent of the energy consumption or the rate of energy consumption.

(*b*) Costs which vary with the amount of energy consumption such as fuel, water, station labor, and so forth.

(*c*) Costs which vary with the maximum rate of energy consumption or maximum demand on the station, principally investment charges, such as interest, depreciation and insurance.

While it is generally conceded that the cost of serving the individual consumer is made up of a portion of each of these costs, there is a wide difference of opinion as to how these costs should be apportioned in the consumer's bill. Consequently, there are in use a great many different methods of charging for electrical service, from the simple flat-rate method where a fixed charge per month is made irrespective of the rate of amount of energy consumption, to the more or less complicated rate systems involving a number of factors which are intended to be proportional to the various classes of costs that make up the total cost of serving the consumer. However, the strictly flat-rate system is being rapidly abandoned in favor of one based at least on the energy consumption or the maximum rate of energy consumption. And in the case of moderate size and large consumers, the practice of charging on both the energy consumption basis and the maximum rate of consumption or maximum demand basis is increasing. Consequently, instruments and methods for measuring the maximum demand, are becoming of greater and greater commercial importance.

270. Definition of Maximum Demand.—Demand and maximum demand, respectively, are defined as follows in the standardization rules of the A. I. E. E. (June, 1916).

"The demand of an installation or system is the load which is drawn from the source of supply at the receiving terminals averaged over a suitable and specified interval of time. Demand is expressed in kilowatts, kilovolt-amperes, amperes or other suitable units.

"The maximum demand of an installation or system is the greatest of all the demands which have occurred during a given period. It is determined by measurement, according to specifications, over a prescribed time interval."

The maximum demand is, therefore, the greatest *load* drawn from the source of supply by an installation, as *averaged* over a suitable and specified interval of time. It is to be noted that "load" and "average" are not defined; so that a statement of maximum demand is not definite unless the quantity involved and the method of averaging is known, as well as the interval of time. It is not commercially practicable to prescribe simply one method of determining maximum demand for all classes of services. On the other hand, assuming that the integrated average of the prescribed quantity (amperes, kilovolt-amperes or kilowatts) in the prescribed time interval is the true demand, it is not commercially possible to use an instrument which is capable of measuring the maximum of such demands under all conditions of load. Therefore, the maximum demand is determined "by measurement according to specifications" agreed upon for each case, in other words, as measured by a particular type of instrument. As a matter of fact a number of devices have been developed, each of which measures a quantity peculiar to itself and differing materially from the integrated average. This distinction is discussed further in connection with time-lagged maximum-demand instruments, described later.

The "Code for Electricity Meters"[1] prescribes that:

"The practical interpretation of maximum demand is as follows: In commercial practice the maximum demand of an installation or a system is given by the record or indication of a demand meter of acceptable type which is correctly installed, properly adjusted, and none of the errors of which exceeds the limits of commercial tolerance."

271. Length of Demand Period—The period of time over which demands are measured varies from 1 min. to 1 hr. Each electricity supply company adopts a time interval which appears

[1] *Q. v.*, section X, 1916.

to be best suited to the particular local conditions. While there are certain local conditions, such as character of the load, which must determine the length of the period in certain cases, an effort is being made to bring about a greater uniformity than there is at present, especially for general-service consumers. The tendency is toward the standardization of the 15-min. interval.

272. Methods of Determining Maximum Demand.—The maximum demand as required for billing purposes may be determined (*a*) by estimation based on previous measurements, (*b*) by occasional measurement, (*c*) from maximum-demand instruments permanently installed.

Estimated demands are usually based on tests made on a large number of installations of each class, the number being sufficiently large to give reasonably reliable average results. A fixed value of maximum demand is then assigned for all consumers in the same class. Obviously, this practice can only be applied to small consumers (connected load, 2 or 3 kw. and less) and where the daily load cycle is known to be substantially uniform. The following table shows the results of some tests of this character made in Chicago.[1]

MAXIMUM DEMAND OF SMALL CONSUMERS

Kind of business	Number of customers	Equivalent connected load, 16-cp. lamps	Ratio of max. to connected
Amusements	245	32,592	56.3
Banks	100	9,102	66.8
Buildings (public)	18	3,152	33.6
Churches	15	3,301	56.0
Clubs	65	12,071	28.8
Flats	24,177	328,939	54.1
Hotels	103	9,972	28.0
Business offices	3,704	77,230	64.2
Pool and billiards	97	2,711	64.5
Restaurants	350	20,846	52.3
Saloons	2,060	50,002	62.6
Barber, inc. boot-black	443	4,883	70.4
Tailor shops	552	16,796	59.3
Livery stables	103	2,612	52.3
Small stores	49	3,069	83.1
Auto garages	204	7,596	60.5

[1] "Method of Determining Maximum Demands for Rate-making Purposes," E. G. RALSTON, Indiana Electrical Association, 1915.

Where the daily load curve is not constant or where the connected load is apt to be changed from time to time, the maximum demand may be occasionally measured by taking readings every few minutes with indicating instruments or with a watt-hour meter during that part of the day when the load is known to be highest. Obviously, a graphic instrument or a maximum-demand instrument may be temporarily installed for this purpose.

Maximum-demand instruments are permanently installed and used for the direct measurement of maximum demands where greater accuracy is desired than can be obtained by the two previously mentioned methods. This chapter is largely devoted to a discussion of these instrumental methods for the direct measurement of maximum demand.

273. Classification of Maximum-demand Instruments.—Maximum-demand instruments may be broadly divided into two groups according to the character of the device, namely: (1) those from which an integrated value of power is obtained; (2) devices which are time-lagged and, therefore, measure a quantity which is not an integrated value.[1] Each class may be further subdivided as follows:[2]

274. Integrated Demand Instruments (Class 1).—The quantity obtained with this general class is usually kilowatts, and is the ratio of the kilowatt-hours consumed in the interval, to the interval expressed in hours. There are four principal subdivisions as follows:

(*a*) Instruments in which a continuous-line record, or chart, of the power is made. These include the ordinary curve-drawing instruments, wattmeters or ammeters. The integrated maximum demand has to be obtained by means of a planimeter (or equivalent integrating operation), that interval being selected by inspection which will give the maximum integrated value. Obviously, this class of instruments gives complete information in regard to the load including in addition to maximum demand, the time at which the maximum demand occurred, maximum demands based on other time intervals, load factor, and so forth. Examples: General Electric Co. and Westinghouse Electric and Manufacturing Co. graphic wattmeters.

[1] Devices which limit the amount of power that can be used by the consumer to a predetermined value are not included in this discussion as they are not, strictly speaking, measuring devices.

[2] Based on the classification in section X of the "Code for Electricity Meters," N. E. L. A. and A. E. I. C., 1916.

(*b*) Instruments which give a record of the demand for each successive time interval as fixed by a clock or other timing device, time being also recorded. The demand for each clock interval, and, therefore, the maximum demand, can be obtained directly from the record. If, however, the maximum demand occurred in an interval beginning at the middle of one clock interval and ending at the middle of the next, it would not be measured by an instrument of this class. For example, in a 15-min. demand instrument, the highest 15-min. demand in the intervals 1.00 to 1.15, 1.15 to 1.30, 1.30 to 1.45, etc., would be shown, but if a higher demand occurred in the interval 1.05 to 1.20, or 1.25 to 1.40 it would not be shown. Examples: General Electric Co. types P and G demand indicators, Westinghouse Electric and Manufacturing Co. type RA recording demand watt-hour meter, Piek demand indicator.

(*c*) Instruments which only indicate and do not record the maximum demand, the time intervals being fixed by a clock or other timing device. No record of time is made. Examples: General Electric Co. demand meters, types M_1, M_4 and M_5.

(*d*) Instruments which make a record on a tape or chart when a certain fixed and predetermined amount of energy has been consumed, time being also recorded. This class differs from class (*b*) in that the demand is shown for any time interval irrespective of when the interval began and ended, thus giving the true maximum demand. In other words, when a predetermined quantity of energy has been consumed a record is made, while in Class (*b*) instruments, a record is made when the predetermined time interval has elapsed. It differs from Class (*a*) in that the record is not a continuous one but periodic. Example: Ingalls demand recorder.

275. Time-lagged Demand Instruments (Class 2).—The value measured by instruments in this general class differs from the integrated value by an amount which depends upon the particular instrument and the character of the load. With a perfectly steady load all forms of instruments of this class would indicate the same value as the integrating instruments, but with a fluctuating load each may give a different result. Hence, when referring to demands measured with instruments in this class, the particular instrument employed should be mentioned.

The principal feature of these instruments is that with a steady load, the full value is not shown until the demand interval has

elapsed. In other words, a retarding device of some kind is used so that a definite period of time must elapse before the full value is indicated. The time interval is independent of clock time. These instruments are indicators only, no record of either time or demand being made. They may be subdivided into two classes, as follows:

(*a*) Instruments in which the rate of motion of the indicator decreases with the time of the deflection. Examples: General Electric type W demand meter; General Electric type H demand meter; Wright demand meter; and Lincoln watt demand meter.

(*b*) Instruments in which the rate of motion of the indicator over the scale is always proportional to the load. Example: Westinghouse type RO maximum demand watt-hour meter.

Descriptions of Demand Instruments.

The following are brief descriptions of the examples mentioned in connection with the preceding classification.

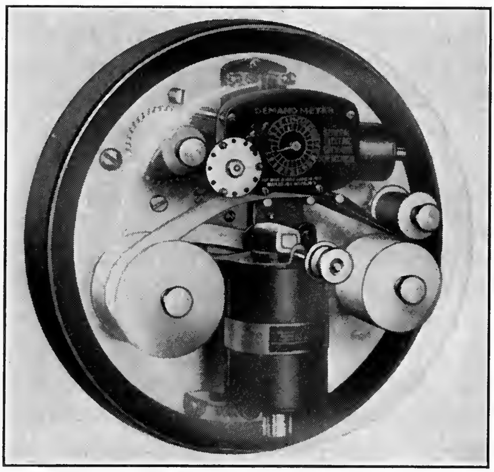


Fig. 136.

276. Integrated Demand Instruments (Class 1*a*).—Standard types of graphic wattmeters and ammeters form this class. They are described in Chapter XVI.

277. Integrated Demand Instruments (Class 1*b*).—*General Electric Co. Type P Demand Indicator.*—This instrument is the successor to the "printometer" formerly manufactured by the

Minerallac Co. It is a device which is installed and used with a watt-hour meter. Fig. 136 is from a photograph and Fig. 137 shows diagrammatically the arrangement of the electrical circuits and the relation of the important features. A set of cyclometer-type wheels in the device are electrically interlocked with the register of the watt-hour meter so that a contact-maker on the first gear shaft of the meter closes a circuit through a small plunger-type solenoid which in turn pushes the cyclometer forward one unit. Thus the cyclometer moves forward at a rate exactly proportional to the speed of the watt-hour meter, or, in

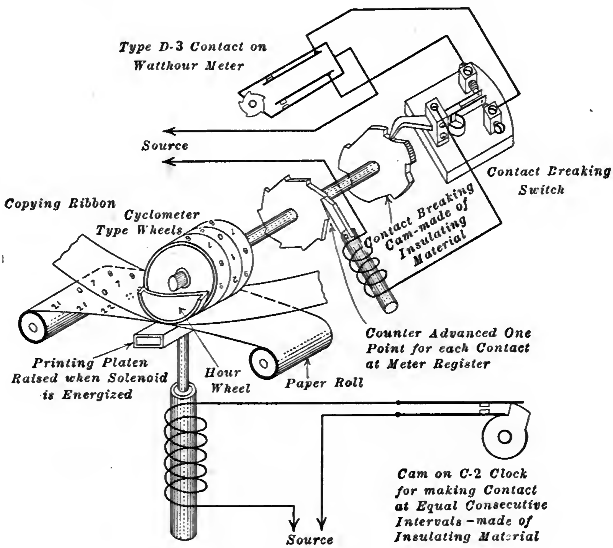


FIG. 137.

other words, to the power. A paper tape is brought into contact with the cyclometer-type wheels by a solenoid-operated printing platen which is operated by a contact-making clock or other timing device at predetermined intervals. The time is simultaneously printed on the tape. Thus the difference between two successive numbers on the tape represents a definite amount of energy in kilowatt-hours consumed in the interval elapsed between the two printings, this interval having been previously assigned and the instrument adjusted accordingly. The maximum demand in kilowatts is of course the greatest difference found between successive numbers multiplied by a

constant (kilowatt-hours per unit of cyclometer) and divided by the interval expressed as a fraction of an hour. Fig. 138 shows a part of a specimen record, the point of maximum demand being indicated by the arrow.

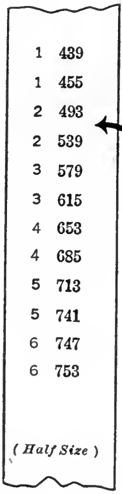


Fig. 138.

General Electric Co. Type G Demand Meter.—This instrument (Fig. 139, case open and cover disc removed) is the successor to the Minerallac Co. "graphometer." Like the type P meter it is installed and used in conjunction with a watt-hour meter. The principle of operation is similar to that of the type P meter. The cyclometer-type wheel arrangement of the latter is replaced by a ratchet and pawl device which moves a steel stylus in a radial direction over a specially prepared, circular paper chart, driven by a clock. The pawl is actuated by a solenoid which receives impulses through a contact-maker on the watt-hour meter shaft. Every impulse moves the stylus outward over the chart from the center a fixed amount (corresponding to one unit on the cyclometer) and at the end of the time interval for which the instrument is adjusted, the stylus is automatically returned instantly to the zero position

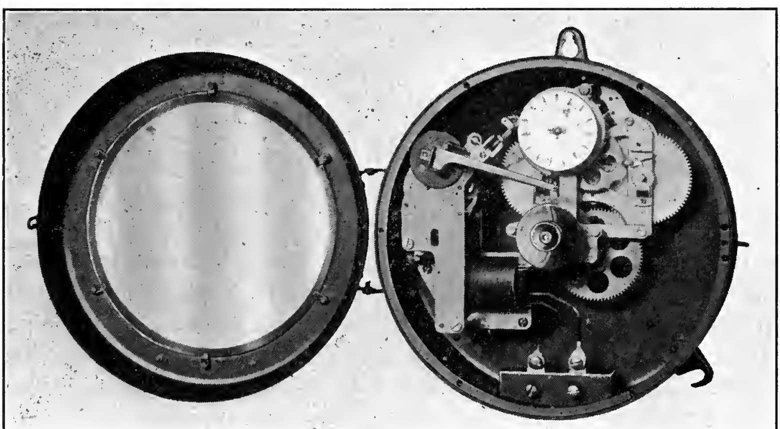


Fig. 139.

by means of a cam. Thus the distance travelled by the stylus between "resettings," that is, the length of the line drawn, is proportional to the energy consumption during that period. The

chart is in continuous rotation and the record is a series of nearly parallel radial curves, one for each interval (Fig. 140). The

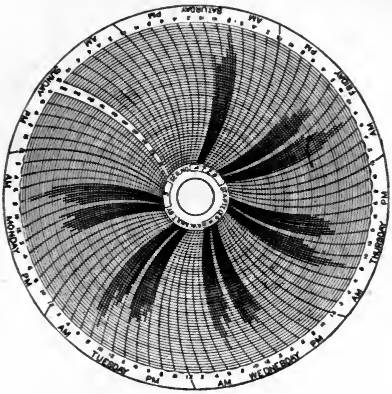


FIG. 140.

longest one corresponds to the maximum demand. Fig. 141 shows diagrammatically the operation of this instrument.

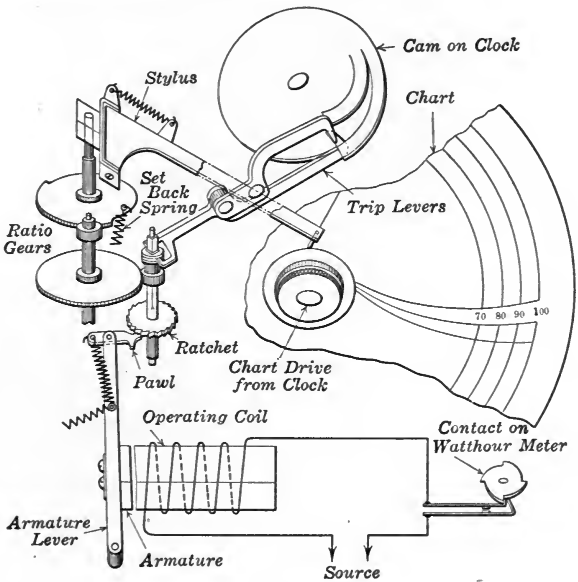


FIG. 141.

The charts are calibrated radially in kilowatt-hours and circumferentially in hours, so that knowing the time interval, the

maximum demand in kilowatts is obtained at once together with the time at which it occurred. The essential difference between the type G and type P instrument is that the former gives a graphic record from which the demand can be read directly without computation and on which the demand can be followed at the time it is occurring. The most pronounced difference between this instrument and its predecessor, the "graphometer," is in the chart which is circular instead of rectilinear and continuous.

Westinghouse Electric and Manufacturing Co. Type RA Demand Watt-hour Meter.—This meter is similar to the General Electric Co. type G instrument in the kind of result obtained and type of record. The record however, is rectilinear (Fig. 142) on a con-

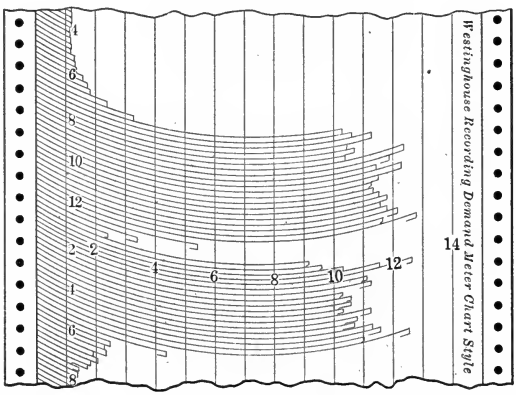


FIG. 142.

tinuous roll of paper, is made with ink and, since the paper is advanced in steps instead of continuously, the successive lines are strictly parallel.

The Piek demand indicator, made for and used by the Niagara Power Co. is similar to the above instruments. The chart is rectilinear, continuous and somewhat larger than that of the Westinghouse instrument.

278. Integrated Demand Instrument (Class 1*c*).—*General Electric Co. Type M_1 Meter.*[1]—This instrument is the successor to the Minerallac Co. "maxicator" (Fig. 143). It is attached to and made part of the watt-hour meter with which it is used, the regular register being replaced by one which has a dial indicating the

[1] This form of the type M meters is no longer listed by the General Electric Co. in its latest bulletins.

power demand, in addition to the standard dials which show the energy consumption. This extra hand is geared to the moving element of the watt-hour meter separately from the other hands and moves forward over a graduated scale at a rate proportional to the rate of energy consumption. At the end of the time interval for which the device is adjusted, the mechanism driving the demand hand is set back to zero by means of a solenoid energized by a contact-making clock (see Fig. 144). The hand itself is, however, left at the highest point which it reached and if, in any succeeding interval, the rate of energy consumption is higher, the hand will be picked up and carried to the new high position

FIG. 143.

where it will remain until returned to zero by hand. This is ordinarily done, in service, by the meter reader after the monthly readings of kilowatts demand have been taken.

General Electric Co. Types M_4 and M_5 Meters.—The principle of these instruments[1] is the same as that of the type M_1 meter just described. The essential differences are that the M_4 and M_5 types are separate from the watt-hour meter but are connected thereto by electrical, instead of mechanical, means. Also, they are mechanically, instead of electrically, connected to the timing device which is in the same case. The diagrammatic plan of operation of these meters is shown in Fig. 144.

[1] Types M_4 and M_5 have succeeded type M_2, forms AA and BA. They include improvements in certain minor mechanical details.

The only difference between type M_4 and type M_5 is the timing device. In the type M_4 meter, a small constant speed induction motor device is used to fix the time intervals. This type is therefore applicable only to alternating-current circuits. The type M_5 for continuous current employs a clock mechanism for timing.

The manufacturers claim the following advantages for the type M_4 and M_5 meters which it is expected will cause them to supersede the type M_1 meter entirely: (*a*) Being a separate device, it can be used with any watt-hour meter and thus greater flexibility

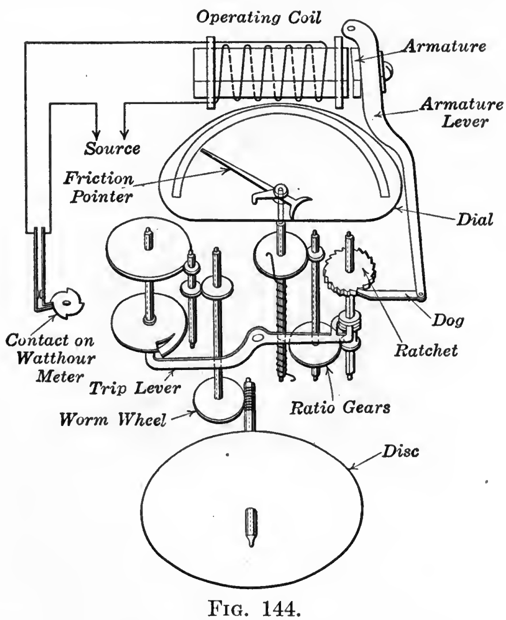


Fig. 144.

in the meter stock of a central station is obtained. (*b*) The resetting is done mechanically and is therefore more reliable. This is particularly important because failure of the solenoid in the type M_1 to operate will cause the instrument to show too high a demand. (*c*) The amount of energy per impulse and therefore the range of the instrument can be varied over a wide range by simply changing a cam wheel.

279. Integrated Demand Instruments (Class 1*d*). *Ingalls Demand Recorder.*—Instruments in this class are obviously relatively complicated and none have been developed or marketed

by any of the leading manufacturers. The Ingalls instrument, made for and used by the Edison Electric Illuminating Co. of Boston, is the only example which is in commercial service to any extent.

The instrument is separate from, but electrically connected to, the watt-hour meter and every block of energy of a predetermined size passing through the watt-hour meter is indicated by a dot on a tape which is moved forward at a constant rate by a clock. Fig. 145 shows diagrammatically the scheme of operation. A printing solenoid, *A*, is energized by means of contacts in the register of the watt-hour meter and a dot is made by a

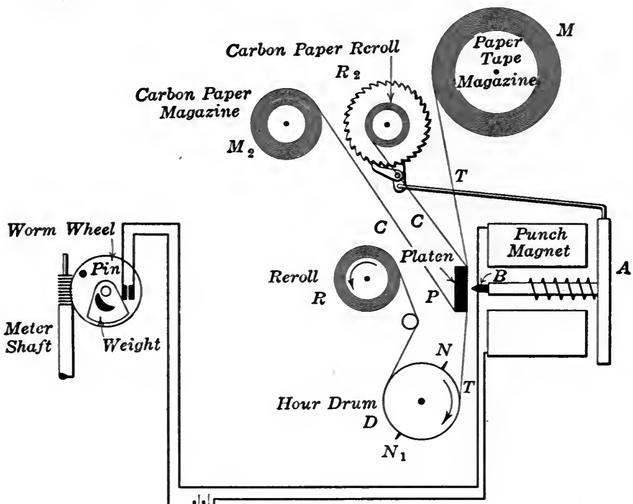


FIG. 145.

sharp point, *P*, on the tape, *T*, at every contact, thus recording the consumption of a fixed amount of energy. The tape is moved at a constant rate by the drum, *D*, attached to the hour shaft of a clock mechanism. The rate of energy consumption is obviously greatest where the dots are most numerous. The length of tape corresponding to the time interval being used which contains the most dots is picked out by inspection and from this part of the record, the maximum demand is quickly calculated. The time of day at which it occurred is also indicated.

280. Time-lagged Demand Instruments (Class 2*a*).—*Wright Demand Indicator.*—This is probably the best-known demand

instrument because it is the first one to be used commercially to any extent. It was developed about 20 years ago and is still in use in substantially the original form. The Wright instrument is a current indicator and operates on a heat storage principle. It consists of a hermetically sealed U-shaped glass tube (Fig. 146), the limbs of which terminate in bulbs of about the same size. One limb, R, is connected near the top to a smaller tube, T, which is graduated. The U-tube is filled to L with sulphuric acid, the rest of the space being filled with air. A resistance wire coil is wound around one bulb, B, and connected in series with the load or across a shunt which is in series with the load. When current flows the resultant heating expands the air in the bulb causing the liquid to rise in tube R and overflow into tube T to a height approximately proportional to the temperature rise in B which in turn varies approximately with the square of the current. Because of the thermal capacity of the air in B, the indication of the instrument, that is, the height of the liquid in T will depend upon the length of time that the current flows. According to average figures, approximately 90 per cent. of a steady load will be indicated after about 4 min., 97 per cent. after 10 min. and 100 per cent. after 30 min. A characteristic time-indication curve is shown in Fig. 153

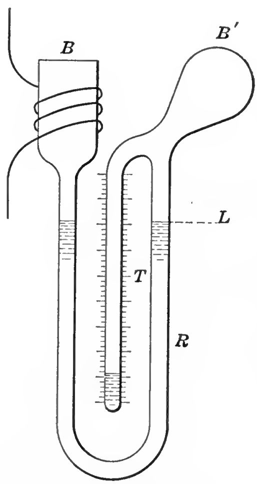


Fig. 146.

Sulphuric acid is used because it has a low temperature coefficient of expansion, "wets" glass readily and flows easily. The effect of variations in room temperature are supposed to be eliminated by the use of a duplicate bulb on tube R, thus making the device operate differentially. The instrument is reset to zero, that is, the index tube is emptied, by raising the support carrying the tubes to a position slightly above the horizontal so that the liquid flows back into the main tube.

The Wright indicator is falling into disuse and is being superseded by other types because of certain defects and limitations, the principal of which are the following: It is an ampere device and a constant voltage has to be assumed; it is applicable only

to unity power-factor loads when used on alternating current; it is affected by the size and the method of connection of the leads to the instrument, because of the variation thus introduced in the amount of heat conducted away from the heater;[1] the use of glass in a more or less fragile form which is easily broken; inaccuracy at low readings.

General Electric Type H Meter.—This instrument is also a differential, thermal, current-indicating device and is of particular interest because of the employment of a unique scheme which results in a device of remarkable simplicity, ruggedness

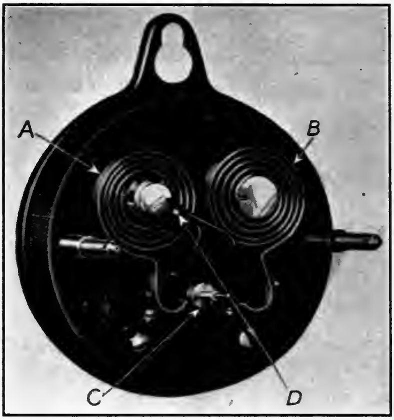


FIG. 147.

and low cost. Two similar small spiral springs are mounted side by side as shown at *A* and *B* in Fig. 147 (cover and dial plate removed). They consist of two strips of metals having different thermal coefficients of expansion and which have been welded together as in the familiar bimetallic thermostat. A heater coil which is in series with the circuit to be measured, is located in the hollow stud, *D*, of one of the coils *A*. When current flows, heat gradually travels along the spring causing it to "unwind."[2] The resulting movement of the end of the coil is transmitted to a pointer by means of a taut wire, which connects the ends of the coils and passes around a drum, *C*, on the pointer

[1] "Rates and Rate-making," P. M. LINCOLN, *Transactions*, A. I. E. E., vol. 34, p. 2279 (1915).

[2] For further information in regard to the "heat flow" principle involved in this instrument, see discussion by C. I. HALL, *Transactions*, A. I. E. E., vol. 34, p. 2326 (1915).

shaft. The pointer is moved forward by a dog, connected to the shaft and is left at the highest position reached, being reset to zero by hand after the monthly reading has been taken. Changes in room temperature have no effect because of the strictly differential action obtained by employing the dummy spring, *B*. The current rating can be changed by changing the heater coils, and variations in the time required to reach a constant indication are obtained by changing the heat storage capacity of the stud. The shape of the time-indication curve can also be adjusted by similar means. A typical time-indication curve of these instruments is shown in Fig. 153.

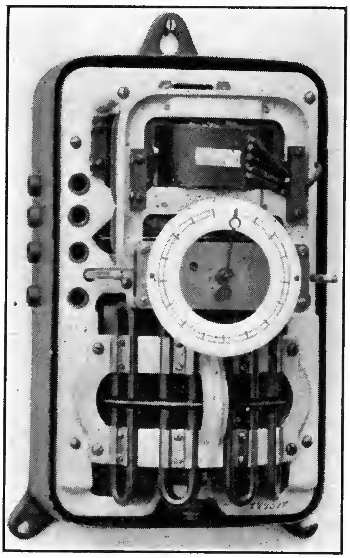


FIG. 148.

General Electric Co. Type W Demand Indicator.—This is one of the first devices, to be used commercially to any considerable extent, which measured the demand in kilowatts. It is, however, falling into disuse because, where the relatively high cost of this instrument is justified, Class 1*a* and 1*b* instruments which give more information, can be employed.

This instrument, Fig. 148, is applicable to alternating-current, polyphase circuits only. It is practically a polyphase watt-hour meter in which both elements act on one disc but continuous rotation is prevented by three long spiral springs connected in series. A large number of drag magnets act on the other disc so that the device is essentially a polyphase induction-type indicating wattmeter so heavily damped that considerable time is required for the movable element to reach its final position. Two pointers are provided, one being geared to the movable element, while the other is carried forward by the active pointer and left at the highest position reached. Thus one pointer indicates the power demand at all times while the other one shows the highest demand on the circuit since the instrument was last reset. By adjusting the length of the springs and the

position of the drag magnets, the time interval can be adjusted over a considerable range. However, it is not feasible to retard the motion of the moving element sufficiently to permit the use of a demand interval longer than 5 min. A typical time-indication curve is shown in Fig. 153. It is customary to rate these instruments on the basis of the steady power which will produce, in the desired time interval, 90 per cent. of the ultimate deflection.

Lincoln Watt Demand Indicator.—This instrument proposed by P. M. Lincoln[1] and being developed by the Westinghouse Electric and Manufacturing Co. employs a differential thermal principle somewhat similar to that of the General Electric type H demand meter. The distinctive features are the method by

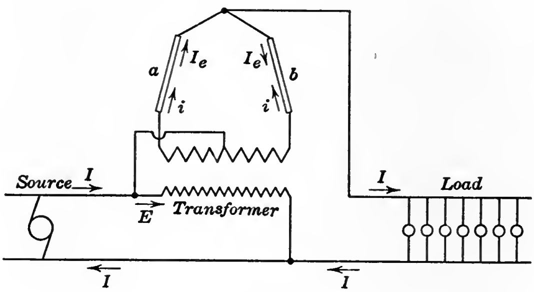


FIG. 149.

means of which the device is made to respond to watts instead of current and the method of utilizing the difference in the amount of heat in the two heat storage elements.

The scheme employed to make the device a watt indicator is shown in Fig. 149 where a and b represent two heater elements connected in parallel and to the ends of the secondary of a small transformer the primary of which is connected across the circuit as indicated. The main circuit is connected to the middle of the secondary and to the common ends of the heater coils. Thus each heater element carries two currents, one due to the load current and one due to the line voltage. In one heater, the two currents are additive and in the other they are subtractive. If the load current is I_i and the voltage current I_e, the heat produced in element a will be proportional to $(I_i + I_e)^2$ and that in

[1] "Rates and Rate-making," P. M. LINCOLN, *Transactions,* A. I. E. E., vol. 34, p. 2279 (1915).

element b will be proportional to $(I_i - I_e)^2$. The difference will be proportional to the product of I_i and I_e or watts.

The method employed to make the difference in heating operate a pointer is shown in Fig. 150. The heater elements, A and B, are immersed in a liquid contained in the cylinders, C and C_1. The expansion of the liquid is transmitted to two rods, E and E_1; respectively, through flexible metallic diaphragms, D and D_1. The movement of the pointer I is, however, proportional only to the difference between the two expansions. The

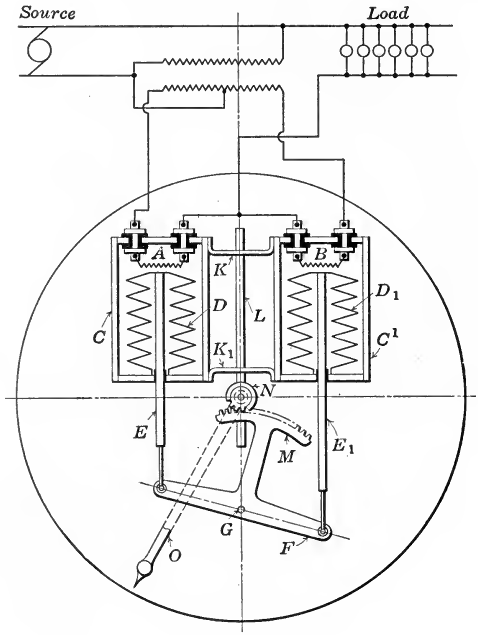


FIG. 150.

thermal capacity of the cylinders and contents provides the necessary time lag which can therefore be varied through a considerable range. The two cylinders being exactly alike, the external temperature has no effect on the indication. The typical time-indication curve is logarithmic in character and similar to the curve for General Electric type H meters.

281. Time-lagged Demand Instrument (Class 2 *b*).—*Westinghouse Electric and Manufacturing Co. Type RO Watt-hour Demand Meter.*—This instrument is the only commercial example of this class, that is, where the time-indication curve is a

straight line. It is a combined watt-hour meter and demand indicator.

The standard type RO watt-hour meter is the basis of the instrument (Fig. 151) and to this is added a power indicator which consists of a peculiarly shaped auxiliary aluminium vane in the air gap just above the watt-hour meter disc as indicated in Fig. 152 which is a schematic diagram of the instrument without the register. Movement of this auxiliary vane is opposed by a spiral

Fig. 151.

spring, and being subjected to forces similar to those acting on the main disc, it would, if not restricted, move to a position corresponding to the power in the circuit. The action would be similar to that of an induction-type wattmeter. However, instantaneous movement is prevented by an eccentric claw (see Fig. 152) working with an escapement wheel and so arranged that the auxiliary vane can only move forward at a rate which is fixed by the speed of the main disc.

When current is passed through the windings of the device,

the escapement wheel is urged forward by the torque of the auxiliary vane but is allowed to move only tooth by tooth by the claw which in turn is operated by means of a cam geared to the main disc. The escapement wheel rotates therefore at the rate permitted by the speed of the eccentric claw which in turn is proportional to the speed of the watt-hour meter or energy element. Hence, for any given power, the auxiliary vane will move forward step by step, until its torque is equalled by the opposing torque of the spiral spring When this condition has been attained, the escapement wheel is no longer urged forward and the claw oscillates idly between its teeth. An increase in the load will renew

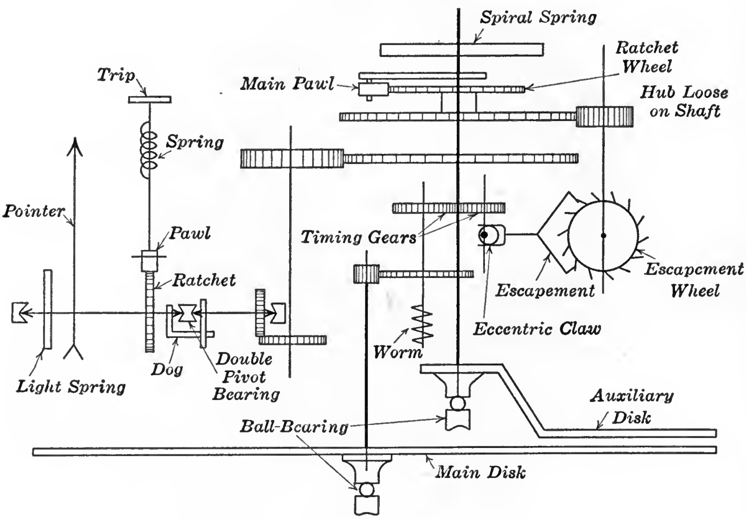


Fig. 152.

the pressure on the wheel and the auxiliary vane or power element will move forward step by step to a new position. The indications of the pointer geared to the auxiliary vane shaft will therefore be proportional to the maximum power.

It will be seen that the time interval can be made perfectly definite because the speed of the watt-hour meter is definite for a given load. For instance, with full load on the disc, the speed of rotation of the main disc will be a definite value and the gearing controlling the frequency of oscillation of the eccentric claw can be so selected that the auxiliary disc will reach the maximum position after any desired number of revolutions or, in other words, after any desired number of minutes.

When the load decreases, the auxiliary disc drops back immediately to a position corresponding to the new value of the load, the pointer being left at its highest indication. As the load again increases, the disc moves forward in the same retarded manner as before and when it reaches its previous position the pointer is picked up and carried forward.

282. Comparison of Characteristics of Time-lagged Instruments.—The characteristic curves of Wright, General Electric type W, General Electric type H and Westinghouse type RO demand instruments are shown in Fig. 153 drawn on the same chart. They show the relation, for any load, between time and

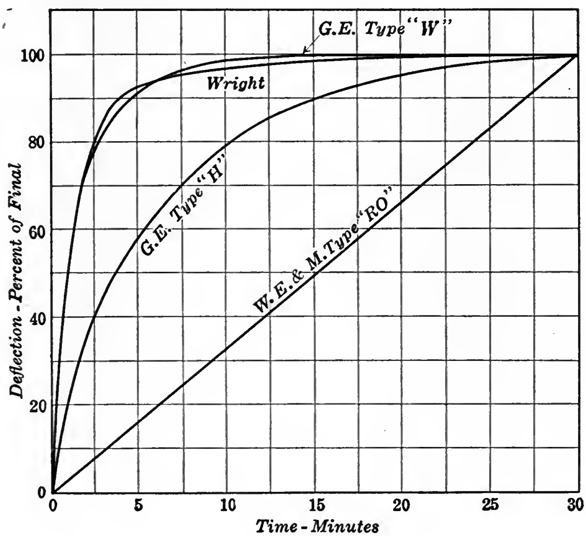


FIG. 153.

the deflection in per cent. of the maximum deflection for that load.

It will be noted that if the load is constant over a sufficiently long period, all of the instruments will show the same demand. If, however, the load is maintained constant only for 7.5 min., for example (which in this particular case is one-quarter of the time required for all instruments to reach 100 per cent. deflection) and then reduced to zero until the moving elements had returned to their initial position, the demand indicated will range from 25 per cent. to 95 per cent. Of course if the type H and the type RO instruments were adjusted for a 15-min. interval instead of a

30-min. interval, the difference in the indications would be less. But it is evident that on a fluctuating load the different types will, theoretically at least, give results which may not only differ among themselves but may not agree with results given by integrating type instruments (Class 1). This difference is due not only to the difference in the curves when the deflection is increasing, but also to the difference between the descending characteristics when the load is reduced below a previous value. For example, the moving element of the type RO instrument drops back instantly to a position corresponding to the new load, while in the other instruments the actuating element moves backward along a curve more or less similar to the ascending curve inverted. The very great difference in the time element thus introduced may make a large difference between the indications of these different types on certain particular types of fluctuating loads.

It is evident that each instrument gives a result peculiar to itself and, therefore, when the maximum demand is to be measured, the kind of instrument to be used should be specified. In other words, contracts or rate schedules involving a maximum-demand charge should preferably state the instrument which is to be used. Fortunately, as a practical matter, the actual differences between the results are much less than would be indicated by the above considerations for the reason that the unsymmetrical variations in most commercial loads tend to eliminate these differences so that over a long period the indications of the various types approach the same value.

283. Application of the Various Classes.—From the standpoint of use, maximum-demand devices may be classified according to the size of the loads on which they are used. This classification in turn is very largely a matter of the investment cost which is justified for a given installation. In general such a classification is about as follows:

1. The small consumer (10 kw. or less) requires an inexpensive instrument. A current indicator is sufficiently correct for such loads and consequently low-price, current-demand devices of which the General Electric type H is an example, are particularly applicable.

2. The medium-size consumer (10 to 100 kw.) requires a more accurate device which means a watt-measuring instrument. A greater instrument charge is warranted and, therefore, such

instruments as the General Electric type M_4 meter and the Westinghouse type RO maximum-demand watt-hour meter may be used.

3. In the case of large consumers (over 100 kw.) more information, than is given by the instruments suggested for consumers in Classes 1 and 2, is desirable. Therefore, the instruments which give a graphic record of the demand such as the General Electric type G, the Westinghouse type and the Piek instruments are applicable.

4. On large loads, the instrument and maintenance costs become secondary considerations. The large amount of money involved requires that the accuracy be a maximum. The scale errors of the instruments suggested for consumers in Class 3 become too important to be neglected in such cases and, therefore, the General Electric type P demand meter is commonly used.

5. On the very largest loads the most complete information in regard to the load is required and, therefore, the curve-drawing wattmeters are usually installed, sometimes in addition to a demand meter of another type.

Specifications for Demand Meters

The following are the essential portions of the specifications for acceptance of types of demand meters which are prescribed in section X of the "Code for Electricity Meters."[1]

284. Curve-drawing Demand Meters (Class 1*a*).

General Conditions.—The instruments shall be mounted on a support free from vibration and shall not be jarred or tapped during the tests.

The tests shall be made with the pen or stylus in operating condition and resting with normal pressure against the chart. The chart shall be moving at its normal speed, and all results shall be taken directly from it, no allowance being made for inaccuracies in chart travel or in the chart itself. Normal speed, when not otherwise specified, shall be the speed corresponding to that indicated on the printed chart or paper.

Note.—In the following tests, polyphase meters shall be tested on a single-phase circuit, with the current windings in series and the voltage windings in parallel.

Test No. 1. *Accuracy of Registration.*—The indication of an acceptable meter after having normal voltage and twenty-five per cent. (25%) rated current applied for at least one (1) hour shall not differ from the true value at twenty-five, fifty, and one hundred per cent. (25, 50 and

[1] *Q. v.*, section X, 1916.

100%) of full scale reading by an amount greater than two per cent. (2%) of full scale indication. The test shall be made first ascending the scale from twenty-five per cent. (25%) to one hundred per cent. (100%), and next descending the scale from one hundred per cent. (100%) to twenty-five per cent. (25%) of full scale indication. The corresponding readings shall not be averaged, but the maximum error shall in each case be the only one considered.

The tests shall be made at calibration voltage, rated frequency and at unity power-factor.

Test No. 2. *Equality of Elements.—Three-wire and Polyphase Meters.* —The current required to give an indication of one-half (½) full scale indication when passed through either current winding alone shall not differ by more than two per cent. (2%) from twice the current required to give the same indication with the current windings in series.

Test No. 3. *Effect of Variation in Voltage.*—The indications of an acceptable meter when subjected to a voltage differing from the calibration voltage by plus or minus ten per cent. (10%) of the calibration voltage, shall not vary from the indications at corresponding loads and calibration voltage by more than one and one-half per cent. (1.5%) of full scale indication. Tests shall be made at approximately fifty and one hundred per cent. (50 and 100%) of full scale indication, at rated frequency and unity power-factor.

Test No. 4. *Effect of Variation in Power-factor—Single-phase and Polyphase Meters.*—The indications of an acceptable meter under the test conditions shall not differ from the indications at corresponding loads and unity power-factor by an amount greater than two per cent. (2%) of the full scale deflection.

The tests shall be made at calibrated voltage and at rated frequency.

Each meter shall be tested at the various values of current and lagging power-factor given below:

Percentage of rated current	Power-factor, per cent.
66	75
100	50
133	75

Test No. 5. *Effect of Variation in Frequency—Single-phase and Polyphase Meters.*—The indications of an acceptable meter tested at ninety-five per cent. (95%) and one hundred and five per cent. (105%) of rated frequency shall not differ from the corresponding indications at rated frequency and corresponding load by an amount greater than one and one-half per cent. (1.5%) of full scale indication.

Tests shall be made at approximately fifty and one hundred per cent. (50 and 100%) of full scale deflection.

Test No. 6. *Effect of Variation in Temperature.*—This test shall be made as prescribed in Section IV, par. 42, of the Meter Code, except

that the test shall be made at fifty per cent. (50%) and one hundred per cent. (100%) of rated current. The temperature co-efficient of an acceptable meter as so determined shall not exceed one-quarter per cent. (0.25%) per degree centigrade.

Test No. 7. *Damping.*—The tests shall be made at calibration voltage, rated frequency and at unity power-factor.

In an acceptable meter, a sudden increase in the load from fifty per cent. (50%) of rated current to seventy-five per cent. (75%) of rated current shall not cause the pen to overtravel by more than ten per cent. (10%) of full scale indication, and the pen shall come to within three per cent. (3%) of its final deflection in a time interval not greater than fifteen (15) seconds.

Test No. 8. *Sensitivity.*—With a steady current supply at the calibration voltage, rated frequency and at unity power-factor, the sensitivity shall be such that the pen responds immediately to a change of current equal to two per cent. (2%) of the current corresponding to full scale deflection of the instrument. The test shall be made at approximately three-quarters (¾) of full scale deflection.

Test No. 9. *Timing.*—The chart shall be set to indicate true time and two (2) records of twenty-four (24) hours each shall be taken, one (1) with the pen at the full scale position, and one (1) with the pen at one-quarter scale position. The time indicated on the chart shall not exceed plus or minus one-quarter per cent. (0.25%) of the elapsed time. The chart shall be reset to indicate correctly the beginning of the second twenty-four (24) hour test.

Test No. 10. *Legibility of Record.*—The deflection of the pen shall be such that at any point on the scale from one-quarter scale to full scale, the record may be read to within one per cent. (1%) of the full scale indication. The travel of the pen from the no-current position to the full scale position shall not be less than two and one-half (2.5) inches.

285. Integrated-demand Meters (Class 1*b*, 1*c* and 1*d*).—*Test No.* 1. *Acceptability of Integrating Meter.*—The integrating meter which constitutes the measuring portion of integrated-demand meters shall be subjected to the tests for acceptance for meters of its class in Section IV. All auxiliary devices necessary for the measurement of demand shall be in train and in full operation during these tests. The limits of allowable variations for an acceptable meter shall be the same as fixed in Section IV for corresponding meters unconnected to registering or recording devices.

Test No. 2. *Steps per Demand Interval.*—The number of steps or their equivalent per demand interval at the rated load of the integrating meter shall be sufficiently great so that in no case the error introduced thereby shall exceed two per cent. (2%).

Test No. 3. *Legibility of Record—Indicating and Graphic Recording Devices.*—The indication or record shall be such that at any point be-

tween one-quarter full scale and full scale it can be read to within one per cent. (1%) of the full scale value.

Test No. 4. *Equality of Energy Intervals.*—The integrating meter shall be operated on a constant load of approximately one hundred per cent. (100%) of its rated capacity during the time necessary for five (5) complete revolutions of the contact wheel which sends energy impulses to the recording or registering device. The times shall be recorded at which each such energy impulse is transmitted, and by subtraction, the time intervals between the energy impulses shall be found. In an acceptable device, no such energy interval shall differ from the mean of all the energy intervals by more than one per cent. (1%) of the time of one (1) complete revolution.

Test No. 5. *Accuracy of Demand Intervals.*—The duration of the demand interval shall be measured throughout three (3) entire cycles of the operation of the timing mechanism. The length of no such interval shall differ from the nominal length by more than thirty-six (36) seconds.

Test No. 6. *Accuracy of Contact Mechanism.*—The integrated-demand meter consisting of an integrating meter with contact mechanism and of the corresponding registering or recording mechanism shall be operated on a load approximately equal to the full load of the integrating meter during a period of time such that not less than five hundred (500) energy impulses are transmitted. A record of these impulses shall be taken on a suitable graphic instrument (*e.g.*, an adapted laboratory chronograph) an inspection of the chart of which will show whether the contact mechanism has failed either by sending out double impulses or by omitting any impulse. In an acceptable type of contact mechanism no such failure shall occur.

Test No. 7. *Accuracy of Registering or Recording Mechanism.*—Using the data of Test No. 6, the total number of energy impulses shown by the registering or recording mechanism shall be compared with the total number as given on the chart of the graphic instrument. In an acceptable type of registering or recording mechanism, these numbers shall be equal. In the case of recorders or indicators which reset, it is evident that the impulses lost during the reset period are not to be considered.

NOTE.—In tests Nos. 6 and 7, the graphic instrument used shall have such electrical constants and shall be connected in the circuit in such a way that it will not interfere with the reliability of operation of either the contact mechanism or the registering or recording mechanism.

Test No. 8. *Clock.*—Two (2) twenty-four (24) hour tests shall be made, one near the beginning and the other near the end of the winding period. In an acceptable meter the gain or loss of the clock in either test shall not exceed two (2) minutes in twenty-four (24) hours.

Test No. 9. *Timing Motors.*—In meters using an electric motor instead of a clock, the timing motors shall be tested as follows:

(*a*) At calibration voltage and at rated frequency, the average rate of the motor during a demand interval shall be such as to correspond to a gain or loss of time not to exceed two per cent. (2%).

(*b*) Effect of Voltage Variation on Timing Motor. A variation of five per cent. (5%) above or below the rated voltage of the timing motor shall not affect the speed of the motor by more than one per cent. (1%).

(*c*) Effect of Frequency Variation. A variation of three per cent. (3%) above or below the rated frequency of the current applied to the timing motor shall not affect the speed of the motor by more than six per cent. (6%).

(*d*) A rise in temperature of ten degrees (10°) centigrade above an initial temperature of twenty degrees (20°) centigrade shall not change the speed of the motor by more than two per cent. (2%).

286. Time-lagged Demand Meters (Class 2*a* and 2*b*).

NOTE.—Polyphase meters shall be tested on a single-phase circuit, with their current windings in series and their voltage windings in parallel.

Test No. 1. *Accuracy of Indication.*—(*a*) Meters in which the speed of the indicator is proportional to the load.

Each meter shall be tested on a load corresponding to twenty-five, fifty and one hundred per cent. (25, 50 and 100%) of full scale indication, at calibration voltage, rated frequency and at unity power-factor as follows:

With the indicator at the no-load position, a load of the prescribed amount shall be suddenly thrown on. The indication shall be noted when one-half (½) of the rated demand interval has elapsed, and again at the end of the demand interval. The indication at the half interval period shall not differ from that corresponding to one-half (½) of the applied load by more than plus or minus three per cent. (3%) of the full scale indication.

The indication at the end of the time interval shall not differ from the true value corresponding to the applied load by more than plus or minus three per cent. (3%) of the full scale indication.

(*b*) Meters in which the speed of the indicator decreases with the deflection.

Each meter shall be tested on loads corresponding to sixty per cent. (60%) and one hundred per cent. (100%) of full scale indication at calibration voltage, rated frequency and at unity power-factor, as follows:

With the indicator at the no-load position, a load of the prescribed amount shall be suddenly thrown on. The time required for the indication to reach ninety per cent. (90%) of full indication shall not be less than the rated demand interval of the instrument. In both of the tests prescribed the loads shall be kept on the instrument until the ultimate

deflection has been reached. The indication at ultimate deflection shall not differ from that corresponding to the actual load by an amount greater than five per cent. (5%) of the full scale indication.

Test No. 2. *Equality of Elements—Three-wire and Polyphase Meters.*—The test shall be made the same as Test No. 2, for Class I*a*, demand meters, and the limits of allowable variation shall be the same.

Test No. 3. *Effect of Variation in Voltage.*—The test shall be the same as Test No. 3, for Class I*a*, demand meters, and the limits of allowable variation shall be the same.

Test No. 4. *Effect of Variation in Power-factor—Single-phase and Polyphase Meters.*—The test shall be the same as Test No. 4, for Class I*a*, demand meters, and the limits of allowable variation shall be the same.

Test No. 5. *Effect of Variation in Frequency—Single-phase and Polyphase Meters.*—The test shall be the same as Test No. 5, for Class I*a*, demand meters, and the limits of allowable variation shall be the same.

Test No. 6. *Effect of Variation in Temperature.*—The test shall be the same as Test No. 6, for Class I*a*, demand meters, and the limits of allowable variation shall be the same.

CHAPTER XI

INDUCTANCE MEASUREMENTS

287. Definitions.—When the current in an electric circuit is altered, an e.m.f. is induced in the circuit by the simultaneous change in the magnetic field which surrounds and interlinks with the circuit. This e.m.f. is opposite in direction to that which produced the change in the current.

This property of a circuit is called inductance. The self-inductance or coefficient of self-induction of a circuit is the constant by which the time rate of change of the current in the circuit must be multiplied to equal the counter e.m.f. induced in the circuit. That is

$$e = L\frac{di}{dt}$$

where L = inductance and e = e.m.f. produced by the current changing at the rate, $\frac{di}{dt}$.

Similarly, the mutual inductance between the two circuits is the constant by which the time rate of the change of current in either circuit must be multiplied to give the e.m.f. thereby induced in the other circuit.

The values of the self-inductance and mutual inductance depend upon the shape and dimensions of the circuit, its arrangement and also upon the nature of the surrounding medium.

288. Units.—The unit of self-inductance and mutual inductance in general use is the international henry. It is equal to 10^9 c.g.s. electromagnetic units[1] of inductance and is the "inductance in a circuit when the e.m.f. induced in this circuit is 1 international volt while the inducing current varies at the rate of 1 amp. per sec." Obviously, for self-inductance the inducing current is in the circuit itself, while for mutual inductance the

[1] The c.g.s. electromagnetic unit is sometimes called the centimeter, inductance having dimensions of length only. Inductance expressed in centimeters may be converted to millihenrys by dividing by 10^6 or to microhenrys by dividing by 10^3.

inducing current is in an external circuit. The henry is, however, too large for ordinary purposes and the millihenry (one-thousandth of a henry) is commonly used.

289. Standards.—Standards of inductance are usually simple coils of copper wire suitably mounted on a non-conducting, non-magnetic frame. The turns are held rigidly in place by shellac, paraffine or other insulating medium. Inductance standards are made in single values like standard resistances, or in combinations, with plug connections, like a subdivided condenser or a resistance box.

In the Ayrton and Perry form of inductance standard (Fig. 154) the inductance can be varied over a wide range. There are

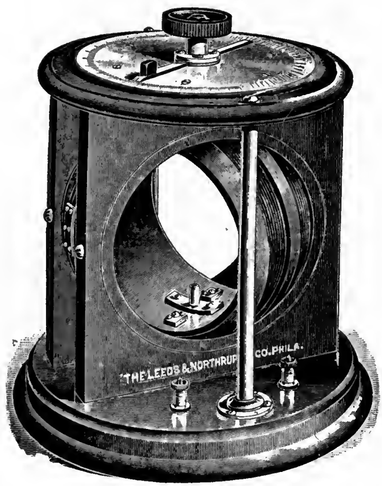


FIG. 154.

two coils, one fixed and the other movable, wound on surfaces which are sections of concentric spheres. When connected in series these coils form a variable inductance, the value of which at any relative position is read from a previously calibrated scale on the top of the supporting frame. In one extreme position of the inner movable coil, the horizontal axes of the two coils coincide, but the current circulates in opposite directions and the inductance is a minimum. In the other extreme position of the inner coil, 180° from the first position, the currents are in the same direction and the inductance is a maximum, thus the change from minimum to maximum inductance is obtained by

infinitely small increments. Additional range is secured by connecting sections of the two coils in series-parallel combinations by means of plugs.

The Ayrton and Perry form of standard is subject to errors due to external magnetic fields. This is a serious defect in measurements of inductances of small value because it is difficult to avoid stray fields, especially those from portions of the test circuits. Taking two sets of readings, direct and reversed, may not entirely eliminate the error. This objection is largely overcome in the variable standard inductor made by Nalder Brothers (London), and in the Mansbridge form (Fig. 155) made by Leeds and Northrup. Both consist of two circular plates of hard rubber

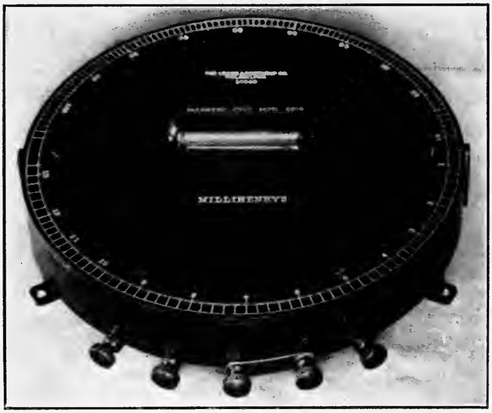


Fig. 155.

(or preferably one of the recent substitutes therefor, which is free from the warping tendency of hard rubber) one above the other, very close together, and with a common axis. The lower one is fixed and contains two fixed coils imbedded therein at opposite ends of a diameter. The upper plate carries two similar coils, similarly placed and is revolvable about a central pivot supported by the lower plate. Thus a practically astatic arrangement is obtained. Maximum and minimum inductance are respectively obtained in the two extreme positions of the upper plate 180° apart where the corresponding coils are coaxial. In the Mansbridge inductor, the coils are not circular but shaped like a letter "D" with the straight sides toward each other. This form of coils gives an inductance scale which is practically uniform.

An improved form of astatic inductor employing four fixed and two movable coils (as in a Kelvin balance) is described by Brooks and Weaver in the *Bulletin* of the Bureau of Standards.[1] Fig. 156 is a cross-sectional view. The coils are link shaped and the two sets of coils are spaced a certain definite distance apart. The result is a high time constant (ratio of inductance to resistance) for the size of the apparatus, and a practically uniform scale.

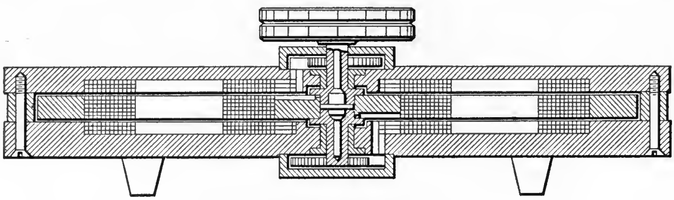


FIG. 156.

Standards of the kind described are usually standardized in terms of resistance and capacitance (see bridge methods of measurement). Standards of simple geometrical form may be evaluated by calculation and consequently standards of very small value are usually a simple circuit such as two parallel straight wires or a circular coil of one turn, the inductance of which can be calculated from the dimensions. The inductance may, however, be determined by measuring the change in the phase angle with change in resistance.[2] This requires that the standard be so made that its resistance can be changed a large amount without changing its inductance. Fig. 157 is a vector diagram applying to such an inductor where R = resistance, ωL = inductive reactance, θ = phase angle, and $\Delta\theta$ = change in phase angle corresponding to a change aR in R. It is evident that

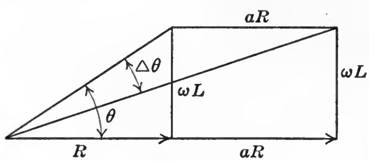


FIG. 157.

$$\tan\theta = \frac{\omega L}{R} \text{ and } \tan(\theta - \Delta\theta) = \frac{\omega L}{R + aR}.$$

[1] "A Variable Self and Mutual Inductor," H. B. BROOKS and F. C. WEAVER, *Bulletin*, Bureau of Standards, vol. 13, p. 569 (1916–1917). This inductor is being put on the market by Leeds & Northrup, Phila.

[2] "Methods of Measuring the Inductance of Low-resistance Standards, WENNER, WEIBEL and SILSBEE," *Bulletin*, Bureau of Standards, vol. 12, p. 11 (1915–16).

The angles involved are very small, hence,

$$\Delta\theta = \frac{\omega L}{R} - \frac{\omega L}{R + aR} = \frac{\omega L}{R}\left(\frac{a}{1 + a}\right)$$

and

$$L = \frac{\Delta\theta R(1 + a)}{\omega a}. \qquad \text{(henrys)}$$

The angle θ is in radians ($1° = 0.01746$ radian, 1 min. $=$ 0.000291 radian), $\omega = 2\pi \times$ frequency, L is in henrys, R is the original resistance in ohms and a is the per cent. increase in R.

The obvious way to employ this scheme is to make the standard inductor of copper and measure the change in phase angle as it is heated up by the passage of current, noting the corresponding increase in resistance. The angle can be readily measured by means of the usual methods for determining the phase angle of current transformers (par. 130) which employ standard resistors. The standard inductor is connected in the primary circuit of the current transformer as the standard resistor, and the apparent phase angle measured at two currents corresponding to two temperatures (resistances) as far apart as feasible. The apparent change of the transformer angle is, of course, the change in the phase angle of the standard.

290. Inductance Formulas.—The following formulas for the more simple and common circuits that might be used as standards or otherwise are from the paper by Rosa and Grover in the *Bulletin* of the Bureau of Standards, vol. 8, p. 1 (1912). This paper is a most comprehensive compilation (230 pages) of the formulas which have been developed for the inductance of various forms of circuits. The paper also includes a thorough discussion of the accuracy of the various formulas, their limitations and application, together with table of constants to reduce the labor involved in the calculating.

In all of the formulas given here, the conductor is assumed to have a circular cross-section and all quantities are in c.g.s. units, that is, lengths are in centimeters and inductances are in centimeters (10^{-3} microhenrys).

(*a*) *Self-inductance of One Circular Turn* (Rayleigh and Niven).—

$$L = 4\pi a\left\{\left(1 + \frac{r^2}{8a^2}\right)\log_\epsilon\frac{8a}{r} + \frac{r^2}{24a^2} - 1.75\right\}$$

where a = mean radius of turn in centimeters, r = radius of wire in centimeters and L = self-inductance in centimeters.

This formula is approximate only, when the ratio $\frac{r}{a}$ is large. However for $\frac{r}{a}$ less than 0.1 (the usual case) the error is less than 1 part in 100,000.

The inductance of coils of more than one turn may be calculated with this formula by including the square of the number of turns, n^2, in the expression before the brackets and correcting for the space occupied by insulation between turns and for the shape of the section.[1] However, a close approximation can be obtained with the following formula (Maxwell),

$$L = 4\pi a n^2 \left(\log_\epsilon \frac{8a}{R} - 2\right)$$

where a = mean radius of coil in centimeters, L = inductance in centimeters, n = total number of turns and R = geometrical mean distance of the cross-section of the coil. When the cross-section is rectangular with sides equal to x cm. and y cm., respectively, $R = 0.2235\,(x + y)$ (approximately); for a circular cross-section with radius $= x$ cm., $R = 0.7768x$.

(*b*) *Mutual Inductance, Two Coaxial, Single Circular Turns of Equal Diameter* (Maxwell).—

$$M = 4\pi a \left\{ \log_\epsilon \frac{8a}{d}\left(1 + \frac{3d^2}{16a^2}\right) - \left(2 + \frac{d^2}{16a^2}\right) \right\}$$

where a = mean radius of turns in centimeters, d = mean distance between turns in centimeters and M = mutual inductance in centimeters.

This formula is not exact but for most purposes is a sufficiently close approximation when the ratio, $\frac{d}{a}$, is small, that is, of the order of 0.1.

(*c*) *Self-inductance of Short Solenoid of One Layer.*—

$$L = nL_1 + 2(n - 1)M_{12} + 2(n - 2)M_{13} \;.\;.\;.\; 2M_{1n}$$

where L = total inductance, L_1 = self-inductance of a single turn, M_{12} = mutual inductance of first and second turns or any two adjacent turns, M_{13} = the mutual inductance of the first

[1] For these corrections see "On the Geometrical Mean Distance of Rectangular Areas and the Calculation of Self-inductance," E. B. ROSA, *Bulletin*, Bureau of Standards, vol. 3, p. 1 (1907).

and third turns or any two turns separated by one turn, and M_{1n} is the mutual inductance of the first and last turns. All L's and M's are in centimeters. L_1 is calculated from formula (*a*) and the M's from formula (*b*).

(*d*). *Mutual Inductance of Two Concentric, Single Layer, Coaxial Solenoids of Equal Length* (Maxwell).—

$$M = 4\pi^2 a^2 n_1 n_2 (l - 2A\alpha)$$

where A and a = mean radii of the outer and inner solenoids respectively in centimeters, n_1 and n_2 = number of turns per centimeter of outer and inner solenoids respectively, l = common length in centimeters, M = mutual inductance in centimeters and

$$\alpha = \frac{A - r + l}{2A} - \frac{a^2}{16A^2}\left(1 - \frac{A^3}{r^3}\right) - \frac{a^4}{64A^4}\left(\frac{1}{2} + 2\frac{A^5}{r^5} - \frac{5}{2}\frac{A^7}{r^7}\right)$$
$$- \frac{35}{2{,}048}\frac{a^6}{A^6}\left(\frac{1}{7} - \frac{8}{7}\frac{A^7}{r^7} + \frac{A^9}{r^9} - 3\frac{A^{11}}{r^{11}}\right)$$

where

$$r = \sqrt{l^2 + A^2}.$$

If the solenoids are very long in comparison with the radii, the expression for α is simplified as follows:

$$\alpha = \frac{1}{2} - \frac{a^2}{16A^2} - \frac{a^4}{128A^4} - \frac{5a^6}{2{,}048A^6}.$$

(*e*) *Self-inductance of a Straight, Cylindrical Conductor* (return circuit neglected).—

$$L = 2\left[l \log_\epsilon \frac{l + \sqrt{l^2 + r^2}}{r} - \sqrt{l^2 + r^2} + \frac{l}{4} + r\right]$$

or approximately

$$L = 2l\left(\log_\epsilon \frac{2l}{r} - \frac{3}{4}\right)$$

where l = length of conductor in centimeters, r = radius of conductor in centimeters and L = inductance in centimeters.

(*f*) *Mutual Inductance of Two Parallel, Straight, Cylindrical Conductors of Equal Length.*—

$$M = \left[l \log_\epsilon \frac{l + \sqrt{l^2 + d^2}}{d} - \sqrt{l^2 + d^2} + d\right]$$

or approximately

$$M = 2l\left(\log_\epsilon \frac{2l}{d} - 1 + \frac{d}{l}\right)$$

where l = length of conductors in centimeters, d = mean distance between conductors in centimeters, M = mutual inductance in centimeters.

It will be noted that r_1, the mean radius of the conductors, does not appear in this formula. It is exact only for conductors of inappreciable cross-section but the error is negligible even for relatively large values of r if l is large compared with d.

291. Classification of Methods of Measuring Inductance.—The various methods generally employed for measuring inductance may be classed as bridge methods or impedance methods. In bridge methods, the inductance is determined by comparison with a known resistance and capacitance, or with another known inductance, in an arrangement of circuits similar to the familiar Wheatstone-bridge network. In impedance methods, the inductance is calculated from measurements of the impedance made with sinusoidal alternating current.

Bridge Methods

292. General.—In the ordinary Wheatstone bridge used for resistance measurements in which all of the arms consist of resistance only, the balance as obtained in the usual manner with continuous current would not be disturbed if a transient or alternating current were substituted for the continuous current. If two of the arms contain inductance in addition to resistance, the continuous-current balance would still be undisturbed but it would not hold for transient current. An adjustment of the arms can be found, however, which will be balanced, in effect, for both continuous and transient current.

293. Detectors and Sources of Current.—Several sources of current and different kinds of detectors can be employed in bridge methods. The usual combinations are briefly as follows:

(*a*) Continuous current and a D'Arsonval galvanometer. The bridge is first balanced with a steady current from a battery. Then with the battery circuit being suddenly opened or closed, the bridge is again balanced, but without disturbing the steady-current ratios of the bridge arms.

(*b*) Continuous current, a D'Arsonval galvanometer and, secohmmeter or rotating double commutator, one commutator in battery circuit and the other in the galvanometer circuit. This commutator simultaneously interrupts and reverses the

currents in the two circuits in such a way that the current in the galvanometer circuit is always in the same direction although that in the bridge is being rapidly reversed. The bridge is balanced for steady current with the commutator at rest and then for transient currents with the commutator revolving.

(*c*) Rapidly interrupted current produced by an induction-coil interrupter with a telephone receiver for the detector.

(*d*) Alternating current of high frequency from a Vreeland oscillator with a telephone receiver as the detector. The Vreeland oscillator delivers an absolutely pure sine-wave current from a continuous-current source and has a range of frequency from about 200 to about 2,000 cycles per second. The advantages of this current over that produced by an interrupter, is the elimination of the high harmonic currents produced by the interrupter which make balancing with a telephone difficult. The principle of the Vreeland oscillator is as follows:[1]

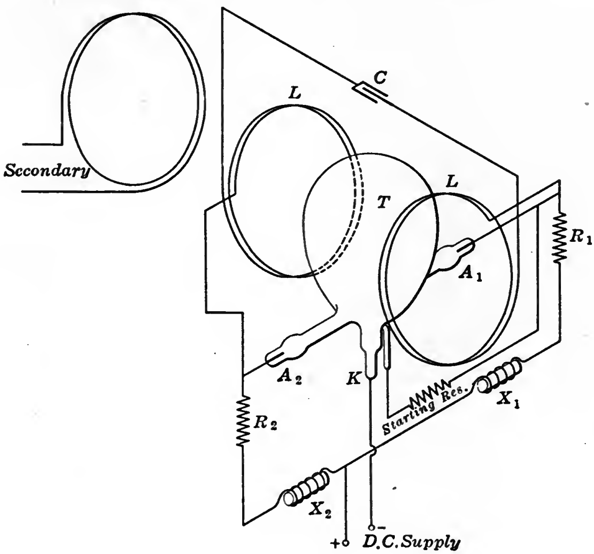


FIG. 158.

Fig. 158 is a schematic diagram showing the general arrangement of the apparatus in isometric projection and the various

[1] "A Sine-wave Electrical Oscillator of the Organ-pipe Type," F. K. VREELAND, *Physical Review*, vol. 27, p. 286, October, 1908. Also pamphlet," The Vreeland Oscillator," published by Western Electric Co., New York.

circuits diagrammatically. In the figure, T, is a pear-shaped mercury vapor tube having a single mercury cathode, K, and two similar anodes, A_1 and A_2. Continuous current is supplied to the tube through a pair of ballast resistances, R_1 and R_2, and a pair of choke coils, X_1 and X_2. The continuous current, therefore, has two paths to follow. Entering at the terminal marked $+$, it will flow through X_1, R_1 and A_1 to the cathode K, or through X_2, R_2 and A_2 to K. Shunted across the anodes is the oscillating circuit comprising a capacitance C and a pair of inductance coils, LL. They encircle the bulb so that the magnetic field will traverse the tube in a direction perpendicular to the plane of the anodes and cathode. Any current through these coils will thus tend to deflect the arc toward one or the other of the anodes, according to the direction of the magnetic field.

The operation is as follows: After the arc has been started, the current is carried between the two anodes and the cathode in two symmetrical streams. The ballast resistances and choke coils serve to maintain equal and constants currents in the two supply branches. Under these conditions the two anodes A_1 and A_2 will be of exactly the same potential and no current will flow through the circuit LCL shunting them. Suppose, however, that through some means, such as a slight unbalance or a residual charge on the condenser, a current is caused to flow through the field coils. This will tend to deflect the arc toward one of the anodes, say A_1, but this cannot result in any material change in the current of either branch of the supply circuit because of the choke coils, X_1 and X_2. The path from A_1 to K has been shortened, while that from A_2 to K has been lengthened. A current will therefore flow through the oscillating circuit LCL from anode A_2 to A_1, and in this way unequal currents will flow from the two anodes to K. The connections of the field coils are such that their current tends still further to deflect the arc toward the anodes, A_1, and thus an unstable condition is established which continues until the condenser C is fully charged. The condenser then discharges, reversing the current in the field coils, reversing the direction of the magnetic flux, and deflecting the arc from the anode A_1 toward the anode A_2. This process is repeated indefinitely, the successive reversals occurring at a frequency dependent upon the natural period of the oscillating circuit, determined by the values of its capacitance and inductance.

It is then merely necessary to adjust the capacitance in this

primary circuit to a value which will give the desired frequency. The inductance being constant, the relation between the frequency and capacity is given by

$$f = A\sqrt{\frac{1}{C}}$$

A being a constant, equal to $\frac{1}{2\pi\sqrt{L}}$. A further adjustment is possible from the fact that a switch is provided to change the field coils from series to parallel connection. Since in the latter case their inductance is one-quarter of its value when the series connection is used, the frequency will be doubled by changing the coils from series to parallel, the capacitance remaining the same.

Power is taken from the apparatus by a secondary coil, which is loosely inductively coupled to the primary coils and mounted so as to permit adjustment of the coupling and therefore the working e.m.f.

(*e*) Alternating current of low frequency (100 cycles per second or less) with a vibration galvanometer or a separately excited electrodynamometer as the detector, or a D'Arsonval galvanometer in conjunction with a synchronously driven reversing key or commutator to rectify the current in the detector circuit.

(*f*) Both continuous and transient (or alternating current) may be used together, the continuous current for resistance balance and transient current for inductance balance. This means, however, a change-over switch in the circuit supplying the bridge and also one in the detector circuit to connect in the proper kind of detector for the corresponding current.

294. Principal Bridge Methods.—A great many forms of bridge methods have been proposed and used for inductance measurements. The following four methods are probably the best known.

With a Standard Inductor.—This method employs a standard inductor. The circuits are arranged as indicated in Fig. 159 where r_1, r_2, r_3 and r_4 are non-inductive resistors, L_s is a standard inductor and L is the inductance to be measured.

If the measurement is being made with continuous current, the bridge is first balanced with a steady current by adjusting r_3 and r_4 (battery key closed and tapping galvanometer key in the usual manner) and then with transient currents (battery key

opened and closed with galvanometer key kept closed) by adjusting r_1 and r_2; the bridge is then rebalanced with steady current, again for transient currents and so on until complete balance under both conditions is obtained. Obviously, if the standard inductor is adjustable, the resistors r_3 and r_4 are not essential and the second balance is obtained by adjusting the standard inductor. Then only two balancing operations are necessary.

If alternating current is used, the resistors r_3 and r_4 are adjusted until the detector shows a minimum indication (or minimum sound if a telephone receiver). Resistors r_1 and r_2 are then adjusted until a new minimum is obtained, the two pairs of resistors being adjusted in succession until complete balance is obtained. As before, if the standard inductor is variable, it can be adjusted to reduce to zero the minimum indication obtained in the first adjustment.

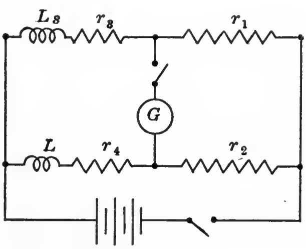


FIG. 159.

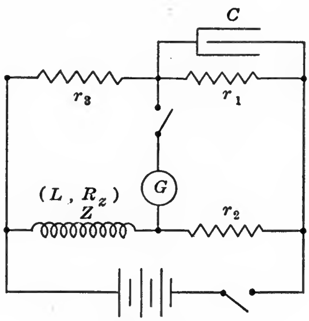


FIG. 160.

With alternating current of low frequency and an electrodynamometer galvanometer as the detector, the latter should preferably have its fixed coils separately excited in order to get greater sensitivity. Furthermore, if the excitation can be shifted in phase 90°, only two settings of the bridge are necessary. First, the fixed coils are excited with current in phase with the current to the bridge. At zero deflection the bridge arm drops are in phase with the bridge current, that is, the resistances are balanced. If the excitation is then shifted in phase 90° (see par. 240) and zero deflection again obtained, the bridge will be balanced for inductive drops.

In all cases at complete balance

$$L = L_s \frac{r_2}{r_1}$$

where r_2 and r_1 are in ohms and L is in the same units as L_s.

With a Known Capacitance.—In the bridge arrangement indicated in Fig. 160, known as Maxwell's method of measuring an inductance, r_1, r_2 and r_3 are non-inductive resistors; C is a condenser in parallel with r; Z is the impedance having the inductance L to be measured and a resistance, R_z.

The procedure is the same as in the preceding method. With continuous current, the bridge is first balanced with steady current by manipulating r_1 and r_3 and then with transient currents by changing r_1 and r_3 simultaneously without disturbing their ratio so that the steady-current balance will be maintained. With alternating current, complete balance must be found by

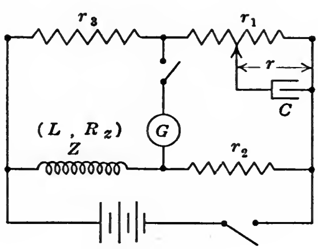


FIG. 161.

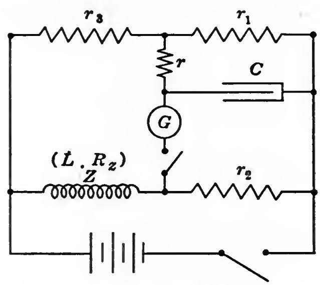


FIG. 162.

continuous trial, first adjusting the ratio r_1 to r_3 for a minimum current in the detector circuit, and then reducing to zero by simultaneous manipulation of r_1 and r_3.

At balance

$$L = R_z r_1 C 10^{-6} \qquad \text{(henrys)}$$

where L is in henrys, R_z and r_1 are in ohms, and C is in microfarads.

With a Known Capacitance. Only Two Balances Necessary.—This method, known as Rimington's modification of Maxwell's method, is distinguished by the elimination of the necessity for making the repeated readjustments which are necessary to get complete balance in Maxwell's method. The diagram of connections, Fig. 161, is the same as Fig. 160 except that the condenser, C, is in parallel with only a portion, r, of the resistance, r_1. The bridge is balanced by first adjusting r_1 and then r. At balance

$$L = \frac{Cr^2R_z}{r_1} 10^{-6} \qquad \text{(henrys)}$$

where L is in henrys, r, r_1 and R_z are in ohms and C is in microfarads.

With a Known Capacitance. Only Two Balances Necessary.—The connections for this, Anderson's method, are indicated in Fig. 162. It will be seen that it differs from the previous method only in that the resistance, r, Fig. 161, is in the detector circuit instead of the bridge arm shunted by the condenser. As in that method, only two adjustments are necessary, first with r_1 (or r_2 or r_3) and then with r, the two adjustments being quite independent of each other. At balance

$$L = C[r(R_z + r_2) + r_2r_3]10^{-6} \qquad \text{(henrys)}$$

where L is in henrys; C is in microfarads, $r_1r_2r_3$ and R_z are in ohms.

This method is a popular one because of its convenience (the two adjustments being independent of each other) and the considerable range of values of L which can be measured by simply changing the range of r without changing the ranges of the bridge arms or the capacitance of the condenser. It has been used extensively at the Bureau of Standards with low-frequency alternating current and a vibration galvanometer, which combination is stated to be "admirably adapted to practical use in measuring inductances of a great range of values" because of its convenience and accuracy.[1]

Impedance Methods

295. General.—Impedance methods employ simply current and potential measurements with sine-wave alternating current, from which the value of the inductance is calculated from the relations

$$Z = \frac{E}{I} = \sqrt{R^2 + (L2\pi f)^2} \text{ or } L = \sqrt{\frac{E^2 - I^2R^2}{(2\pi f I)^2}} \qquad \text{(henrys)}$$

where Z = impedance in ohms, L = inductance in henrys, E = drop in volts, I = current in amperes, R = resistance in ohms, and f = frequency in cycles per second. Being more indirect, these methods are not, in general, as accurate nor in many cases as convenient as bridge methods. However, where the current which is permissible and the potential drops obtainable are within the range of available instruments, an accuracy which is usually ample for commercial purposes is easily and conveniently attained.

[1] "Measurement of Inductance by Anderson's Method Using Alternating Currents and a Vibration Galvanometer," E. B. Rosa and F. W. Grover, *Bulletin*, Bureau of Standards, vol. 1, p. 291 (1904–05).

296. With Ammeter and Voltmeter.—The fall of potential in the circuit containing the inductance is measured when a measured current of known frequency is passed through it. Having previously measured the resistance with continuous current, L is calculated from the formula given above. Obviously, due allowance should be made for the current taken by the voltmeter where it is of sufficient magnitude.

297. With Three Voltmeters.—The inductance to be measured is connected in series with a non-inductive resistance as shown

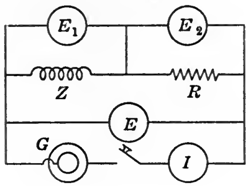


FIG. 163.

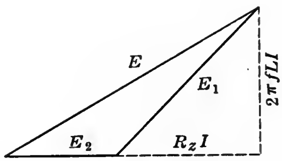


FIG. 164.

in Fig. 163. The current, I, is measured; also the total volts, the volts across the inductance Z, and the resistance R. From these readings a triangle is constructed, Fig. 164. If R is known, the quantity $2\pi fLI$ can be calculated from the triangle. If R is unknown, $2\pi fLI$ can be obtained by graphical construction. I and f being known, L is obtained by calculation.

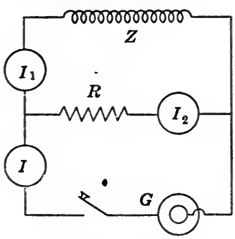


FIG. 165.

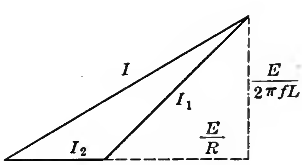


FIG. 166.

298. With Three Ammeters.—This method is similar to the one using three voltmeters. The connections are shown in Fig. 165 and from the three currents, Fig. 166 is constructed. The e.m.f., E, can be measured directly or, if R is known, by calculation from the relation $E = RI_2$. The value of L is then computed from the quantity, $E/2\pi fL$.

Mutual-inductance Measurements

The mutual inductance between two circuits may be measured by several methods. The following are more typical.

299. Self-inductance of One Coil Known.—This method is known as Heaviside's bridge method. The connections are indicated in Fig. 167. The bridge is so balanced by any of the methods described under self-inductance measurements (par. 293) that the detector will show no current for either steady continuous current or transient (or alternating) current. Then

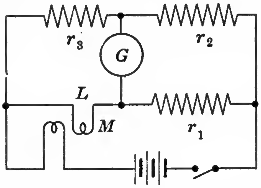


Fig. 167.

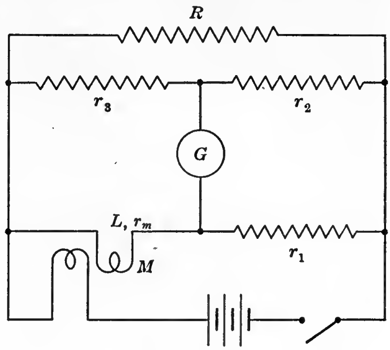


Fig. 168.

$$M = -\frac{Lr_2}{r_1 + r_2}$$

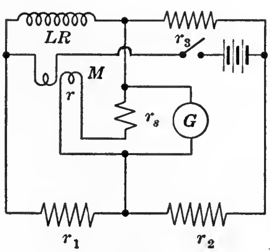


Fig. 169.

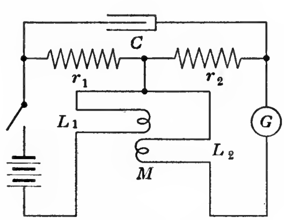


Fig. 170.

where M and L are in the same units of inductance, r_1 and r_2 are in ohms.

In the above method the final balance is a matter of several trials,—first with steady continuous current and then with transient (or alternating) current, repeating each adjustment until a balance is found which holds for both conditions. These trials may be reduced to two by shunting the bridge with a non-inductive resistance R, as shown in Fig. 168. The steady

continuous current balance is obtained as before by adjusting r_1, r_2, and r_3. Then R is adjusted until balance is again obtained with transient (or alternating) current. Since this latter adjustment does not disturb the first balance, complete balance is obtained with two trials and

$$M = -\frac{Lr_3R}{(r_3 + r_m)R + (r_m + r_1)r_3}$$

300. With a Known Inductor.—The circuits are arranged as indicated in Fig. 169. When complete balance is attained (adjusting r_s for transient current balance),

$$M = -L\frac{r_1(r_s + r)}{(R + r_1)^2}$$

where M and L are in the same units of inductance, R, r, r_s and r_1 are in ohms. The resistance, r, is the resistance of the secondary of the mutual inductance and R is the resistance of the self-inductance L.

301. With a Known Capacitance.—Fig. 170 shows, diagrammatically, the arrangement of the circuits. Balance is obtained by adjusting r_1 and r_2. At complete balance, when the switch is opened or closed (or with alternating current),

$$M = Cr_1r_2 10^{-6} \qquad \text{(henrys)}$$

where M = mutual inductance in henrys, r_1 and r_2 are in ohms, and C = capacitance in microfarads.

302. With Ballistic Galvanometer.—The secondary of the mutual inductance is connected in series with a ballistic galvanometer. The primary circuit is then closed through a known resistance, R (ohms), and the deflection of the galvanometer is observed. The steady value of the primary current, I (amperes), is also noted. The quantity of electricity, Q (coulombs), corresponding to the deflection is obtained from the constant of the galvanometer or by calibration with a standard condenser (see par. 34).

The mutual inductance is

$$M = \frac{QR}{I} \qquad \text{(henrys)}$$

303. Measurement of Small Inductances of Large Current Capacity.[1]—The measurement of small inductances involves

[1] See "The Measurement of the Inductances of Resistance Coils," F. W. Grover and H. L. Curtis, *Bulletin*, Bureau of Standards, vol. 8, p. 455 (1912–1913).

difficulties, particularly because of the inductance of connecting wires and the mutual inductance with other parts of the testing circuits. Such measurements are becoming increasingly important, because of the greater demand for accurate measurements involving alternating current and power. For example, resistors of large current capacity are used in connection with the accurate calibration of current transformers which are used for power and energy measurements. If such a 0.0001 ohm resistor is assumed to be non-inductive but has an inductance as small as 0.001 microhenry, the error in the determination of the phase angle of the transformer will be 2.15° at 60 cycles which corresponds to an error of 3.7 per cent. at 0.71 power-factor.

The Anderson form is the most suitable of the bridge methods (par. 294) for measurements of this character where suitable variable condensers are available. The determination should be made by difference, that is, the bridge is first balanced with the unknown inductance in place and then with it removed, due account being taken of the inductance of the conductor closing the gap. The standard inductor used in such measurements would naturally have to have a very small value. Such a standard may be one whose value is obtained by calculation from the dimensions (par. 290) or one in which the inductance can be determined from the change in phase angle with change in resistance (par. 289).

304. Precautions in Inductance Measurements.—Care should be taken that the effect of stray fields, either from other parts of the circuit, from the standard inductor or from outside sources, is eliminated by arranging connecting wires in twisted pairs, locating standard inductors at a distance, interchanging the ratio arms in bridge measurements, and so forth.

Obviously, the resistors should be non-inductive and free from capacitance. The standard forms of standard resistors in resistance boxes, bridges, etc., are made nominally non-inductive by winding the wire back on itself once only, that is, it is wound bifilar. In all ordinary engineering measurements where the inductance to be measured is not very small and the resistances used are only a few ohms, these standards may be used without appreciable error. That there may be some inductance and capacitance should be kept in mind, however, particularly in the larger resistors. A method of winding resistance coils which will be practically non-inductive and anti-capacitance has been

devised at the Bureau of Standards,[1] the principle being the reversal of the winding at every turn. The following table gives the inductance of some typical standard coils and of these new coils. The effective inductance given in the table is that inductance which would produce the same phase angle as the residual reactance of the resistor. If inductance predominates in the residual reactance, the sign is positive and if capacitance predominates, the sign is negative.

INDUCTANCE OF RESISTANCE COILS[2]

Nominal resistance of coil, ohms	Effective inductance, microhenrys at 1,200 cycles		
	Standard coils		New form
	American make	German make	
0.1	+ 0.14	+0.18	+ 0.005
1.0	+ 0.4	+0.5	+ 0.05
10.0	+ 0.9	+1.0	+ 0.3
100.0	− 5.0	−2.0	− 1.6
1,000.0	−400.0	−100.0	− 16.0
5,000.0	....	−27,500.0	+ 30.0
10,000.0	....	−100,000.0	+100.0

Similar low inductance and capacitance results are obtained in a type of high resistors devised by Dudell and Mather and made by P. W. Paul. The resistance is in the form of a flat card, the wire being interwoven with silk fibers with the wire as the woof and the silk as the warp.

When measuring very small inductances, the residual inductances and capacitances in the test circuits of bridge methods may be eliminated by first balancing with the unknown reactance short-circuited with a non-inductive resistance equal to the resistance of the reactance. The bridge is then rebalanced with the short-circuit removed.

Where the magnitude of the currents employed is appreciable and the resistances are small, heating of the copper circuits

[1] "Resistance Coils for Alternating-current Work," H. L. CURTIS and and F. W. GROVER, *Bulletin*, Bureau of Standards, vol. 8, p. 512 (1912–13).

[2] "Resistance Coils for Alternating-current Work," H. L. CURTIS and F. W. GROVER, *Bulletin*, Bureau of Standards, vol. 8, p. 514 (1912–1913).

See also Catalogue No. 48 (1916), Leeds & Northrup Co., from whom these new resistors may be obtained.

should be guarded against because the resistance of copper changes approximately 0.4 per cent. per degree C. change of temperature.

If the inductance to be measured is in a circuit involving iron, the measurement must be made with alternating current having a sine-curve wave form, a known frequency and measured intensity because the inductance of such a circuit will vary with the frequency and intensity of the current.

CHAPTER XII

CAPACITANCE MEASUREMENTS

305. General.—When a difference of electrical potential is applied to two conductors which are insulated from each other a charge or quantity of electricity is transferred to the conductors. The ratio of the quantity of electricity to the potential is the electrostatic capacity or capacitance.[1]

306. Condensers.—A condenser is an arrangement of two or more conductors separated by insulation and constructed primarily to provide capacitance. The capacitance of a condenser depends upon the surface area of the conductors, the distance between the conductors, the nature of the dielectric, the temperature and the pressure.

An ideal condenser has zero resistance in the connectors and the conductor plates, the same capacitance under all conditions, infinite resistance between the conductor plates and is capable of absorbing a charge instantly, that is, the dielectric has no absorption characteristics.

Condensers employed in commercial and engineering work are usually made with the conductors in the form of flat sheets and with air, oil, glass, paper or mica as the dielectric. Of these only the air type of condenser can be made to approach the ideal condenser and thus be used as a standard of capacitance. Condensers with other insulation than air depart materially from the ideal conditions particularly in that the phenomenon of absorption is present, that is, the condenser does not receive its full charge instantly but continues to absorb electricity for a greater or less time after the potential is applied (see par. 316). This means energy loss when used with alternating potentials. Furthermore, the charging current with an e.m.f. which is sinusoidal will not be exactly 90° ahead of the e.m.f., a matter of importance in alternating-current work. Also, in the case of certain dielectrics, the capacitance may vary considerably with temperature.

[1] The term "capacitance" is preferable to "capacity" because of the ambiguity of the latter term. The use of "capacitance" rather than electrostatic capacity is recommended by the A. I. E. E. standardization rules.

High-grade condensers are made with tin foil and mica or silvered mica, compressed under high pressure for the purpose of excluding air. Such a condenser has low absorption, small energy loss and a negligible temperature coefficient. The principal condenser of commerce is the paraffined-paper type which is used in enormous quantities in telephone and signalling work where its inferior qualities are not objectionable.

Commercial air condensers[1] usually consist of sets of interleaved metallic plates with air insulation between plates. The plates of a set are connected together, thus making the equivalent of two large plates separated by a layer of air. The solid insulation necessary for the proper supporting of the two sets of plates is so disposed that it is not included in the dielectric between the plates. This type is frequently made variable by having the plates semicircular in shape, one set being mounted on a shaft and arranged to interleave, by revolving, with the other set which is stationary. Air condensers are, of course, very limited in capacitance, rarely exceeding 0.02 or 0.03 microfarad. The variable type referred to may be as small as 0.0001 microfarad maximum and 0.00001 microfarad minimum.

Where greater capacitances than those obtainable with air condensers are required and the potentials employed are relatively small, condensers are made with (*a*) tin foil with mica insulation, (*b*) mica sheets with silver coatings on each side which serve as the conductor plates and (*c*) tin foil with paraffined paper for the insulation. With these dielectrics, the plates can be much closer together than with air and the specific inductive capacitance (par. 318) is greater, so that capacitances several thousand times larger than that of an air condenser of the same volume can be obtained.

Where condensers are to be used at high potentials, they are made with glass and oil as the dielectric. The latter has the advantage that it is not, necessarily, permanently injured when the insulation breaks down because the puncture is "self-healing." Also, a good grade of oil, though having a lower specific inductive capacitance, has much less absorption than glass. Condensers employing compressed air as the dielectric

[1] Much of the following matter pertaining to commercial condensers is from Bureau of Standards *Circular* No. 36, "The Testing and Properties of Electric Condensers" (June 30, 1912) to which the reader, who is especially interested in condensers, should refer.

are sometimes used for high-voltage work, the voltage at which brush discharge takes place (and, therefore, leakage) being raised by compressing the air.

307. Combinations of Condensers.—The capacitance of a group of condensers connected in series is

$$C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \cdots\cdots\cdots\cdots \frac{1}{C_n}}$$

When the condensers are connected in parallel, the capacitance is

$$C = C_1 + C_2 + C_3 \cdots\cdots\cdots\cdots C_n$$

where C_1, C_2, C_3 are the capacitances of the various condensers.

308. Unit of Capacitance.—The unit of capacitance usually used in electrical measurements is the international microfarad which is one-millionth of a farad. A farad is the capacitance of a condenser charged to a potential of 1 international volt by 1 international coulomb of electricity. Very small capacitances are sometimes expressed in "centimeters" because the dimensional formula of capacitance contains only length and a constant. A "centimeter" of capacitance is equal to 1.11×10^{-6} microfarad.

309. Standards.—The standards of capacitance are some form of condenser. As stated above, air condensers can be made to comply practically with the requirements for an ideal condenser. Such condensers are, therefore, used for primary standards of capacitance. They may be constructed in some simple geometric form so that the capacitance can be calculated from the dimensions, or, if the construction is made more complex in order to increase the capacitance, the true value is determined by measurement.

The following are the formulas for the capacitance of the more simple forms of air condensers whose capacity can be calculated from the dimensions.

(*a*) *Condenser Formed by Two, Flat, Parallel Metal Plates.*[1]—

$$C = \frac{S}{4\pi d} \times 1.11 \times 10^{-6} \qquad \text{(microfarads)}$$

where C = capacitance in microfarads, S = area in square centimeters of that surface of the smaller plate which is toward the other plate, d = distance between plates in centimeters.

[1] "Absolute Measurements in Electricity and Magnetism," ALEXANDER GRAY, vol. 1, p. 57.

(*b*) *Condenser Formed by "N" Flat, Parallel, Uniformly Spaced Metal Plates of Same Shape.*—

For series connection

$$C = \frac{S}{4\pi d\ (N - 1)} \times 1.11 \times 10^{-6} \quad \text{(microfarads)}$$

For parallel connetion

$$C = \frac{S\ (N - 1)}{4\pi d} \times 1.11 \times 10^{-6} \quad \text{(microfarads)}$$

C, S and d have the same significance as in formula (*a*). N = total number of plates.

It is to be noted that the formulas (*a*) and (*b*) are correct only when the distance d is small compared with the size of the plates because of the non-uniform field at the edge of the plates and the protrusion of the field beyond the edges of the plates (fringe effect, see par. 319).[1]

(*c*) *Condenser Formed by Two Equal Parallel Wires.*—Length great compared with interaxial distance.[2]

$$C = \frac{0.0894}{\log_\epsilon \left\{ \frac{d}{2a} + \sqrt{\left(\frac{d}{2a}\right)^2 - 1} \right\}} \quad \text{(microfarads)}$$

where C = capacitance per mile of single wire in microfarads, d = distance between the axes in feet, a = radius of wires in inches.

If d is great compared with a,

$$C = \frac{1}{2 \log_\epsilon \frac{d}{a}} \quad \text{(microfarads)}$$

where C, d and a have the same significance as in the first formula.

Mica condensers, when well-made and properly used, rank next to air condensers in approximating the characteristics of the ideal condenser (par. 306). The capacitance is very much greater for the same volume and a much greater range is readily obtained, consequently mica condensers are extensively used as standards. Obviously they have to be standardized experimentally.

[1] For correction factor for this edge effect in the case of circular plates see "Calculation of Alternating-current Problems," LOUIS COHEN, p. 88.

[2] "Calculation of Alternating-current Problems," LOUIS COHEN, p. 101, The reader is referred to this book for formulas for other variations of this case.

Methods of Measuring Capacitance

310. Bridge Method.—In this method the unknown capacitance is measured by comparison with another known capacitance. Fig. 171 indicates the arrangement of the circuits where C_1 is the condenser of known capacitance and C_2 is the unknown capacitance. The bridge is balanced by adjusting either r_1, r_2 or C_1 when the latter is adjustable. The condition of balance is indicated, when using continuous current, by the absence of a "kick" of the galvanometer when the reversing switch is suddenly thrown from one position to the other.

Obviously, the two capacitances C_1 and C_2 should be as nearly alike as possible in order to get maximum sensitivity. The time constants of the two condensers are likely to be very different, as, for example, when the standard is a mica condenser and the unknown capacitance is a length of paper-covered cable. Care should, therefore, be taken to operate the switch slowly so that the condensers will be completely charged each time.

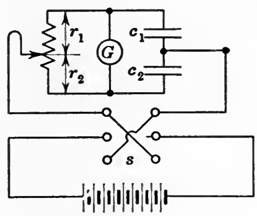


Fig. 171.

Transient or alternating current with suitable detectors, as described in par. 293 for inductance measurements, may also be used. With a Vreeland oscillator as the source of current, a telephone detector and an adjustable standard air or oil condenser, a wide range of very small capacitances can be quickly and accurately measured.

When using alternating current for the bridge, any difference in absorption or leakage characteristics of the two condensers will make it impossible to get absolute balance because of the resulting differences in phase angles, that is, the condenser currents will differ by different amounts from the theoretical angular position of 90° from the impressed e.m.f. To get complete balance, an adjustable resistance is inserted in series with each condenser which permits adjusting the two power-factors to equality. The procedure is to adjust for a minimum as before and then for a new minimum by adjusting one or the other of the added resistances. The first adjustment is repeated and so on until complete balance is obtained.

In all cases

$$C_2 = C_1 \frac{r_1}{r_2}.$$

where C_2 and C_1 are in the same units and r_1, r_2 are also in the same units. The auxiliary resistances used to get complete balance in the alternating-current bridge do not enter the formula.

311. Ballistic Galvanometer Method.—This is a substitution method. First, the unknown capacitance is discharged through the galvanometer, immediately after having been charged at a known potential, and the deflection observed. A known capacitance, that is, a standard condenser, is substituted for the unknown capacitance and the operation repeated. The deflections are kept about the same by adjusting the standard condenser or its charging potential. The unknown capacitance is computed from the relation

$$C_2 = C_1 \frac{E_1}{E_2} \times \frac{d_2}{d_1}$$

where C_2 and C_1 are the unknown and standard capacitances, respectively; E_2 and E_1 are the corresponding charging potentials; and d_2 and d_1 are the corresponding deflections. The two capacitances C_2 and C_1 are in the same units, and E_2 and E_1 are in the same units.

This method is particularly suited to the measurement of large capacitances such as cables and is, therefore, commonly employed in cable factories.

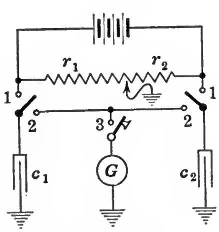


Fig. 172.

312. Method of Mixtures (*Thomson*).—The circuits are arranged as shown in Fig. 172 where C_2 is the unknown capacitance which is shown to be a cable, transmission line or other capacitance in which one conductor is grounded. A standard condenser is represented by C_1. First, the switches are closed at 1, 1 and the two condensers charged to potentials corresponding to r_2 and r_1, respectively. After complete charge (a cable may require several seconds), the switches are shifted to 2, 2 and the charges equalized. The switch at 3 is then closed and a deflection of the galvanometer will be obtained which is proportional to the differences between the charges. This operation is repeated with

various ratios of r_2 to r_1 until there is no deflection when switch 3 is closed.

Then
$$C_2 = C_1 \frac{r_1}{r_2}$$
where C_2 is in the same units as C_1; r_1 and r_2 also being in the same units.

For maximum sensitivity, C_2 and C_1 should be as nearly equal as possible. The resistances should be large and the battery potential as high as permissible, particularly in the final adjustments.

313. Loss-of-charge Method.—In this method, Fig. 173, the condenser to be measured, C, is first completely charged by moving switch b to a and then immediately discharged through a ballistic galvanometer by moving b to c. The condenser is again charged and allowed to discharge through a known high resistance, R, by closing the switch, s, a given number of seconds,

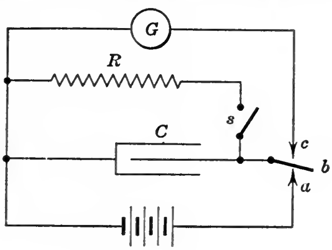


FIG. 173.

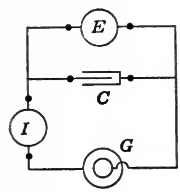


FIG. 174.

t, before being connected to the galvanometer a second time. The capacitance in microfarads is
$$C = \frac{t}{R \log_{10} \left(\frac{d_1}{d_2}\right) 2.303} \qquad \text{(microfarards)}$$
where d_1 = first deflection, d_2 = second deflection and R = resistance in megohms (millions of ohms). This method is applicable only where the resistance of the condenser being measured is very high, such as porcelain insulators. Obviously all parts of the circuit must be highly insulated.

314. Impedance Method.—The capacitance is computed from the reactance as measured with an ammeter and a voltmeter, with alternating current of known frequency, as shown in Fig. 174. Then
$$C = \frac{I}{2\pi f E} \times 10^6 \qquad \text{(microfarads)}$$

where C = capacitance in microfarads, E = volts, I = amperes, and f = frequency in cycles per second. Unless the voltmeter is an electrostatic instrument, it should be disconnected when taking current readings. With small capacitances, where I is small, care should be taken that the high inductance of a low-reading ammeter does not introduce an error.

The capacitance given by this method depends upon the wave form. Therefore, the test should be made from the circuits to which the apparatus being measured will be connected in service. The method assumes, of course, that the power-factor is zero, an assumption that is not strictly justified (see par. 317) but which introduces an error that is negligible in many cases.

315. Absolute Measurements of Capacitance.—A capacitance may be measured in terms of the fundamental units by several methods. The ohm and the second are the units used in the method indicated in Fig. 175 where G is an ordinary D'Arsonval galvanometer, F is a vibrating switch or a tuning fork carrying a suitable contactor, such as a platinum stylus, which charges and discharges the condenser to be measured, C, once per complete oscillation. If the fork makes n vibrations per second, the mean current measured by the galvanometer will be

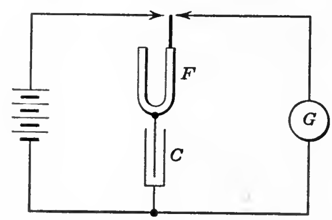


FIG. 175.

$$i = nEC$$

or

$$\frac{1}{nC} = \frac{E}{i} = R$$

The quantity, $\frac{1}{nC}$, is thus equal to a certain resistance, R, the value of which can be obtained by connecting the galvanometer to the same potential and noting the resistance (including that of the galvanometer) which will make the deflection the same as before.

Then

$$C = \frac{1}{nR} \times 10^6 \qquad \text{(microfarads)}$$

where C = capacitance in microfarads, R = resistance in ohms and n = number of discharges per second.

The method usually employed[1] is Maxwell's bridge method. It also is based on the ohm and second. The bridge is arranged as shown in Fig. 176 where the condenser, C, is continuously charged and discharged by means of a tuning fork, a rotating switch or a rotating commutator, F. The exact formula for the capacitance, when balance is obtained by adjustment of the bridge-arm resistances, is

$$C = \frac{r_2}{nr_1r_3}\left\{\frac{1 - \dfrac{r_2^2}{(b + r_2 + r_3)(g + r_1 + r_2)}}{\left(1 + \dfrac{br_2}{r_1(b + r_2 + r_3)}\right)\left(1 + \dfrac{gr_2}{r_3(g + r_1 + r_2)}\right)}\right\} \times 10^6 \quad \text{(microfarads)}$$

where C = capacitance in microfarads; n = number of times condenser is charged and discharged per second; r_1, r_2, r_3 = bridge arm resistances in ohms; b = battery resistance in ohms; and g = galvanometer resistance in ohms.

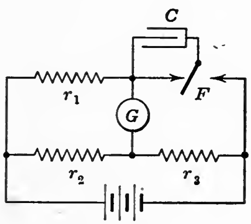


Fig. 176.

316. The Measured Capacitance of Condensers.[2]—As previously pointed out, no type of condenser except air condensers, absorbs its charge instantly because of the characteristic known as absorption.[3] That is, when a potential is applied to a condenser, a certain quantity of electricity will instantly flow, but a much smaller and continuously decreasing current will flow for some time. Similarly, a condenser will not discharge instantly. Therefore, the total charge which a condenser (other than an air condenser) will take depends upon the time of application of the potential, and the measured capacitance will depend upon the frequency of the potential if alternating, or the time of charge and discharge, if the potential is continuous.

The theoretical capacitance of an air condenser as calculated from its dimensions is called the geometric capacitance of the

[1] "The Absolute Measurement of Capacity," E. B. Rosa and F. W. Grover, *Bulletin*, Bureau of Standards, vol. 1, p. 153 (1904–1905).

[2] For a comprehensive, practical discussion of condensers and their properties see "The Testing and Properties of Electrical Condensers," *Circular* No. 36, Bureau of Standards (June 30, 1912).

[3] For a discussion of theories of absorption see "The Capacity and Phase Difference of Paraffined-paper Condensers as Functions of Temperature and Frequency," F. W. Grover, *Bulletin*, Bureau of Standards, vol. 7, p. 495 (1911).

condenser. Since such a condenser takes its charge instantly, the geometric capacitance of any condenser is the ratio of the charge taken in an infinitely short period of time to the applied potential. This obviously cannot be measured directly (although the apparent capacitance at 1,200 cycles, alternating, is practically equal to the geometric capacitance), but may be determined indirectly by measuring the capacitance with various periods of discharge and extrapolating the plotted curve backward to zero time. However, the geometric value of the capacitance is of little value practically because it would not be obtained under the conditions of use. Therefore, if the conditions under which the condenser is to be used are known, the capacitance should be measured with the corresponding periods of charge and discharge. If these conditions are not known, the measurement should be made with some standard period or at several periods. The Bureau of Standards has adapted 100 cycles for measurements with alternating current, and 0.6-sec. charge with 0.1-sec. and 1.0-sec. discharge, for measurements with continuous current.[1]

Temperature may have a considerable effect on the apparent capacitance, especially if paraffine is used, due to the expansion and contraction which changes the spacing between the conductors. Temperature also affects the amount of absorption. Consequently the temperature of the condenser should be noted when it is being measured.

317. Power-factor of Condensers.—The power-factor of an ideal condenser is zero, that is, with a sinusoidal e.m.f. the charging current is 90° ahead of the potential. This condition exists in an air condenser if the applied potential is below the corona value. In other forms of condensers, the phase angle is always less than 90° because of the absorption phenomenon and the energy absorbed due to the ohmic resistance of the leads, plates and the dielectric. It is frequently important to know the amount of the departure from 90° particularly in the low power-factor measurements where condensers are employed. The difference between 90° and the actual angle is usually referred to as the phase angle of the condenser.

The power-factor of a condenser may be measured by comparison with a standard condenser by one of several bridge methods

[1] "The Testing and Properties of Electric Condensers," *Circular* No. 36, Bureau of Standards, p. 25 (June 30, 1912).

described in the *Bulletin* of the Bureau of Standards.[1] The procedure is similar to that for the comparison of capacitances by bridge methods, the essential difference being the addition of a variable resistance or inductance in each of the condenser arms of the bridge to balance the energy components of the condenser currents. The value of the resistance or inductance is noted at balance.

In an alternating-current bridge method used by Thomas[2] for investigating the capacitance and power-factor characteristics of simple two-plate condensers made up with various insulating materials, the arrangement indicated in Fig. 177 was employed where C represents preferably a variable air condenser; K a standard subdivided mica condenser of known power-factor; c, the condenser to be tested; R and r non-inductive resistors; and G a suitable alternating-current detector such as a vibration or dynamometer galvanometer. With alternating potential of the proper potential and frequency, and with K disconnected, r and R are adjusted for a minimum deflection of G. Then C is adjusted until r and R are as nearly equal and as large as possible when the deflection is a minimum. K is then connected and adjusted until a second minimum deflection is obtained. R and r are slightly altered in succession until the deflection becomes practically zero. If C and K are practically zero power factor, the angle of c is obtained from the formulas

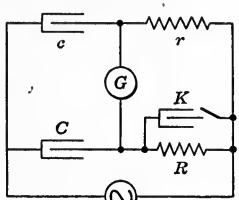


Fig. 177.

$$\tan \phi = \frac{1}{2\pi f R K} \times 10^6$$

$$\text{Power-factor} = 100 \cos \phi$$

and the phase angle of $c = 90° - \phi$. R is in ohms and K in microfarads.

When the capacitance to be measured is 0.0005 microfarad and less, serious errors will be produced by any phase angle in the resistors or standard condensers, and also by the capacitance

[1] For further details and formulas see "Simultaneous Measurement of the Capacity and Power-factor of Condensers," F. W. Grover, *Bulletin* Bureau of Standards, vol. 3, p. 371 (1907).

[2] "A Quality Test for Sheet Insulation," Phillips Thomas, *Electric Journal*, vol. 11, p. 628 (November, 1914).

of various parts of the testing circuits to each other and to ground. It is best to use the substitution method, that is, first balance with a third condenser of known characteristics and having about the same capacitance as that of the condenser being measured. Then substitute the unknown capacitance and correct the result obtained for any error in the bridge as shown by the first measurement.

A wattmeter method which is very convenient employs an electrodynamometer instrument or wattmeter with separate fixed and moving coils. The method[1] is shown diagrammatically

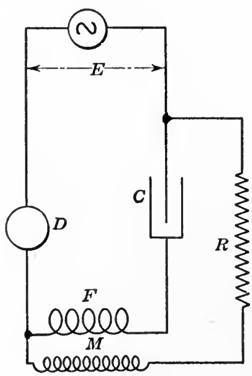


FIG. 178.

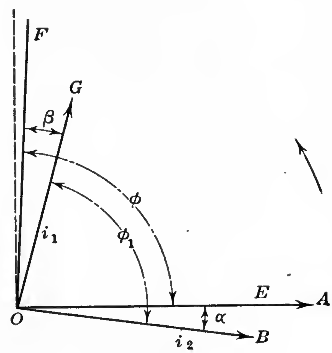


FIG. 179.

in Fig. 178 where F is the fixed coil of the wattmeter, M the moving coil, D a dynamometer ammeter and R a high non-inductive, anti-capacitance resistor, and C is the condenser to be measured. The corresponding vector diagram is shown in Fig. 179 where OA is the impressed e.m.f., E, OB is the current, i_2, in the potential circuit which differs in phase from OA by the angle α; OG is the current through the condenser and fixed coil, i_1, which differs in phase from i_2 by the angle ϕ_1; and OF (angle β from OG) is the position i_1 would have taken if the fixed coil had no resistance. The angle α is the result of the inductance of the moving coil and the residual capacitance of the resistor R. It may be either lagging, as indicated, or leading.

The true angle is $\phi = \phi_1 - \alpha + \beta$. The values of these

[1] "Wattmeter Methods of Measuring Power Expended upon Condensers and Circuits of Low Power-factor," E. B. ROSA, *Bulletin*, Bureau of Standards, vol. 1, No. 3, p. 383 (1904–05).

various angles are found as follows: If d is the deflection of the wattmeter and K its constant,

$$\frac{dK}{i_1 i_2} = \cos \phi_1$$

where $i_2 = \frac{E}{R}$ and $i_1 = i$, the current measured by D, minus i_2;

$$\frac{2\pi f L}{R} = \tan \alpha$$

$$2\pi f C r = \tan \beta$$

where d = frequency in cycles per second, L = effective inductance of moving-coil circuit in henrys, C = capacitance of condenser in farads, r = resistance of fixed coil and connections in ohms, R = resistance of moving-coil circuit (including the moving coil) in ohms.

This method may be made a zero one, that is, the deflection reduced to zero by inserting an additional variable inductor in the moving-coil circuit and adjusting until the deflection is zero. Then the angle ϕ_1 becomes 90° and if the added inductance is L_2,

$$\frac{2\pi f(L + L_2)}{R} = \tan \alpha,$$

$$2\pi f C r = \tan \beta$$

and

$$\phi = 90 - \alpha + \beta$$

The notation is the same as in the preceding formulas.

318. Specific Inductive Capacitance.—The specific inductive capacitance of materials is the ratio of the capacitance of a condenser with the given material as a dielectric, to that of the same condenser with air as a dielectric. In the case of solids in the form of sheets or plates, a condenser is readily made by attaching similar square or circular pieces of tin foil of known area to opposite sides of the specimen and measuring the capacitance by one of the methods described. Care should be taken that there is no deflection when the condenser has no charge. Certain materials in thin, sheet form may show a deflection, due to the galvanic action of the paste used to apply the tin foil.

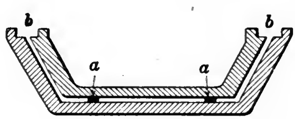


FIG. 180.

Liquid dielectrics are conveniently measured by the apparatus devised by H. W. Fisher and shown in Fig. 180. This apparatus consists simply of two accurately turned metal bowls, one of which fits into the other in such a manner that at any spacing, the distance between the bowls is the same at all points. Three small pieces of glass, *a*, *a* (third not shown), placed in the bottom serve as spacers and the liquid to be measured is poured into the intervening space. The upper edges are cut back at *b*, *b*, in order to reduce the "fringe" effect referred to in the next paragraph.

The capacitance of the corresponding air condenser may, of course, be determined by measurement before the unknown dielectric is inserted between the conductor plates. The value obtained can be checked by calculation from the formula for two parallel plates (par. 309*a*).

319. "Fringe" Effect.—In condensers of the parallel-plate form, such as those referred to in the two preceding paragraphs, the electrostatic field extends beyond the edges of the two conductor surfaces, thus making the area to be used in the calculation

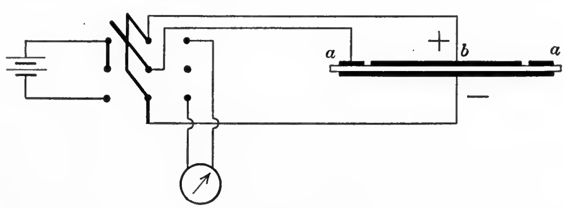


FIG. 181.

of the specific inductive capacitance greater than that of the conductor surfaces by an uncertain amount. Further, the length of the path of the flux beyond the edge is greater than that between the plates. This error may be eliminated by the use of a "guard ring" *a*, *a*, Fig. 181, a ring of sheet metal slightly larger than one plate, *b*. The lower plate is made at least as large as the upper plate plus the ring. The ring is charged to the same potential as the plate *b*, but the connections are so arranged that only the current to *b* is measured (see also guard ring for insulation esistance measurements, par. 162).

320. Precautions.—In general, all parts of the testing circuit should be well insulated, which means all insulating supports should be not only made of a good insulating material but that

their surfaces be thoroughly clean and dry. Wires should preferably be run through the air with a minimum number of points of support.

In the measurement of small capacitances, the capacitance of connecting wires, terminal blocks and auxiliary apparatus may be appreciable and should be considered. In many measurements, this can be corrected for, as previously pointed out, by using the substitution method, that is, measuring the unknown capacitance and then a known capacitance (of about the same value) substituted for the unknown.

CHAPTER XIII

FREQUENCY AND SLIP MEASUREMENTS

Frequency Measurements

321. General.—The frequency of an alternating current is the number of complete cycles per second, or the number of alternations per second multiplied by two. When the source of the alternating current is the usual dynamo-electric machine, the frequency is

$$f = \frac{nr}{2} \qquad \text{(cycles)}$$

where f = frequency in cycles per second, n = number of poles on the generator and r = revolutions of the armature or field per second.

The simplest way to determine the frequency of the current produced by ordinary alternators is obviously to note the number of poles of the generator, or any synchronous machine, being operated by the current and measure the speed. Several types of instruments have been developed for indicating frequency directly. The principal ones are described in the following paragraphs.

322. Frequency Meters.—In the reed type as made by Hartmann and Braun and by Siemens and Halske, there are numerous

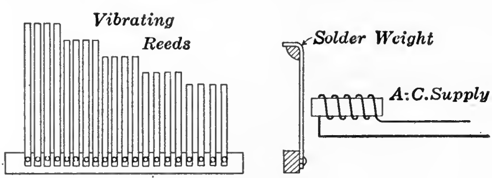


Fig. 182.

steel strips of different lengths, each rigidly fastened at one end and free to vibrate at the other. The strips are placed in the field of an electromagnet which is energized from the circuit to be measured, as indicated in Fig. 182. The strips have different natural periods, and the one with a period corresponding to the alternations of the magnetic field will be set in vibration. The

ends are turned up and painted white so that the particular reed in a row, which is vibrating, will be indicated by a white band or blur. Each reed is carefully adjusted to an exact period by attaching minute weights. These meters are made in various ranges and with reeds adjusted from 0.25 cycle to 2 cycles apart. Provision is frequently made for doubling the range by superposing a continuous-current field on the alternating field. The effect is to neutralize the alternating flux in one alternation or half cycle and amplify it in the next alternation, thus reducing the number of magnetic pulsations one-half.

323. The Westinghouse frequency meter consists of two voltmeter movements, mechanically so interconnected that they tend to rotate the pointer in opposite directions. A non-in-

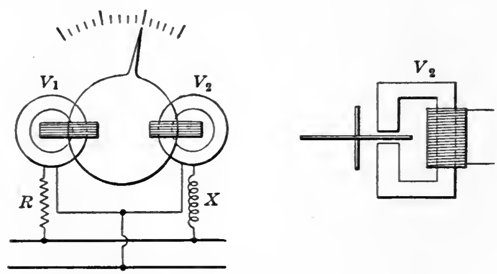


FIG. 183.

ductive resistance R is connected in series with one movement V_1 (Fig. 183), and an inductance X is connected in series with the other element, V_2. The apparent resistance of the inductive movement varies with the frequency and thus varies the amount of current taken by it. Therefore, each frequency will cause the pointer to take up a different position. The scale is empirically graduated with current of known wave form and frequency, which is the same as that of the system upon which the instrument is to be used.

324. The Weston frequency meter is shown in Fig. 184 where 1, 1 and 2, 2 are fixed coils, 90° apart, and c, c is the movable element consisting of a simple, soft-iron core mounted on a shaft, with no control of any kind. One coil, 2, 2, is connected in series with a non-inductive resistance, R_2, and the other coil 1, 1, in series with an inductance, X_1. A second non-inductive resistance R_1 is connected in parallel with 1, 1 and X_1. A second

inductance, X_2, is connected in parallel with 2, 2 and R_2. The soft-iron core takes up the position of the resultant field produced by the two coils. When the frequency increases, the current decreases in 1, 1 and increases in 2, 2, thus shifting the direction of the resultant field and the position of c, c to which the pointer is attached. The opposite effect takes place when the frequency is decreased. The series inductance, X, serves merely to dampen the higher harmonics.

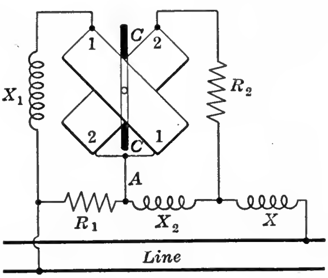


FIG. 184.

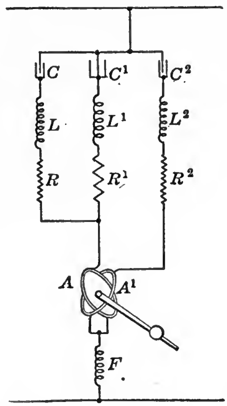


FIG. 185.

325. High-sensitivity Frequency Indicator.—Pratt and Price[1] have devised a frequency indicator which can be made very sensitive. The principle of resonance (see succeeding paragraph) is employed and the electrical circuits are indicated diagrammatically in Fig. 185. In a 60-cycle instrument, one main circuit is adjusted for resonance at about 70 cycles, another at about 58 cycles and the third circuit at about 36 cycles. The two latter are connected in parallel, and then in series with coil A; the first circuit is in series with coil A', both coils being in series with the field F. With the center of a 6-in. (15-cm.) scale marked for 60 cycles, half-scale deflection is obtained for a variation of only 5 cycles either way. It is possible to adjust the instrument for a full scale range of only 1 cycle.

326. Measurement of High Frequencies.—The usual types of electrodynamometer instruments are not suitable for measuring frequencies much in excess of 500 cycles per second. Furthermore, some sources of high-frequency current such as a Vreeland

[1] "A Resonant-circuit Frequency Indicator," W. H. PRATT and D. R. PRICE, *Transactions*, A. I. E. E., vol. 31, p. 1595 (1912).

oscillator, for example, have insufficient power capacity to operate an electromagnetic instrument. It is, in general, necessary to employ a resonant circuit for measuring high frequencies.

When a variable condenser and a variable inductor are connected in series in a circuit which is connected to a constant potential source, there is a certain frequency at which the potential across the condenser becomes a maximum, and equal and opposite in sign to that across the inductor. The current then rises to a maximum value which is limited only by the resistance of the circuit. Such a condition is called electrical resonance. The frequency of the current in any circuit can thus be determined by connecting to the circuit another circuit containing a suitable resistance, a variable condenser, a variable inductor and a suitable indicator, connected to show maximum current or maximum potential. The condenser and the inductor are adjusted until the indicator shows a maximum. Then

$$f = \frac{1}{2\pi\sqrt{LC}}$$

where f = frequency in cycles per second, L = inductance in henrys and C = farads.

For frequencies not over 2,000 or 2,500 cycles, a telephone receiver can be used as the detector if the current is made sufficiently small by the addition of resistance. In wireless telegraphy, various forms of indicators are used including thermogalvanometers and bolometers (see par. 36).

Slip Measurements

327. General.—The difference, if any, between the speed of a rotating alternating-current machine and the synchronous speed is called the slip. It is usually stated in percentage, that is, the difference in speed divided by the synchronous speed and multiplied by 100. Obviously, slip may be determined from the measured speed and the synchronous speed, as calculated from the frequency and the number of poles in the machine, but this method is likely to be inaccurate because the result is a small difference between two relatively large quantities. It is, therefore, customary to measure slip directly by one of the several methods which have been devised. When it is desired to show zero slip or synchronism, synchronism indicators are usually employed.

328. Dooley Method.[1]—This device is shown diagrammatically in Fig. 186. A small cylinder made of conducting material, and in two parts, each insulated from the other, is mounted on a frame. Four small brushes, 1, 2, 3 and 4, bear upon the cylinder as shown. The brushes, 3, 4, are connected through a resistance, r, across one phase of the supply circuit and the brushes 1, 2, are connected to a low-reading continuous-current ammeter, I. Each time the brushes, 1, 2, bridge the insulating strip as the cylinder rotates, the circuit is completed in alternate directions through the ammeter. The cylinder should have as many segments as the motor has poles. The ammeter will indicate a constant current at synchronous speed, and an oscillating current for any speed above or below synchronism, because the impulses of current through the brushes, 1, 2, will occur at the same point on the wave at synchronous speed, and at constantly advancing or retreating points for other speeds. Thus, the ammeter will be reversed each time the motor loses one complete cycle. If the motor loses n cycles per minute, then the slip in per cent. is

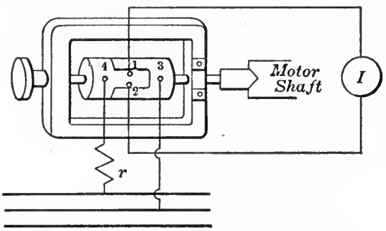


FIG. 186.

$$S = \frac{100n}{60f} \qquad \text{(per cent.)}$$

where f = frequency of the system in cycles per second which must be simultaneously observed. The apparatus can be made self-registering, or recording, by making use of any recording ammeter that can be operated through a relay. A different cylinder is required for each different number of poles.

329. Stroboscopic Method.—The principle of this method is shown in Fig. 187. A black disc with white sectors, symmetrically located, is attached to the end of the induction motor or other machine to be tested. To the end of the shaft of a small synchronous motor, is attached another disc in which sectors have been cut out to correspond to the white sectors in the first disc. A revolution counter is so arranged that it can instantly be thrown in or out of gear with the synchronous motor. The first

[1] "A Slip Indicator," C. R. DOOLEY, *Electric Journal*, vol. 1, p. 590 (1904).

disc is then observed through the second one and if the two machines are running at the same speed, the two discs will appear as one solid disc. But if the speed of the rear disc is less than the front one, it will appear to rotate backward and the slip will be

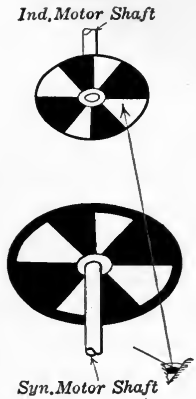


FIG. 187.

$$S = \frac{\frac{n}{n_s}}{R} \times 100 \qquad \text{(per cent.)}$$

where n = number of passages of a white sector past a given point during any interval, n_s = number of openings in the front disc and R = revolutions during the period of observation of n. Any number of white sectors may be used provided there are the same number of openings in the front disc, but the larger the number, the more difficult it is to count n. With very large slips it may be necessary to reduce n_s to one, in which case n is the slip in revolutions.

330. Springer's Method.—A scheme proposed by Prof. Springer[1] is indicated in Fig. 188. The device consists of one shaft, S_2, connected to a small synchronous motor and another

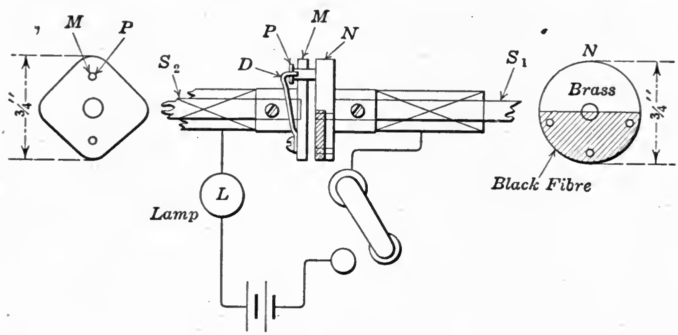


FIG. 188.

shaft, S_1, in line with S_2, suitably geared to the machine being tested. The two discs, M and N, mounted on these two shafts respectively, rotate in the same direction and, when in synchronism, at the same speed. A pin brush is attached to M. One-half of N is cut away and replaced with insulating material

[1] "An Electric Slip Meter," F. W. SPRINGER, *Electrical World*, June 30 1910, p. 1712.

such as hard fiber. An electrical circuit including a flash light battery and lamp is arranged as shown. For each revolution if M gains on N the miniature lamp L will flash once. Thus, if the synchronous motor is an eight-pole machine, each flash represents a slip of 4 cycles. With a revolution counter attached to the synchronous motor and a stop watch, the observer can measure the frequency and slip simultaneously. Counting should start at the exact beginning or end of a flash.

331. Direct-reading Device.—A direct-reading slip-measuring device is shown in Fig. 189. D is a carefully turned and hardened conical drum driven by the motor being tested through a flexible shaft, S. The long screw, H, moves a carriage, C, parallel to the surface of D. This carriage carries a wheel, d, also carefully turned and hardened, which has a line edge and is kept in contact at all times with the surface of D by means of a light spring.

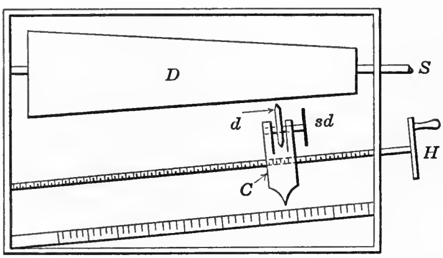


Fig. 189.

Thus it revolves with D. On the same shaft is a disc, sd, with alternate black and white sectors painted on it, the number of sectors being equal to the poles of the motor being tested. The diameter of d is made equal to that of the small end of D. The scale, along which the carriage moves, is marked zero at the setting corresponding to the small end of D. This corresponds to synchronous speed. As the speed of D decreases, it is necessary to move d toward the large end of D in order to keep the speed of d the same as at synchronous speed. This distance is a measure of the slip. The synchronous speed of sd is indicated when it appears to stand still when illuminated by an arc lamp connected to the same circuit to which the motor is connected. Thus, if D is 2 in. (5.1 cm.) in diameter at the small end, 2.5 in. (6.35 cm.) at the large end and 5 in. (12.7 cm.) long, slips from 0 to 25 per cent. can be measured with a precision of 0.2 per cent.

A more sensitive detector consists of a commutator in place of *sd*, connected in series with a continuous-current voltmeter and the circuit to which the motor is connected. The indication is a maximum at synchronous speed.

SYNCHRONISM INDICATORS

332. General.—When it is desired to connect any synchronous alternating-current machine in parallel with another machine or to a circuit, it is necessary not only to have the two voltages equal but the two frequencies must be equal and the two waves must be in phase. That is, the speed of the machine must be so adjusted that the corresponding instantaneous values of the two voltage waves are reached at the same instant and are, therefore, in exact phase or synchronism. This process is called synchronizing and instruments or devices for indicating when synchronism has been obtained are called synchronism indicators. With polyphase machines, the direction of phase rotation must also be made the same. However, if the machine is permanently installed, this feature is taken care of once for all when the machine is installed by properly connecting the leads to the machine.

333. Lamp Method.—The lamp method of synchronizing is the simplest. The principle of lamp synchronizers is shown in Fig. 190, where a, a_1 are the sources being connected in parallel and t, t_1 are transformers, the secondaries of which are connected in opposition through incandescent lamps, l, l'. When the two sources are in synchronism, the secondary e.m.fs. neutralize each other and the lamps will be "dark." As the phase difference increases, the current through the lamps will increase, reaching a maximum at a phase difference of 180°. If the secondary of one transformer is reversed, the lamps will be brightest at synchronism and dark at 180° of phase difference. The former connection is preferable because the point of total "darkness" is more easily detected than the point of maximum "brightness." A voltmeter may be substituted for the lamps by connecting it so that synchronism is indicated when the reading is a maximum.

The disadvantage of this method is that it does not show which frequency is the higher, that is, in which direction the speed of the machine being synchronized has to be adjusted. This has to be determined by trial by passing through synchronism. Commercial synchronism-indicating instruments not only over-

come this objection but indicate the point of exact synchronism more accurately. This is important when synchronizing large apparatus where the disturbances on a system, due to a momentary excessive current caused by an inequality of frequency, are objectionable.

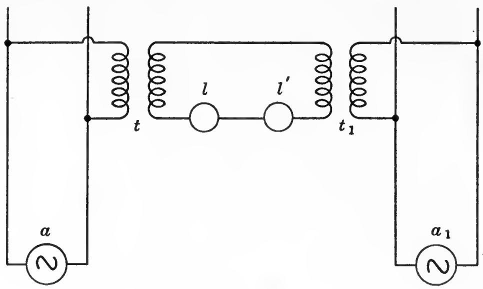


FIG. 190.

334. Westinghouse Synchronizer.—The principle of the Westinghouse synchronizer is shown in Fig. 191. The two coils, M and N, which are both in the plane of the shaft of the moving element but at right angles to each other, are connected to the buses through an inductor P, and a resistor Q, respectively.

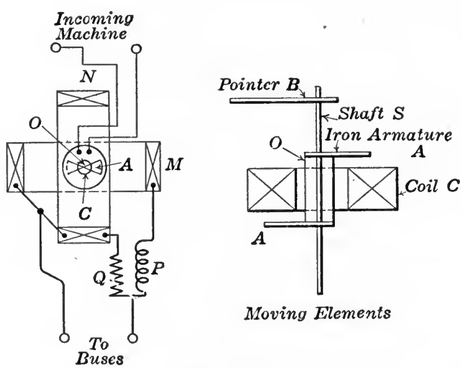


FIG. 191.

The uncontrolled moving element consists of a soft-iron cylinder O with projecting soft-iron vanes, AA, at the top and bottom. Around this so-called armature and close to it, is a coil C which is connected to the incoming machine.

The action of the instrument may be explained as follows: The armature is magnetized by the coil C, first in one direction

and then in the other. When the magnetic flux wave produced by coil *C* is in phase with that produced by coil *N*, the armature will take up a position in the direction of the field produced by *N*. The field produced by coil *M*, being 90° in phase from the field produced by coil *N*, will exert no torque on the armature and consequently the pointer attached to the latter will remain stationary at a point on the scale corresponding to exact synchronism. If the bus frequency and the frequency of the incoming machine are the same, but there is a difference in phase of 90°, the armature will take a position in the direction of the field from coil *M*. For any smaller phase difference, the armature will take a stationary position corresponding to the resultant torque produced by the two coils, *M* and *N*. But if there is a difference in frequency, the phase difference will continually change at a constant rate and the armature will revolve at a corresponding rate. The direction of rotation will, of course, indicate which frequency is the higher.

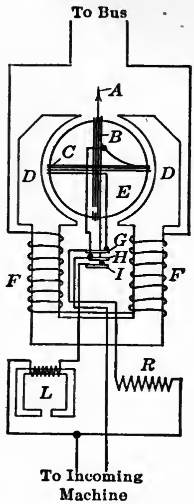


Fig. 192.

335. General Electric Synchroscope.—This instrument (Fig. 192) operates on the same principle as the Westinghouse instrument. The split-phase winding, *C*, *B*, is mounted on the movable element, *E*, and connected to the incoming machine through slip rings.

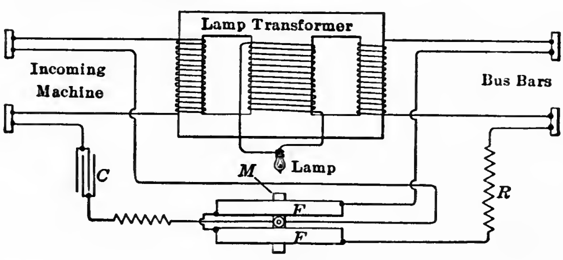


Fig. 193.

336. Weston Synchroscope.—The principle of this instrument is shown in Fig. 193. There is no iron in the instrument and the moving element is not allowed to rotate. The elements are practically the same as in an electrodynamometer wattmeter. The fixed coils, *F*, *F*, are connected in series with the resistor

R and to the buses. The moving coil, *M*, is connected in series with a condenser, *C*, and the incoming machine. The two circuits are adjusted to exactly 90° difference in phase. At synchronism there is no torque and *M* is held at the zero position by the control spring. If the frequencies are the same, but there is a phase difference, a torque will be exerted and *M* will move to a position of balance at the right or left ("fast" or "slow"). If the frequencies are different, the torque will continually vary and the pointer will oscillate over the dial. A synchronizing lamp connected to the three-legged transformer as indicated, simultaneously illuminates the dial and the direction of the apparent rotation of the pointer indicates which frequency is the higher.

337. High-voltage Synchronizer.—A relatively inexpensive synchronizing device for direct connection to three-phase circuits up to 110,000 volts utilizes the luminous discharge which takes place in a rarefied gas under the influence of a high voltage.[1] Each "glower" consists of a round glass bulb, similar to an incandescent lamp, which contains two electrodes and a rarified gas. The two terminals of one glower are connected through suitable insulators to the same phase of the incoming and the "live" circuits respectively. The other two glowers are connected to dissimilar phases of the two remaining phases. If the two circuits are not in synchronism, the glowers will flicker and produce a rotating effect, the direction of which will indicate which circuit is at the lower frequency. When in synchronism, one glower will remain dark and the other two will glow steadily at half brilliancy.

[1] *Bulletin* No. 46,022, General Electric Co., February, 1916.

CHAPTER XIV

WAVE-FORM DETERMINATIONS

338. General.—A curve showing the relation between time and the instantaneous values of a varying electrical e.m.f. or potential is referred to as the wave form of that e.m.f. or potential. The wave form actually determined is usually, however, that of a small derived current which is exactly proportional at all instants to the voltage being investigated. Similarly, the wave form of a current is the same as that of the drop across a shunt in the circuit and the wave form of a magnetic flux is the same as that of the e.m.f. induced in a loop cut by that flux.

The instantaneous variations may be periodic and recurring as in an ordinary alternating-current circuit, or they may be transient as, for example, the momentary voltage and current oscillations which are set up when a "dead" transmission line is connected to a high-voltage system.

Wave forms may be obtained by any one of several "point-by-point" methods or by means of an oscillograph. In point-by-point methods, the wave form is obtained by plotting against time the instantaneous values measured at suitable intervals of time. The curve is ordinarily plotted by hand on cross-section paper but instruments have been developed by means of which the curve is plotted mechanically or photographically.

With the oscillograph, the wave form is continuously traced photographically by means of a beam of light which is made to follow exactly the instantaneous variations of a current through a suitable galvanometer element.

The point-by-point methods are applicable only to phenomena which are strictly periodic and recurring. The oscillograph may be used with both periodic and transient phenomena. However, where a wave form is to be analyzed, a curve obtained by a point-by-point method is much more accurate and is usually more convenient where any calculations are to be made.

POINT-BY-POINT METHODS

339. With a Condenser, Discharging.—The feature of these schemes is the use of a condenser which is charged by the instantaneous voltage corresponding to a point on the wave and then discharged through an indicating instrument. One of the earliest methods employed for obtaining the wave form of a voltage is shown diagrammatically in Fig. 194. A point contact, *c*, is rotated in synchronism with the source of the voltage being investigated. The brush, *b*, is attached to a fixed collar which is concentric with the shaft and provided with an evenly divided scale. This brush can be rotated to any angular position, so that

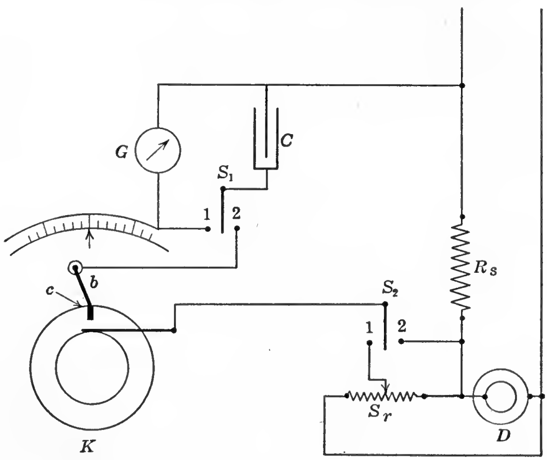


FIG. 194.

contact can be made at any instant during a cycle. For current waves, switch S_2 is placed in position 2 and then the drop across the shunt R_s is measured. For potential waves, the switch is placed in position 1. The resistor S_r permits adjusting the potential to the contactor to a suitable fraction of the line potential.

The procedure is as follows: Switch S_1 is turned to position 2 and the condenser charged. Then the switch is turned to position 1 and the condenser discharged through a ballistic galvanometer G. A deflection is obtained which is proportional to the instantaneous voltage (or current) at that particular point of the wave. The apparatus is calibrated by passing a known continuous current through the shunt, r_s. The wave form is obtained by plotting

the deflection (reduced to amperes or volts) as ordinates, and the angular positions as abscissas.

A more convenient arrangement is indicated in Fig. 195. A contactor, K, consisting of four equal and separate segments set in the periphery of a disc of insulating material, is direct-connected to a synchronous motor. The segments are connected in pairs to two slip rings on the same shaft as shown. The brush which makes contact with the segments is mounted on a collar which can be revolved around the shaft so that contact is made at any angular position the same as in Fig. 194. This brush and the two on the slip rings are connected to the circuit as indicated in the figure, C being a condenser and V an ordinary continuous-

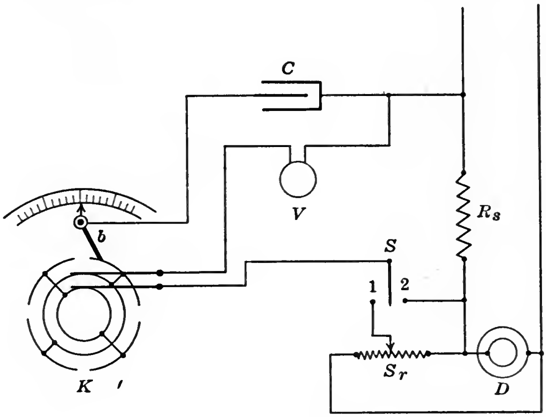


FIG. 195.

current voltmeter. When the contactor rotates, the condenser is first charged and then immediately discharged through the voltmeter, the impulses to the latter following each other so rapidly that a steady deflection results. Voltage readings are obtained with the switch S on 1 and current readings with the switch on 2.

It will be noted that the contacts are of equal length. It was found that the charging segment does not have to be of minimum length as intimated by the scheme of Fig. 194 because the charge taken by the condenser is proportional to the potential at the final instant only when the segment leaves the brush. The length of the segment is, therefore, immaterial and the charging segments may be of the same length as the discharging segments if desirable for structural reasons. Neither does the number of pairs of

segments have to be the same as the number of pairs of poles. In the figure (195), a four-part contactor is shown. If it is driven by a four-pole synchronous motor, one impulse will be obtained per cycle and the contact brush would have to be shifted through 180 mechanical degrees to get a complete cycle. But if an eight-pole motor was used, a complete cycle would be obtained in 90 mechanical degrees. A two-segment contactor would give the same result although an impulse would be obtained only every second cycle and the deflection of the instrument might not be as steady.

The condenser in all cases should be one which is free from absorption (par. 316), such as an air condenser or a high-grade mica condenser. Also, the time constant of the discharge circuit should always be less than the discharge period in order to permit complete discharge of the condenser. That is, the product of the capacitance of the condenser in farads and the resistance of the detecting-instrument circuit in ohms should be less than the time in seconds corresponding to the length of the discharge segments.

The appartus is calibrated by substituting continuous current for the alternating current, and, with the contactor rotating at normal speed, noting the deflections with various values of current and potential.

The "wave meter" made by the General Electric Co. operates on this principle. The arrangement of the circuits is the same as in Fig. 194, except that there are two pairs of slip rings and eight segments on the contactor with two brushes set 45 mechanical degrees apart. One set of four segments is connected to each pair of rings. Thus two waves may be examined simultaneously, each brush being connected, as in Fig. 194, to a condenser and indicating instrument suitable for the quantity being determined. Provision is made for photographing the wave form by allowing a photographic plate to move in front of two galvanometers which reflect a sharply focused beam of light from an incandescent lamp. The galvanometers replace the indicating instruments. The plate holder is connected by a cord to the yoke carrying the two contactor brushes so that when the plate holder is moved at a slow rate by means of a weight and dashpot, the contactor brushes are automatically moved at the same rate. Thus curves of the successive deflections of the galvanometer are obtained.

Dr. Louis Duncan devised a scheme for taking the wave form of

several quantities simultaneously with one contactor. The arrangement is shown in Fig. 196. The indicators, D and D', are electrodynamometer instruments, two being indicated in the diagram, one for voltage and one for current. The movable fine-wire coils are connected in series and to one slip ring of the contactor. A battery, B, and condenser, C, are connected as shown. The condenser is successively charged from the battery through brush b, and then discharged through the two movable coils, thus producing steady deflections which are proportional to the potential and current, respectively, at the point on the wave corresponding to the setting of the contact-brush yoke.

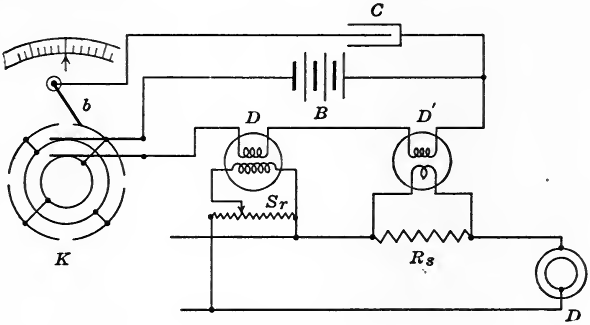


FIG. 196.

It is to be specially noted that the preceding methods give the complete wave form of a complete cycle, that is, they are applicable to unsymmetrical waves.

340. With a Condenser, Charging.—The charging current of a condenser is employed to determine the wave form of an e.m.f. or current by means of a synchronous commutator.[1] A synchronous commutator is the synchronous contactor of Figs. 195 and 196 arranged to commutate the alternating wave, that is, rectify every other half cycle. The arrangement of the circuit is indicated diagrammatically in Fig. 197 where K is a four-part commutator driven by a four-pole synchronous motor (or eight parts for an eight-pole motor). The two commutator brushes,

[1] "The Use of the Synchronous Commutator in Alternating-current Measurements," FREDERICK BEDELL, *Journal Franklin Institute,* October, 1913, p. 397.

"Condenser-current Method for the Determination of Alternating Wave Forms," FREDERICK BEDELL, *Electrical World,* Aug. 23, 1913, p. 378.

B_1 and B_2, are mounted on a revolvable yoke exactly opposite each other. These brushes are connected to a continuous current instrument, G, of suitable range for measuring the current taken by the condenser (an ammeter, milliammeter, or voltmeter). The slip-ring brushes are connected to the circuit through a condenser, C.

The value of the e.m.f. at the instant corresponding to the angular position of the commutator brushes is

$$e = \frac{A}{4fC} \times 10^6 \qquad \text{(volts)}$$

where A = current in amperes corresponding to indication of G, f = frequency and C = capacitance of condenser in micro-

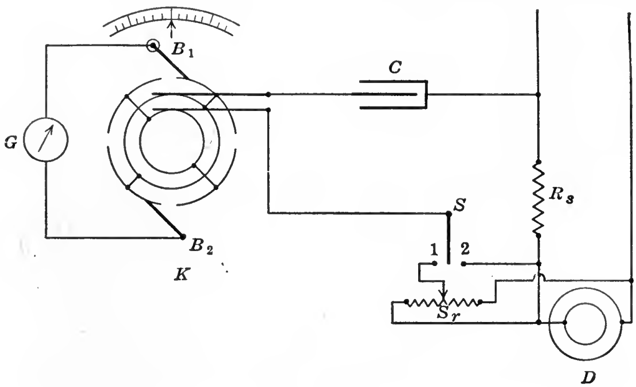


FIG. 197.

farads. For e.m.f. readings the switch S is turned to position 1, and for current readings to position 2, R_s being a non-inductive shunt and S_r a potential divider as in Fig. 194.

This method is applicable only to symmetrical waves, that is, waves with positive and negative sides exactly alike.

341. Opposition Method.—This method is probably the most convenient of the point-by-point methods, because of the simplicity of the apparatus and the ease of application. Furthermore, being a zero method, it is extremely accurate.

The principle of the scheme is shown in Fig. 198. A single point contactor like the one in Fig. 194 is connected in series with the e.m.f. being investigated and the impulses opposed to the drop along a resistor, m, n, which is connected across a source of continuous potential such as a battery, B. The contact, X, is ad-

justed until the detector, G, shows no deflection. The instantaneous value is then equal to the drop in nx.

The detector may be any suitable continuous-current instrument such as a D'Arsonval galvanometer, millivoltmeter or voltmeter. A telephone receiver may also be used. For small potentials, the resistor, mn, may very conveniently be a straight wire laid on a graduated scale with a metal point or stylus for the contact, x. The switch, S, provides for both current and e.m.f. measurements as in the previous methods and S_1 serves to reverse the continuous potential when the wave reverses. This latter switch may be dispensed with by connecting the fixed alternating-current terminal to the middle of ab instead of at a.

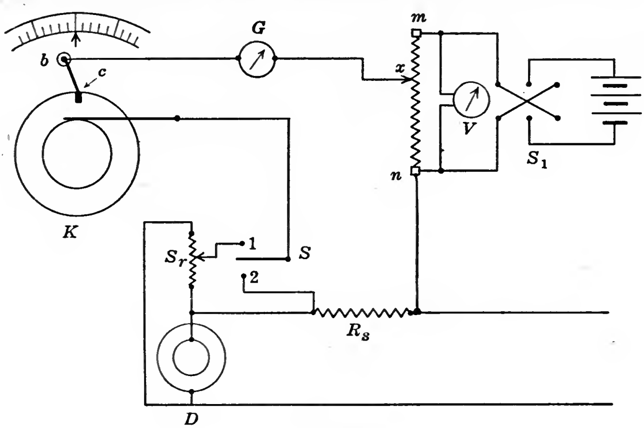


FIG. 198.

The Rosa curve tracer[1] employs the opposition principle. The scheme is exactly the same as that indicated in Fig. 198 with the addition of a cylindrical drum alongside the potential divider ab and a contact device, x, so arranged that a mark can be made on a sheet of paper on the drum when balance is obtained. After a point is obtained, the drum is advanced a fraction of a revolution corresponding to the fraction of a cycle that the contact brush b is advanced.

OSCILLOGRAPHS

342. General.—An oscillograph is a form of galvanometer in which the natural period of the moving system is so small that

[1] For further details, see *Physical Review*, vol. 6, p. 17 (1898).

the deflections will always be proportional to the instantaneous value of the current flowing through the coil. The indicator is a beam of light from an arc lamp or other high-intensity source, reflected from an extremely small mirror attached to the moving system. The path of the beam is determined visually or photographically. Recurrent or periodic waves may be rendered stationary, and therefore visible, by suitable optical systems. Transient phenomena must be photographed as they occur.

The various forms of oscillographs which have been developed are distinguished principally by the kind of moving system employed and the method used to produce the oscillations. One of the earliest forms, devised by Blondel, had a moving system consisting of a very small piece of soft iron suspended in a strong magnetic field which thus formed a polarized magnet. On either side of this small magnet and close to it was placed a small coil. These two coils were connected in series and carried the current to be investigated. The alternating field produced was superposed on the fixed field and caused the magnet to oscillate with the alternating current in the two small coils. The moving-coil type, first developed by Duddell, has, however, probably been the most extensively used in general engineering work, especially in this country.

343. Moving-coil Oscillographs.—The moving-coil type of oscillograph consists of a single-turn coil formed by passing a phosphor-bronze strip over a pulley suspended by a spring, and between the poles of a powerful electromagnet as indicated diagrammatically in Fig. 199. This type is prominently represented by the General Electric oscillograph, a very simple, rugged and practical instrument. Three elements or vibrators are usually provided and three waves may be taken simultaneously.

The details of one vibrator are shown in Fig. 200 where BB_1 are bridges for supporting that portion of the strip which is in the magnetic field; P is an ivory pulley around which the strip passes; SB a spring balance which indicates the tension on the strip; M a mirror; TS a screw for adjusting the tension on the strip, and TT_1 the terminals of the strip. The complete vibrator, as shown in Fig. 200, is immersed in a well which has a glass front and is filled with oil to dampen the oscillations. The disc, C, serves as the cover of the well.

The natural frequency is of the order of 5,000 vibrations per second and with the electromagnets excited at the normal value,

the current sensitivity is of the order of 0.005 amp. per millimeter deflection on the screen furnished with the instrument. With a given field strength, the sensitivity and period are determined by the tension on the strip and the spacing of the bridges, BB_1. Too low a tension causes the vibrating strip to overshoot and distort the wave. A frequency of the order of 1,500 cycles per second is ordinarily about the upper limit for reliable results.

The moving-coil type has a lower inductance than the moving-iron type and therefore is more generally useful. A hot-wire instrument devised by J. T. Irwin[1] and made by R. W. Paul has practically no inductance. It utilizes the unequal expansion of

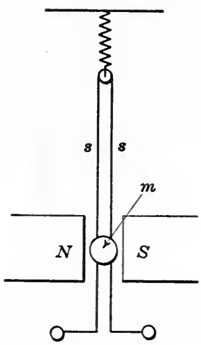


FIG. 199.

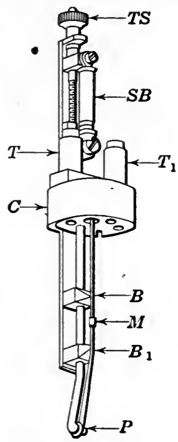


FIG. 200.

the two sides of a loop, one side of which carries a current proportional to the difference between the current being investigated and a steady battery current. The other side carries a current proportional to the sum of the same two currents. The thermal lag is reduced by means of condensers. The natural frequency is about 6,000 per second.

344. Visualizing Devices.—Various receiving devices are used to make the wave visible to the eye. One scheme employs a rotating mirror driven by a synchronous motor connected to the circuit being tested, but the following description of the method used in the General Electric Co.'s oscillograph will illustrate the principles usually employed.

[1] *Transactions*, Inst. E. E., vol. 34, p. 617 (1907).

In the General Electric oscillograph the beam of light from the arc, *a*, Fig. 201, is concentrated by a lens, *b*, upon the mirror, *l*, of the vibrator which is oscillating about an axis perpendicular to the beam. The mirror, *m*, is driven by a cam, *d*, which gives a comparatively slow forward stroke and a quick backward stroke. When the mirror, *m*, is stationary, the image of the reflected beam at a screen, *s*, appears as a straight line, the length of which is proportional to twice the angle of deflection of the vibrator from the zero position, or in other words, proportional to twice the amplitude of the wave. If the mirror is rotated forward during a period corresponding to exactly one cycle, the image at *s* is spread out and a tracing of the wave is obtained. Then by cutting off the light during the return stroke of the mirror, the wave will appear stationary. This is all accomplished automatically by means of a synchronous motor connected to the source of the e.m.f. or current being investigated. This motor oscillates the

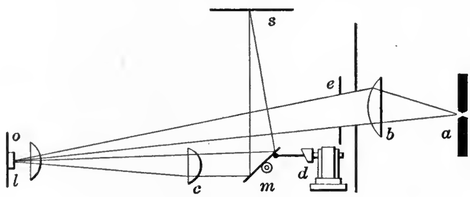


FIG. 201.

mirror, *m*, and rotates the shutter, *e*, in such a manner that the mirror is moved forward during one cycle and the shutter is interposed during the next cycle while the mirror is returning to its initial position.

345. Oscillograph Records.—Permanent records may be made by tracing the wave on transparent paper or tracing cloth placed on the screen, *s*. For photograph records, or oscillograms, the visual attachment is replaced by a light-proof cylinder containing the film and fitting over an opening in the oscillograph case through which the beam of light passes to the film. The film is unrolled from one spool on to another and past the opening by a motor-driven mechanism; its speed is adjusted to suit the conditions. In the case of transient phenomena it is often necessary to arrange special devices to start the film automatically at the proper time to get the phenomena on the film.

A record of the time in continuous-current phenomena can be

obtained by connecting one of the elements to a source of alternating current of known frequency, or by means of an electrically driven tuning fork of known frequency. A small mirror mounted on the end of one prong of the tuning fork, and so placed that it will reflect a part of the beam, will give an excellent time record and without interfering with the use of all elements for taking simultaneous wave forms. Often, however, an alternating potential of known frequency can be superposed in the circuit to one element without interfering with the record being obtained.

The scale of ordinates is obtained by calibration on continuous current, the deflection of the spot of light corresponding to a known current, or voltage, being noted.

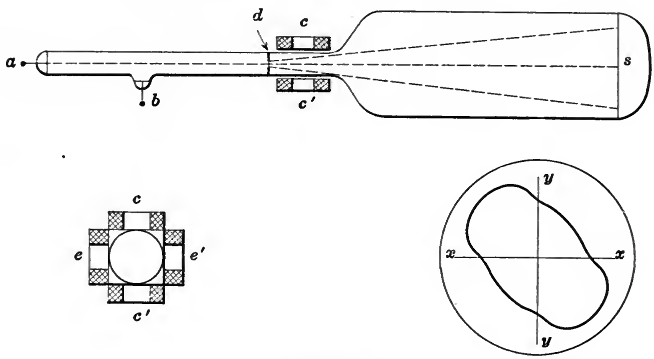


FIG. 202.

346. Cathode Ray Indicator.—Prof. Harris J. Ryan has developed an inertialess type of oscillograph by utilizing the Braun type of cathode ray tube.[1] The construction of the tube is shown in the upper part of Fig. 202. It is a glass vacuum tube with cathode and anode electrodes at *a* and *b*, respectively, which are connected to a source of continuous, high e.m.f., such as an electrostatic machine or high-voltage rectifier. When the tube is excited, a stream of cathode particles are projected along the tube. The diaphragm, *d*, intercepts all but a ray which passes through a small hole at the center and impinges on the fluorescent screen, *s*. If two coils, *c*, c^1, are placed on opposite sides of

[1] "The Cathode Ray Alternating-current Wave Indicator," HARRIS J. RYAN, *Transactions*, A. I. E. E., vol. 22, p. 538 (1903). See also par. 200 and Fig. 117.

the tube as indicated, and the current is passed through them, the cathode ray will be deflected an amount strictly proportional to the current. If two sets of coils are arranged, as shown at the lower left-hand corner of the figure, and one carries a true sine-wave current while the other is connected to the unknown alternating-current wave, the ray will trace a curve on the screen *s* of the character of the example shown at the lower right-hand corner. This curve can then be converted to rectangular coördinates and the unknown wave shape determined.

Magnetic Flux-wave Forms

347. Determination of Wave Form.—The wave form of a magnetic flux may be very easily determined by means of the four-part synchronous commutator[1] of Fig. 197. The slip-ring brushes are connected to a test coil placed where it interlinks with the flux being investigated, and the commutator brushes are connected to a continuous-current voltmeter of suitable range. With the driving motor connected to the source of the flux, the instantaneous value of the flux corresponding to the angular position of the brushes is

$$\phi = \frac{V \times 10^8}{4fT} \qquad \text{(maxwells)}$$

where ϕ = total flux which links with the coil, V = indication of voltmeter in volts, f = frequency and T = turns in test coil. The flux density, in gausses, is obtained by dividing ϕ by the mean area of the coil in sq. cm. That is, the value of the flux at any instant is proportional to the algebraic average of the induced e.m.f. during the succeeding half cycle. The method is not applicable where the two halves of the wave are not exactly the same.

Wave Forms in High-voltage Circuits

348. Potential Wave Forms.—The shape of a high-potential wave can be obtained with modern, standard types of instrument

[1] "An Apparatus for the Determination of the Form of a Wave of Magnetic Flux," M. G. Lloyd and J. V. S. Fisher, *Bulletin*, Bureau of Standards, vol. 4, p. 467 (1907–1908).

"The Use of the Synchronous Commutator in Alternating-current Measurements," Frederick Bedell, *Journal Franklin Institute*, October, 1913, p. 393.

potential transformers in conjunction with the various methods described for small potentials. There is no appreciable distortion in a properly designed transformer, consequently the accuracy is ample for all practical purposes. The resistance of the secondary circuit should, however, be kept as high as possible so that the transformer will be as near the no-load condition as possible.

The shape of a high-voltage wave may also be determined directly by the scheme shown diagrammatically in Fig. 203[1] where $C + C_1$ is a series of condensers connected across the high-tension circuit and serving as a voltage divid r. The end condenser C_1 is grounded and across it is connected a synchronous contactor R_1 which is a device exactly the same as the synchronous commutators of Figs. 195, 196 and so forth, except that the length of the segments is reduced to a minimum, that is, merely pieces of thin brass strip set radially in the edge of the disc. The slip-ring brushes are connected to C_1 and the contact brushes are connected to an electrostatic voltmeter, V. The contacts are set exactly one polar space apart and the effect is to connect the voltmeter to the circuit for an instant every half cycle, the connections being reversed every second half cycle. The electrostatic voltmeter is fully charged as a condenser in a few revolutions and its final indication is proportional to the instantaneous value of the voltage corresponding to the position of the contact brushes. If the actual voltage is required, the instrument can be calibrated against a sphere gap across the circuit, the contactor being at rest with the brushes resting on the contacts. The multiplier is the ratio of the capacitance of $C + C_1$ to the capacitance of C_1. The capacitance of V should be small compared with that of C_1.

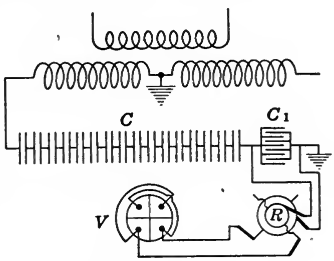


FIG. 203.

349. Current Wave Forms.—For current measurements, the standard instrument current transformers are not generally suitable because the e.m.f. available in the secondary is too small when the resistance in the secondary circuit has been made

[1] "Measurement of Maximum Values in High-voltage Testing," C. H. SHARP and F. M. FARMER, *Transactions*, A. I. E. E., vol. 31, p. 1617 (1912).

sufficiently small to eliminate distortion of the current wave. An air-core transformer is, however, quite free from this objection. Its primary is connected in series with the circuit and the secondary is connected in series with a continuous-current voltmeter through a synchronous commutator. The instantaneous value of current at any position of the commutator brushes is

$$i = \frac{V}{4fM} \qquad \text{(amperes)}$$

where V = indication of voltmeter in volts, M = mutual inductance of transformer in henrys and f = frequency. The resistance of the secondary circuit should be kept sufficiently high to make the circuit practically non-inductive.

Other methods, particularly the opposition methods, can also be used with an air-core transformer for current determinations.

Deviation of Wave Form from the Standard

350. General.—The sine curve is considered the standard of wave form. Obviously it is not feasible to "measure" a form directly in terms of the standard and express the result in a single numerical quantity. The deviation may, however, be expressed by the ratio of certain constants of the unknown wave to similar constants of an equivalent sine wave—that is, a wave having the same frequency and the same root-mean-square value.[1] Unfortunately, none of these ratios completely defines the deviation from the standard but each emphasizes the deviation in certain particulars. For example, the ratio of the peak factors (ratio of maximum value to root-mean-square value) might be a satisfactory measure where only the stress in insulation is concerned. Again, the ratio of form factors (ratio of root-mean-square value to average value) might serve where only hysteresis losses in iron are considered.

351. Deviation Factor.—The present official method of the A. I. E. E. for indicating the divergence of a wave form from the standard is by means of the "deviation factor." This factor is obtained by superposing the actual wave on an equivalent sine wave in such a manner as to give the smallest differences between corresponding ordinates. The maximum difference between

[1] "Distortion Factors," F. Bedell, R. Brown and C. L. Swisher, *Proceedings*, A. I. E. E., June, 1915, p. 1059.

corresponding ordinates, divided by the maximum ordinate of the equivalent sine wave, is the deviation factor. The standardization rules of the A. I. E. E. recommend that this factor be not more than 10 per cent.

This method necessitates the actual determination of the wave shape by one of the methods described in the preceding paragraphs and therefore involves a considerable amount of labor. Furthermore this factor does not sufficiently emphasize the presence of the higher harmonics in a distorted wave, a matter of great importance in modern engineering practice. Several other factors have therefore been proposed which could be obtained by simple testing methods involving the use of only ordinary indicating instruments.

352. Differential Distortion Factor.—Of the other factors which have been proposed, the one which probably meets present commercial requirements better than any other, is the "differential distortion factor." It is the ratio of the root-mean-square value of the first derivative of the wave with respect to time to the root mean-square value of the first derivative of the equivalent sine wave. Or, in other words, differential distortion factor of a wave is the ratio of the admittance of a condenser on that wave to the admittance on an equivalent sine wave.[1]

There are objections to this factor, principal among which is the fact that high harmonics have too great influence and low harmonics too little. It seems probable, however, that there will eventually be adopted a factor which is obtained by measurements with some form of condenser circuit, such a circuit for example, as one containing inductance and resistance in addition to capacitance. A circuit of this general character can be adjusted to give the lower and the higher harmonics the desired respective weights in the factor obtained.[2]

Analysis of Wave Forms

353. General.—The most complex alternating e.m.f. or current wave form may be considered as made up of a fundamental sine wave and one or more sine waves of higher frequency. The

[1] "A Proposed Wave Shape Standard," C. M. Davis, *Transactions*, A. I. E. E., vol. 32, p. 775 (1913).

[2] "Characteristics of Admittance Types of Wave Form Standard," Frederick Bedell, *Proceedings*, A. I. E. E., August, 1916, p. 1171.

complete definition of a wave form expressed numerically is, therefore, an equation which indicates the amplitude of the fundamental, together with the amplitude, frequency and phase position of each component wave with reference to the fundamental. It is often only required to know that a wave does not deviate from the standard sine wave by more than a prescribed amount as determined by means of the factors discussed in the preceding paragraphs. On the other hand, it is frequently desired to know what harmonics are present which cause the departure from the sine shape, their magnitude and their phase position. This information is obtained by analyzing the wave form, that is, determining the component waves. Such an analysis may be made mathematically and by means of mechanical devices which have been developed for that purpose.

354. Mathematical Methods.—Mathematical methods are based on the fact that any alternating e.m.f. or current wave may be expressed as a Fourier's series as follows:

$$e = C_1 \sin (x + \phi_1) + C_2 \sin 2(x + \phi_2) + C_3 \sin 3(x + \phi_3) \ . \ . \ .$$

where the first term represents the fundamental wave; the other terms represent harmonic waves of 2, 3 . . ., times the frequency of the fundamental wave; C_1, C_2, C_3, . . ., are the corresponding amplitudes; and ϕ_1, ϕ_2, ϕ_3, . . ., are the corresponding phase positions referred to the origin from which the angle x is measured. The analysis of a wave consists in determining the amplitudes, C_1, C_2, C_3, . . ., and the angles ϕ_1, ϕ_2, ϕ_3, ... The actual labor of making the large number of calculations involved, even with the shorter methods which have been devised, is very great and as the average engineer is seldom called upon to make such an analysis, space will not be taken here to present the details of the procedure. The reader who is especially interested is referred to the numerous articles in the technical literature on the subject. The following articles are particularly concise:

"Direct- and Alternating-current Manual," 2d edition, Bedell and Pierce, p. 331. Gives details of the method of 18 ordinates (odd harmonics only) with numerical example. Includes the origin and proof of the method. Runge's method of 12 ordinates for both odd and even harmonics is also given.

"Electric Waves," W. S. Franklin, p. 217. A chapter is devoted to a detailed discussion of the analysis of harmonic waves and the application of Fourier's theorem.

"Analysis of Alternating-current Waves," F. W. GROVER, *Bulletin*, Bureau of Standards, vol. 9, p. 567 (1913–1914). Analysis of waves by the method of Fourier with special reference to methods for facilitating the computations. Gives schedules for the arrangement of the computations, also numerous numerical examples and helpful multiplication tables for use in the computations.

"Experimental Electrical Engineering," V. KARAPETOFF, vol. 2, p. 222. A discussion of wave analysis with specimen tables for convenient arrangement of data.

"Alternating-curve Wave Form Analysis," S. M. KINTNER, *Electrical World*, May 28, 1904, p. 1023. Brief discussion of the theory and methods based on Fourier's series with detailed instructions for carrying out the analysis of an ordinary wave such as those usually encountered in practice. Includes tables of working constants.

"Wave Form Analysis," P. M. LINCOLN, *Electric Journal*, vol. 5, p. 386 (1908). Describes the theory and application of the Fischer-Hinnen method, with illustrative example.

"The Separation of an Alternating-current Wave into Its Harmonic Components," C. A. PIERCE and WILLIAN ANDERSON, *Electrical World*, Oct. 21, 1911, p. 1007. A comparison of the Fischer-Hinnen and the Runge methods (two of the shorter methods) and simplified means of applying the latter.

"Graphical Computations of Fourier's Constants for Alternating-current Waves," C. S. SLICHTER, *Electrical World*, July 15, 1909, p. 146. A short graphical method for the ordinary waves, that is, waves with odd harmonics only and not exceeding the ninth or eleventh.

"American Handbook for Electrical Engineers," p. 1829. Brief description of Fischer-Hinnen method.

"Standard Handbook for Electrical Engineers," 4th edition, p. 101. Brief description of Fischer-Hinnen method.

355. Mechanical Methods.—A number of mechanical devices have been devised for analyzing harmonic waves.[1] The most recent and generally useful is the Chubb harmonic analyzer[2] manufactured by the Westinghouse Electric and Manufacturing Co. The principle of this device is the measurement of the sine and cosine components of each harmonic, by integrating a curve formed by a point oscillated by a template the shape of which is that of the curve, plotted in polar coördinates. The apparatus

[1] For a description of the principle of one of the earliest and also one of the simplest methods (perfected by Lord Kelvin) see "Electric Waves," W. S. FRANKLIN, p. 240.

[2] "The Analysis of Periodic Waves," L. W. CHUBB, *Electric Journal*, vol. 11, p. 91 (1914).

is shown in Fig. 204 and the general construction is as follows: A template of cardboard, X, is cut out exactly the shape of the wave to be analyzed, drawn in polar coördinates.[1] This template is mounted on the turntable T, which is carried backward and forward on a carriage C sliding on rails RR. This oscillating motion is produced by crank pin P on worm gear G mounted on the base. The gear is driven by a worm on shaft D. The pin P

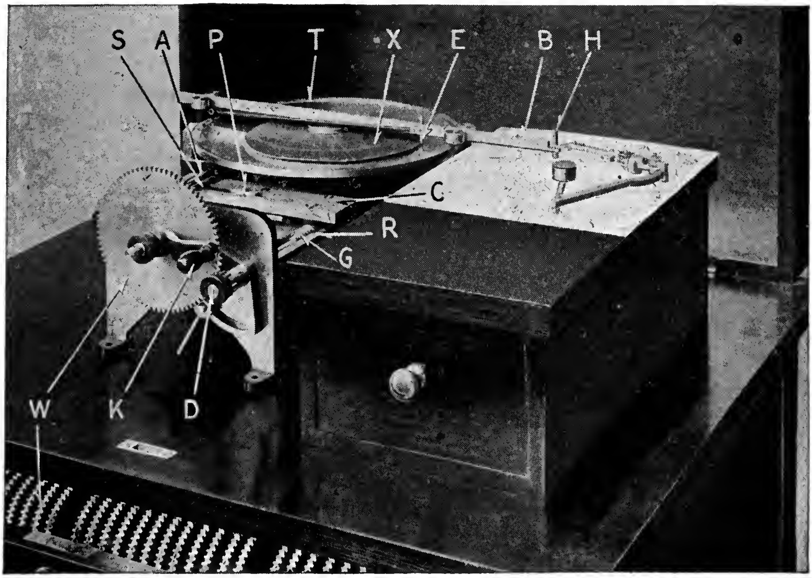


FIG. 204.

runs in a transverse slot S in the carriage C and thus gives a simple harmonic motion to the carriage.

The rotational motion of the turntable and template is produced by suitable worm wheels. As the template revolves, the crossbar B is given a transverse motion due to the contact point E which is in contact with the edge of the template and against which it is held by springs. The point H of a polar planimeter is carried by the end of the bar E and the weight furnished with the planimeter is placed over this point instead of the usual position on the planimeter bar. The shafts A and D are driven

[1] For description of an attachment to standard oscillographs by means of which oscillograms can be obtained directly in polar coördinates, see "Polar and Circular Oscillograms and Their Practical Application," L. W. CHUBB, *Electric Journal*, vol. 11, p. 262 (1914).

by gears rigidly connected to the crank K. Gears (W) are provided for all even and odd harmonics from 1 to 50, directions being provided for the selection and insertion of the proper gears for any harmonic.

With the proper gears in place and the planimeter set at zero, the crank is turned exactly one revolution. The planimeter integrates the area of the figure described by its point. The reading divided by a constant and the order of the harmonic is the amplitude of one component of that harmonic. The amplitude of the sine component is obtained with the carriage started at the near end and the amplitude of the cosine component is obtained with the carriage started at the center and moved toward the rear.

Having the amplitudes of the sine and cosine components of a harmonic, the amplitudes of the harmonics (C_1, C_2, C_3, . . .) are obtained from the relation:

$$C_n = \sqrt{A_n^2 + B_n^2}$$

and the phase position from the relation:

$$\phi_n = \frac{\tan^{-1}\left\{\dfrac{B_n}{A_n}\right\}}{n}$$

where C_n = amplitude of the harmonic of the n^{th} order, A and B = amplitude of sine and cosine components, respectively, ϕ_n = phase position of the harmonic of the n^{th} order.

An arrangement which is much simpler but limited in range and less accurate is proposed by C. A. Pierce.[1] It employs a synchronous commutator with commutators having, 2, 6, 10, 14 and 18 segments, respectively, and all mounted on one shaft. All are connected in parallel with a pair of slip rings to which the alternating e.m.f. or potential, E, is connected. A pair of brushes is set at the proper angle on each commutator and shifted until a maximum indication is obtained on a direct-current voltmeter. This indication corresponds to the average value of E of the complete wave when taken from the two-part commutator, the average value of the third harmonic in the case of the six-part commutator, fifth harmonic for the ten-part commutator and seventh harmonic for the fourteen-part commutator. The phase positions of the various harmonics are given by the position of the brushes at maximum indication.

[1] "A Mechanical Alternating-curve Wave Analyzer," C. A. PIERCE, *Electrical World*, April 13, 1911, p. 916.

CHAPTER XV

MAGNETIC MEASUREMENTS

356. General.—The magnetizing force, or intensity of the magnetic field in a magnetic circuit, is the m.m.f. per unit length of the path of the field and may be considered as the rate of change of magnetic potential along the path of the field. It is represented by the symbol H and is measured in gilberts per centimeter.

The intensity of the magnetic field in a long straight solenoid is

$$H = \frac{4\pi NI}{10l} \qquad \text{(gilberts per centimeter)}$$

where I = current in amperes, N = number of turns of wire on the solenoid and l = length of the solenoid in centimeters. In other words, the m.m.f. is proportional to the ampere-turns on the solenoid and the field intensity or magnetizing force is proportional to the ampere-turns per unit length of the solenoid. This is the fundamental equation of a magnetic circuit.

The induction or density of the magnetic flux produced by the magnetic field is measured in gausses (lines of flux per square centimeter) and is represented by the symbol B. When the substance in which the magnetizing force exists in non-magnetic, 1 gilbert per centimeter will produce 1 gauss. Therefore, the flux density is *numerically* equal to H, that is,

$$B = H = \frac{4\pi NI}{10l} \qquad \text{(gausses)}$$

When the substance is magnetic, B becomes much greater than H because of the lower reluctance or magnetic resistance of the path of the field. The ratio, B/H, is called the permeability.

There are no concrete standards of magnetic units, similar, for example, to that of the volt, which is the standard cell. Measurements of magnetic quantities are made by means of laws which connect the magnetic quantity with other quantities which can be compared with known standards. For instance, H is

measured in terms of the ampere by means of the law stated in the previous paragraph and B is measured in terms of either the coulomb or the unit of force, the dyne.

The quantities most frequently determined in magnetic measurements are the following:

Magnetizing force or field intensity, H.

Flux density, B, produced by a magnetizing force, H.

Permeability, the ratio of B to H.

Residual magnetism, the density of the flux which persists in a magnetic circuit after the magnetizing current has been reduced to zero.

Coercive force, the value of H in a magnetic circuit which is necessary to reduce the residual flux to zero.

Hysteresis loop, the closed curve obtained when B is plotted against H for various successive values of H between a given maximum positive value and the same maximum negative value, and back again to the original maximum positive value.

Core loss, the total energy losses which take place in a magnetic material when subjected to an alternating m.m.f.

Hysteresis loss, that part of the energy losses which take place in a magnetic material when subjected to an alternating m.m.f., which is expended in overcoming the residual magnetism twice each cycle.

Eddy-current loss, that part of the energy loss in a magnetic material subjected to an alternating m.m.f., which is due to the parasitic electric currents set up in the materials by the e.m.fs. induced by the constantly changing magnetic flux.

Field Intensity

The intensity of a magnetic field may be measured (in air) by inductive methods, with an oscillating bar magnet or with a small coil of bismuth wire.

357. Inductive Methods.—A coil of wire of a known number of turns and a known area is so arranged that it can be made to cut the flux produced by the field, in a direction perpendicular to the flux, either by rotation or translation. Or the flux is made to cut the coil by opening or closing the magnetizing-circuit current.

The e.m.f. generated in the coil, and therefore the field intensity or flux density is determined from the quantity of electricity discharged through a ballistic galvanometer connected to the coil terminals, according to the following relation:

$$H\ (= B) = \frac{dKr}{K'an} \times 10^8 \text{ (gilberts per centimeter or gausses)}$$

where d = deflection of galvanometer, K = constant of galvanometer in coulombs per scale division, r = total resistance of galvanometer circuit and n = number of turns in the test coil. If the coil is rotated 180° $K' = 2$ (flux enclosed by coil being cut twice, once by each side of the coil) and a = mean area of the coil in square centimeters. If the coil is moved across the flux in one plane, $K' = 1$ and a = the area in square centimeters cut over by the active side of the coil, that is, the side which cuts the flux. It is to be noted that the value of H obtained is the *average* value for the area concerned.

When there is sufficient space, the best method of manipulating the coil is to rotate it through exactly 180° about an axis perpendicular to the direction of the field. Very weak fields, such as that of the earth and those due to the current in neighboring conductors, may be conveniently measured in this way by using a sufficiently large coil.

If the space is limited, as in the air gaps of electrical machines, one side of a rectangular coil may be located outside of the field and the opposite, or active, side in the field at a point where all the flux in a known area will cut that side when the magnetizing-current circuit is opened or closed. The field intensity in any particular area would be obtained from the difference in measurements made at those two boundaries of the area which are perpendicular to the flux, taking care that the flux collapses in the same direction in the two cases.

The field strength in air gaps may also be measured by arranging a coil so that it can be quickly moved across the entire gap perpendicular to the field or through a definite portion of the gap. If the flux in the entire gap is to be included, the coil must be sufficiently large so that when in the starting position it includes *all* of the flux, otherwise opposite sides of the coil would cut the flux in the same direction and the e.m.fs. induced in the two sides of the coil would neutralize each other. Any particular portion of the field in the gap can be measured by arranging the coil so that one side moves a definite and known distance perpendicular to the field, the other side of the coil being in a region where there is no flux. Obviously, in a measurement of this kind, a square coil is more convenient because the area cut can be more easily calculated.

The total flux emanating from a permanent magnet may be measured by winding a loosely fitting coil around it (around one limb of a horseshoe magnet) and then quickly slipping it off over the end. The coil cuts all of the flux and produces a deflection in the galvanometer proportional thereto.

The Grassot fluxmeter is a portable instrument for the direct measurement of field strengths by an inductive method, which has certain advantages where a large amount of work requiring only moderate accuracy is to be done. These are portability, elimination of the element of time which is involved with a ballistic galvanometer (it is essential that the discharge from the test coil be complete before the galvanometer coil begins to move) and a

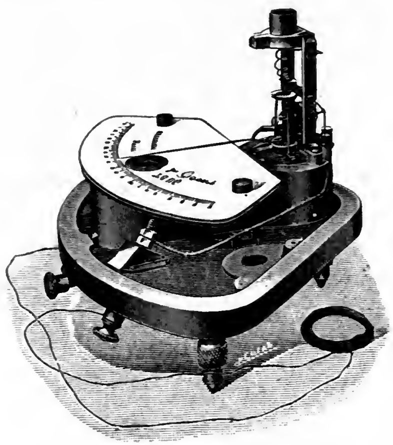


FIG. 205.

fixed indication, the pointer remaining at the position corresponding to the flux being measured. The instrument (Fig. 205, cover removed), is essentially a portable ballistic galvanometer with the important exception that the moving element is suspended by a torsionless suspension. Thus, the final deflection is the same whether the flux through the test coil is changed slowly or rapidly and the pointer remains at the final deflection. The instrument is connected to a test coil which may be used in the various ways described in the previous paragraphs.

358. Oscillating-magnet Method.—In the oscillating-magnet method, a small, simple, bar magnet is suspended by untwisted silk fibers. The magnet is set to vibrating through an angle of

about 5° and the period of oscillation determined. The average of at least three observations should be taken. The field strength is

$$H = \frac{4\pi^2 K}{MT^2} \quad \text{(gilberts per centimeter)}$$

where K = moment of inertia computed from the mass and dimensions, M = magnetic moment, and T = period of oscillation. M may be determined with a magnetometer,[1] or by calibration in a known field (Helmholtz coil, par. 242). This method is suitable only for weak fields, such as that of the earth. If mounted in a wooden box with a glass front, it will be protected from air currents and will be found convenient for making magnetic surveys.

359. Bismuth Spiral Method.—The resistance of bismuth increases when placed in a magnetic field. This property is utilized by noting the increase in resistance of a flat spiral coil of bismuth wire when placed in the field to be measured. The leading-in wires are arranged non-inductively so that slight fluctuations in the flux do not affect the detecting instrument used with the bridge, by means of which the resistance changes are measured. The device is calibrated with known field strengths. It is particularly suitable for exploring small air gaps such as those in motors and generators.

B–H or Normal Induction Data

360. General.—In general, it is not only desirable to have the permeability of a given kind of magnetic material as high as possible, but the variation of the permeability with the magnetizing force is important. Consequently, it is seldom that the flux density or induction corresponding to only one value of field intensity is required but a curve showing the relation between B and H for several values of H. Such a curve of a magnetic material, plotted between the magnetizing force or field intensity and the corresponding flux density or induction produced in the material when it is in a neutral or normal condition, is called the normal induction or B–H curve. It is the locus of the end points of the hysteresis loops corresponding to successive values of magnetizing force as indicated by the dotted line in Fig. 206. This figure illustrates in fact, the process of obtaining a normal induction curve by any of the various methods described below. It is cus-

[1] See any text-book on Physics for description of magnetometer.

tomary, however, to plot only the positive half of a normal induction curve because both halves are alike. A permeability curve is a curve plotted between the permeability and either B or H.

The various methods of determining normal induction data are distinguished principally by the manner employed to measure B, for in all methods H is determined from the magnetizing coil in accordance with formula in par. 356. B can be measured directly as in the methods which employ a ballistic galvanometer, or indirectly with permeameters. Ballistic methods are the more accurate and are the most generally employed. Permeameters

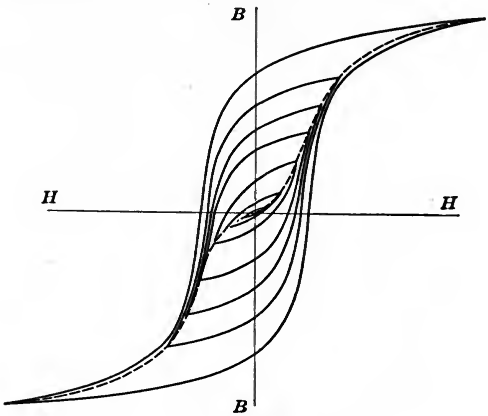


FIG. 206.

are suitable for factory testing where approximate results are usually sufficient.

There are several methods which use a ballistic galvanometer connected to a test coil for measuring the induction B. The principal ones are the ring method, the long straight-bar method, the Hopkinson or divided-bar method and the Burrows or double-bar, double-yoke method.

361. Ring Method.—The ring method, devised by Rowland, is one of the earliest methods of measuring the permeability and the hysteresis of iron and steel. The connections are shown diagrammatically in Fig. 207, where T is the test specimen. The latter is an annular ring, either solid or built up of punchings of sheet metal, with a diameter preferably 8 or 10 times the radial thickness. After covering with a thin layer of insulation, a test coil of

very fine double silk-covered wire is wound on a portion of the ring. The magnetizing coil is wound over the test coil, and distributed uniformly over the entire ring; it is usually composed of double cotton-covered wire, of sufficient size to carry the maximum current without raising the temperature of the iron appreciably. A variable resistance is inserted in series with the ballistic galvanometer in order to keep the deflection at a reasonably large and constant value.

After demagnetizing (par. 379) the exciting current is adjusted to a small value. It is then reversed by means of the reversing switch, the current and deflection being quickly noted. This deflection is proportional to the total change in induction and half of the deflection is therefore proportional to the induction existing before the switch was reversed. The operation is

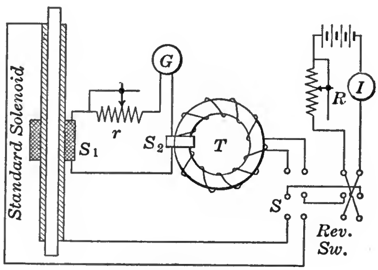


FIG. 207.

repeated for various values of exciting current, up to the maximum value of H desired.

The magnetizing force is computed from the formula

$$H = \frac{4\pi NI}{10l} \qquad \text{(gilberts per centimeter)}$$

where N = total turns of exciting coil, I = current in amperes, l = mean circumference of ring in centimeters.

The induction is computed from the relation

$$B = \frac{dKr}{2an} \times 10^8 \qquad \text{(gausses)}$$

where d = deflection, r = total resistance of test-coil circuit, K = galvanometer constant, a = area of ring section in square centimeters and n = turns in test coil.

The constant K may be determined with a standard condenser

or a standard mutual inductance or solenoid (see par. 34). The latter method is convenient and in practice more accurate, because the secondary of the mutual inductance can be left in the galvanometer circuit throughout the test, so that the conditions during calibration are the same as during the test. Such an arrangement is shown in Fig. 207, where S_1 is the secondary of a mutual inductance.

The ring method was formerly very generally used for magnetic tests but it has been superseded by other methods which are much more conveinent and also, in general, more accurate. The windings have to be put on each specimen separately and by hand, thus making the preparation of the test specimen a tedious and expensive operation. The magnetizing force is not uniform throughout the cross-section of the specimen because of the difference between the length of the magnetic path on the inside of the ring and that on the outside of the ring. Although the mean circumference can be used in computing the average magnetizing force, the average induction as obtained from the deflection of the galvanometer does not necessarily correspond to this average magnetizing force because of the variation in the permeability from the inner to the outer edge of the specimen. It may more nearly correspond to the magnetizing force at the "harmonic mean" radius.[1] However, this error is rendered negligible for all commercial purposes, if the mean diameter is made 8 or 10 times the radial thickness of the specimen.[2]

362. Divided-bar Method.—The divided-bar method devised by Hopkinson avoids the necessity of winding each specimen separately and permits the use of a more convenient test piece. The device consists of a test piece, *BC* (Fig. 208), in the form of a bar about 15 in. (38.1 cm.) long and 0.5 in. (1.27 cm.) diameter, which is divided at *A* and inserted in a massive frame, *F*. The secondary coil, *S*, is so arranged that it will be thrown clear of the yoke by a spring when the part, *AB*, of the test bar is suddenly withdrawn. In calculating *H*, the length of the magnetic circuit is that between the inside faces of the yoke, the reluctance of the yoke being considered negligible. This introduces

[1] "Magnetic Tests of Ring-formed Samples," EDY VELANDER, *Elek. und Masch.* (Vienna), Jan. 2, 1916. See abstract, *Electrical World*, April 1, 1916, p. 779.

[2] "Errors in Magnetic Testing with Ring Specimens," M. G. LLOYD, *Bulletin*, Bureau of Standards, vol. 5, p. 435 (1908–1909).

an indeterminable error and therefore the method is suitable only for rough measurements. The leakage error due to flux through the coil, but not through the bar, can be determined by preliminary test with a non-magnetic bar. The formulas for H and B are the same as in the ring method, except that the denominator in the expression for B is an and not $2an$ because the flux interlinks with the coil only once.

363. Long Straight-bar Method.—This method utilizes the principle of a solenoid of infinite length. A specimen with a length at least 200 times the diameter is required. It is placed inside of a solenoid of the same length, at the center of which the test coil is wound. The deflection is noted, both with and without the test piece in the solenoid, when the magnetizing current

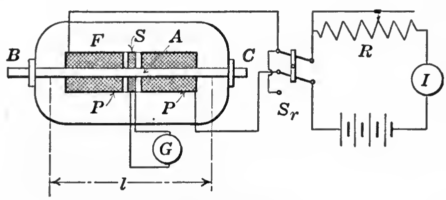


FIG. 208.

is suddenly reversed. The magnetizing force, H, is calculated as in the previous methods, and B is obtained from the relation,

$$B = \frac{d}{d_1}$$

where d = deflection with bar in place and d_1 = deflection without bar. This method is not always practicable because of the difficulty in preparing long samples of small uniform diameter.

364. Double-bar, Double-yoke Method.—This method, devised by C. W. Burrows[1] of the Bureau of Standards, is the standard method adopted by the American Society for Testing Materials for the determination of normal induction and hysteresis data. It is also the standard method used at the Bureau of Standards for both solid and sheet specimens. Not only is the precision high, but the method is rapid and convenient when the observer is experienced.

[1] "The Determination of the Magnetic Induction in Straight Bars," C. W. BURROWS, *Bulletin*, Bureau of Standards, vol. 6, p. 31 (1909–10). See also *Transactions*, A. S. T. M., vol. 11, p. 31 (1909) and *Circular* No. 17," Magnetic Testing," Bureau of Standards (1916).

The method employs permanent magnetizing and test coils wound on very thin, slotted brass tubes; but the distinctive feature is the distribution of the magnetizing winding in sections, which permits the independent adjustment of the magnetizing force in various parts of the magnetic circuit. Thus the effect of non-uniform reluctance at joints, etc., can be overcome and the induction made uniform throughout the entire magnetic circuit.

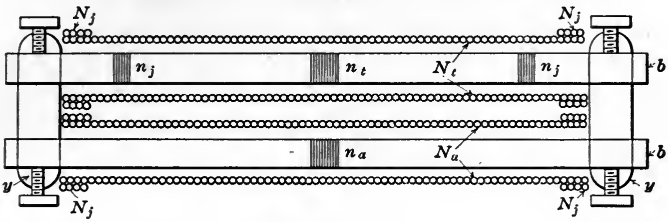


FIG. 209.

Exploring coils are placed at various positions so that the uniformity of the induction can be tested.

The scheme is shown in Fig. 209 where b, b are two bars (standard size, 1 cm. diameter, 35 cm. long), one of which is the bar to be tested and the other is an auxiliary bar of similar material. Y, y are yokes of Norway iron about 15 cm. long and about 4 to 5 cm. diameter, into which the bars are clamped. The

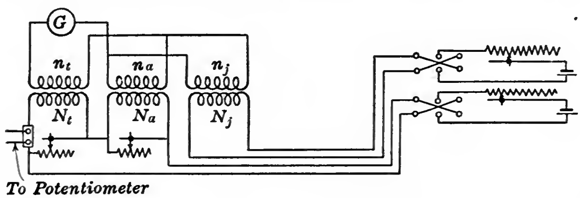


FIG. 210.

m.m.f. is applied in three sections; one coil, N_t, is placed over the test specimen; another, N_a, over the auxiliary rod; and the third, N_j, is divided into four parts, one near each joint. The corresponding exploring coils are n_t, n_a and n_j. These coils all have the same number of turns, that is, $n_t = n_a =$ the two parts of n_j in series. They are so connected to a switch that n_a and n_j may be connected through the ballistic galvanometer in opposition to n_t, thus providing a zero method of determining the condition of uniformity of flux. The electrical connections

are shown in Fig. 210. The magnetizing coils are connected to special reversing switches so constructed that they can be operated simultaneously.

The method of procedure is as follows: After demagnetizing (par. 379) the current in N_t is adjusted to the value of H required. The current in all magnetizing coils is then simultaneously reversed several times to get the specimen in a cyclic condition, the current in N_a and N_j being adjusted during the process until the flux is uniform as indicated by zero deflection when n_a and n_j are successively opposed to n_t. The conditions may then be assumed to be the same as in a long straight bar. The galvanometer is connected to n and the deflection noted when the currents in the various magnetizing coils are reversed simultaneously. Then

$$H = \frac{4\pi NI}{10l} \quad \text{(gilberts per centimeter)}$$

$$B = \frac{dKr}{2an} \times 10^8 - \left(\frac{t - A}{A}\right) \quad \text{(gausses)}$$

where the symbols have the same significance and are expressed in the same units as in the previous paragraphs.

The quantity in the parenthesis is the correction factor for the space between the surface of the bar and the test coil. A = area of the bar and a = area of the test coil. Ordinarily this correction is very small because the brass tube is made very thin and the test coil is wound underneath the magnetizing coil.

It has been proposed to make this a zero method by opposing the test coil to a standard mutual inductance, the primary of which is in series with the magnetizing coils.[1] This would also eliminate calibration of the galvanometer. It has been found impossible however, to get a true balance because of the difference in the time constants of the two circuits, one of which contains iron and the other none.

365. Permeameters.—Permeameters are commercial instruments for the rapid testing of iron and steel for permeability. Thompson and DuBois permeameters employ the tractive force exerted between the pole of a magnetized bar and a piece of steel in direct contact with the pole. This force in dynes is

$$F = \frac{B^2a}{8\pi} \quad \text{(dynes)}$$

[1] "The Determination of the Magnetic Induction in Straight Bars," C. W. Burrows, *Bulletin*, Bureau of Standards, vol. 6, p. 75 (1909–10).

where B = induction in the bar in gausses and a = area of bar in square centimeters.

The principal advantages of traction-type permeameters are their ruggedness, and simplicity, which are important features in shop testing where rapidity is essential and only comparative data are required.

Koepsel, Esterline and Picout permeameters are induction-type instruments.

S. P. Thompson Permeameters.—This instrument is shown schematically in Fig. 211, where AB is the test specimen in the form of a rod which passes through a hole in the top of a heavy yoke, F. The surfaces of the end of the rod and the yoke at F are carefully machined. The force necessary to move the rod is measured with a spring balance at S. The induction is

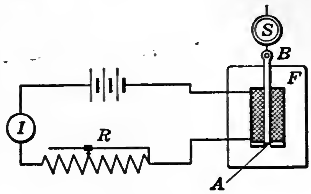


FIG. 211.

$$B = 156.9\sqrt{\frac{P}{a}} + H \qquad \text{(gausses)}$$

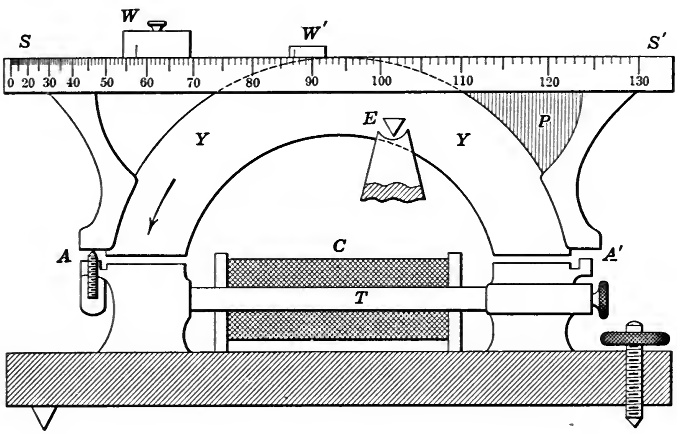


FIG. 212.

where B = induction in gausses (lines per square centimeter), P = pull in grams and a = area in square centimeters.

DuBois Permeameter.—In the DuBois permeameter or magnetic balance, the force between the parts of a yoke which complete the magnetic circuit is measured, instead of the force between the yokes and the test piece. A sketch of a section is

shown in Fig. 212, where T is a test specimen exactly 1.128 cm. diameter (= 1 sq. cm. area), and 4 cm. long between the yoke arms. Y is the yoke separated by the air gaps A and A' and supported by a knife edge E eccentrically located. A scale, S, S', and sliding weights, W, W', are provided on top of the yoke so that when the coil B is excited, the unequal pulls at A and A' can be counter-balanced. The position of balance is kept the same by means of a stop so arranged that a bell circuit is closed when contact is made. The instrument is calibrated with a standard test piece.

Koepsel Permeameter.—The Koepsel permeameter, as made by Siemens and Halske, is shown schematically in Fig. 213, where JJ' is a massive yoke divided at the center to admit a moving

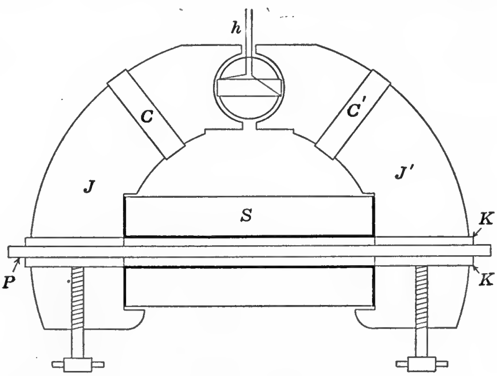


FIG. 213.

coil, h, to which a pointer is attached, the arrangement being similar to standard D'Arsonval-type, continuous-current instruments. This pointer moves over a scale graduated directly in lines per square centimeter. The magnetic circuit is completed through the test piece, P, firmly clamped between the ends of the yoke with soft-iron bushings, KK'. The value of B corresponding to various magnetizing currents in S, the main magnetizing coil, is indicated directly by the deflection with a known small current through h. The constants are such that the magnetizing force in gausses is

$$H = 100I \qquad \text{(gausses)}$$

where I = current in the coil S, in amperes.

Separate coils C, C' are placed on the yoke pieces JJ', by means of which the reluctance of the various gaps is approxi-

mately compensated. But even with these coils there is a flux leakage so that correction or "shearing" curves have to be used. These curves, obtained by test with standardized specimens, are furnished with the instrument as are also standardized test specimens with which the device can be checked from time to time. Either round or square-bar specimens can be tested by using suitable bushings. Sheet material is tested by building up a bar of strips.

Burrows[1] states that it is essential that a shearing curve be prepared for each size and kind of specimen tested, but that, with care, "the apparatus is capable of giving quantitative re-

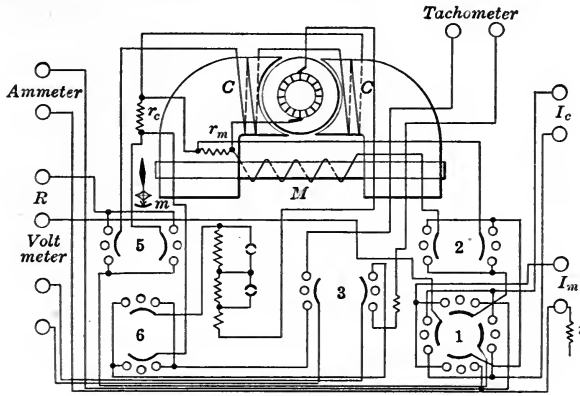


FIG. 214.

sults within 5 per cent. of the true value of the magnetizing force for a given induction."[2]

Esterline Permeameter.[3]—This device is a modification of the Koepsel apparatus. The moving coil is replaced by an armature driven at constant speed, the e.m.f. generated being a measure of the flux. This instrument has the advantage that no corrections are necessary. A diagrammatic sketch is shown in Fig. 214, where M is a magnetizing coil surrounding the test specimen, and C, C are compensating coils on the pole tips. Switch 1 is arranged to reverse the magnetizing and compensating coils

[1] "The Determination of the Magnetic Induction in Straight Bars," C. W. BURROWS, *Bulletin*, Bureau of Standards, vol. 6, p. 75 (1909–10).

[2] "An Experimental Study of the Koepsel Permeameter," C. S. BURROWS, *Bulletin*, Bureau of Standards, vol. 11, p. 101 (1914–15).

[3] J. W. ESTERLINE, *Transactions*, A. S. T. M., vol. 3, p. 288 (1903), vol. 8, p. 190 (1908).

simultaneously (when testing for complete compensation), and switch 2 reverses the magnetizing current only. The ammeter in the latter circuit is graduated directly in terms of H.

The speed is indicated by an electric generator or tachometer which is connected by switch 3 to the same voltmeter used to measure B. Complete compensation, that is, practically zero reluctance in the yoke circuit, is indicated by a small magnetometer (compass), m, placed at the end of the test bar. Noting the position when there is no current or test bar in the apparatus, the compensating current is adjusted until the magnetometer returns to this "zero" position. Then there is no leakage and all of the flux induced in the test specimen by the magnetizing coil M is passing around the yoke and will be measured by the revolving armature.

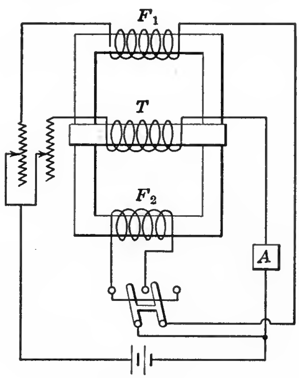


FIG. 215.

Both round and square specimens can be measured by using suitable bushings. Sheets are measured by building up a bar of strips.

Picout Permeameter.—The Picout permeameter is a double-yoke single-bar instrument in which the reluctance in the junction between the test specimen and the rest of the magnetic circuit is compensated for. The scheme is indicated in Fig. 215, where F_1, F_2 are the two yokes. The square-bar test specimen T is clamped between the ends of the yolk as shown. The reluctance of the joints is eliminated and the condition of a long test specimen obtained by first connecting two auxiliary magnetizing coils on F_1, F_2, in such a manner that their m.m.fs. are in series. There is then supposed to be no flux in the test bar, its magnetizing coil being out of circuit. The two coils on F_1, F_2 are then connected in opposition and the current in the test-bar coil adjusted until the flux in F_1, F_2 is the same as before, as indicated by a ballistic galvanometer or some other suitable method. The conditions in the test bar are then supposed to be the same as in a long, uniformly magnetized bar. The m.m.f. of the test-bar coil simply overcomes the reluctances of the test bar, and H is calculated from the current and the constants of the coil by the usual formula, B being measured with a test coil placed over the test bar.

366. Measurement of Inductions at High Densities.[1]—The apparatus employed with standard methods for obtaining B-H date over the usual ranges is ordinarily not suitable for high magnetizing forces because of the large magnetizing currents involved. On the other hand, if the current capacity was sufficient, the accuracy at small magnetizing values would be too low. Consequently, the experimental determination of B-H data is usually more or less troublesome.

Such data can, however, be obtained by extrapolation of normal data with an accuracy which is usually high enough for all practical purposes. The procedure is as follows: A curve is plotted between H and the true or metallic reluctivity, r, which is the ratio of H to B-H (not to the total reluctivity or ratio of H to B). The straight portion of this curve, which includes the greater part from the upper end down, is prolonged to cut the "y" axis. The value of the "y" intercept, a, is noted, and also the slope of the line, b, expressed as the ratio of r to H. The equation for the induction at high values is then.

$$B = \frac{H}{a + bH} + H \qquad \text{(gausses)}$$

where B is in gausses and H in gilberts per centimeter.

Hysteresis Curves

A hysteresis curve or loop is obtained by most of the methods described above for normal induction curves, the only difference being in the procedure.

367. "Step-by-step" Method.—In the methods employing ballistic galvanometers this method is carried out as follows: After demagnetizing the specimen, the current in the magnetizing circuit is adjusted to a value corresponding to maximum B. It is then reversed a few times to get the specimen in a cyclic state. By means of a sectionalized rheostat provided with switches for short-circuiting the various sections, or some similar arrangement, a resistance is suddenly cut into the circuit, thus reducing H to a new value which is carefully noted. The corresponding change in B is determined with a ballistic galvanometer in the usual manner. This process is continued step by step until zero

[1] "Some Notes on Magnetization Curves," J. D. Ball, *General Electric Review*, January, 1915, p. 31.

Circular No. 17, "Magnetic Testing," Bureau of Standards, p. 36 (3d. edition, 1916).

current is reached, when the circuit is reversed and the resistance cut out step by step until the same value of *H* is reached in the opposite direction. The whole process may then be repeated and the other side of the loop obtained by direct measurement if desired. It should exactly duplicate the first side.

The value of *B* corresponding to the initial value of *H* is computed in the usual manner as explained in the measurements for normal induction. The second point on the curve is found by subtracting from the initial value the change in induction corresponding to the galvanometer deflection when the first decrease in *H* was made. The third point is similarly obtained by subtracting from the second point the change in induction corresponding to the second decrease in current and so on.

With direct-indicating permeameters, such as the Koepsel and the Esterline instruments, *B* is observed directly for various values of *H*, first descending from a positive maximum to a negative maximum and then ascending in the opposite direction.

368. "Repeated-maximum" Method.—The "step-by-step" method is defective in that the errors are accumulative, since an error at any one step is retained in all of the others. This objection is removed by returning to the initial positive value of *H* after obtaining each point. Incidentally, this permits occasional checking of the initial value of *B* which shows whether conditions are remaining constant. This method also has the advantage that it more nearly approaches the conditions actually existing in commercial practice. In general, it is the most satisfactory method and the one most used.

369. Residual Magnetism and Coercive Force.—The residual magnetism of a magnetic material is the induction which remains when the magnetizing force is reduced to zero. The coercive force is the reversed m.m.f. which must be applied to reduce this residual induction to zero. These two quantities are respectively the "*y*" and "*x*" intercepts of the hysteresis curve and, therefore, depend upon the maximum m.m.f. to which the material has been subjected (see Fig. 206).

The value of the residual induction corresponding to a given m.m.f. may be determined experimentally by first mesuring the induction corresponding to the m.m.f. and then the change in the induction when the m.m.f. is reduced to zero by opening the magnetizing circuit. The difference between these two quantities is the residual induction.

The coercive force corresponding to a given m.m.f. is obtained by first determining the m.m.f. which it is necessary to apply in the opposite direction in order to produce a change in induction just equal to the induction corresponding to the initial m.m.f. The difference between these two m.m.fs. will be the coercive force.

Hysteresis Loss

370. General.—The energy expended in a magnetic material due to magnetization when the m.m.f is changed from a given value in the positive direction to the same value in the negative and then back again, is proportional to the area of the hysteresis loop thus formed. If the loop is plotted with B in gausses and H in gilberts, the energy in ergs per cubic centimeter is the area of the loop divided by 4π. The area may be measured by a planimeter or otherwise, the unit of area being the product of 1 gauss and 1 gilbert per centimeter.

This method of determining the hysteresis loss is much too slow and expensive for ordinary purposes and the following are some of several methods which have been advised for measuring the loss directly by electrical or mechanical means.

371. Robinson's Method.—The Epstein method for measuring core loss (see pars. 374 and 375) can be used for measuring hysteresis loss by using alternating current of very low frequency and a sufficiently sensitive wattmeter. The method proposed by Robinson[1] employs this principle but it is much more economical of time and expense in preparing the sample. While probably not as accurate as the Epstein method, the accuracy is ample for factory purposes (about $\pm$ 5 per cent.).

The specimen is a bundle of strips, cut from the sheets to be measured, 0.5 in. (1.27 cm.) wide by 10 in. (25.4 cm.) long and weighing 1 lb. (0.45 kg.). It is placed in a simple straight solenoid with the current coil of the wattmeter (reflecting electrodynamometer) in series with the magnetizing winding. As in core-loss measurements the induction at which the hysteresis loss is to measured is determined by means of a voltmeter connected to a separate, secondary winding at the middle of the solenoid. Because of the effect of the ends, the induction is not uniform throughout the specimen but is greatest at the center.

[1] "Commercial Testing of Sheet Iron for Hysteresis Loss," L. T. Robinson, *Transactions*, A. I. E. E., vol. 30, p. 741 (1911).

However, the ratio of the maximum (where the secondary coil is located) to the average is found by experiment and the voltmeter indications corrected accordingly when adjusting the magnetizing current for a given average flux density.

The potential coil of the wattmeter is connected to a third winding having the same number of turns as, and wound with, the magnetizing winding. Both windings are larger than the specimen so that the latter does not have to be accurately centered.

The frequency employed is not over 10 cycles. A correction for the eddy-current loss as determined by experiment has to be made and also a correction for the losses in the two secondary or instrument circuts.

372. Holden and Ewing Hysteresis Meters.—These instruments are examples of methods in which the loss is measured by mechanical means. In the Holden meter the test specimen, a ring of laminations about 1 by 2 cm. (0.4 by 0.79 in.) cross-section, and 9 cm. (3.55 in.) diameter, is placed between the poles of a pair of revolving magnets. The torque exerted on the specimen is resisted by a spiral spring. The amount which it is necessary to deflect the spring in order to bring the specimen back to the zero position is a measure of the loss in ergs per cycle.

The Ewing apparatus is based on a similar principle, but the specimen is rotated instead of the magnets. The specimen is a bundle of strips 5/8 in. (1.6 cm.) square and 3 in. (7.6 cm.) long.

Core-loss Measurements

373. General.—When a magnetic material is subjected to a constantly reversing m.m.f., energy is expended in the material in two ways. One is due to the hysteresis of the material as discussed in the preceding paragraphs and the other is due to the parasitic or eddy currents in the material which are caused by the e.m.fs. induced in the metal by changing flux. The sum of these losses constitutes the core loss.

Core-loss measurements are usually made directly with a wattmeter in conjunction with coils and a specimen arranged something like a transformer. The Hopkinson ring specimen can be used but the Epstein form of apparatus is much more convenient. Furthermore, it is the method used in most laboratories

for precision measurements and has been adopted as standard by the American Society for Testing Materials.[1]

374. A. S. T. M. Epstein Method.—The scheme of the particular arrangement of the Epstein method recommended by the A. S. T. M. is shown diagrammatically in Fig. 216 and the specifications are as follows:

The specimen is made up of four parts arranged in the form of a rectangle. The magnetizing winding is divided into four solenoids, each being wound on a form into which one side of the rectangular specimen is placed. The form is non-magnetic, non-conducting and has the following dimensions: inside cross-section, 4 cm. (1.57 in.) by 4 cm.; thickness of wall not over 0.3 cm. (0.12 in.); winding length, 42 cm. (16.5 in.). Each section of the

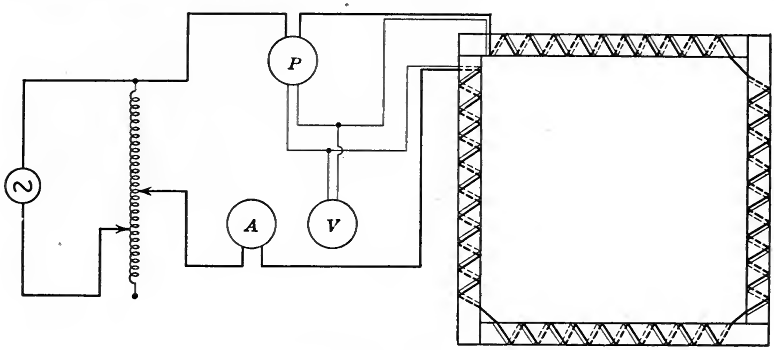


FIG. 216.

specimen consists of 2.5 kg. (5.5 lb.) of strips, 3 cm. (1.18 in.) wide and 50 cm. (19.7 in.) long, bound tightly together in a bundle with tape. Two of the bundles are made up of strips cut in the direction of rolling and two at right angles to the direction of rolling. The bundles form butt joints at the corners separated by tough paper 0.01 cm. (0.004 in.) thick and are held firmly in position by clamps placed at the corners.

The magnetizing winding on each solenoid consists of 150 turns of wire uniformly distributed over the 42 cm. (16.5 in.) winding length, and has a resistance of between 0.075 and 0.125 ohm. A secondary winding is uniformly wound underneath the first; it also contains 150 turns in each solenoid, and energizes the potential circuit of the wattmeter and also the voltmeter

[1] "Standard Tests for Magnetic Properties of Iron and Steel," Specification No. A-34-14, A. S. T. M. Standards, p. 243 (1916).

with which the induction is measured. The resistance of this winding should not exceed 0.25 ohm per solenoid.

With a sine-wave e.m.f. impressed on the magnetizing winding, the maximum induction is

$$B = \frac{E4lD10^8}{4fNnM} \qquad \text{(gausses)}$$

where E = volts indicated by voltmeter, l = length of specimen, D = specific gravity of specimen, f = form factor of magnetizing e.m.f. (1.11 for a sine wave), N = total secondary turns, n = cycles per second, and M = total mass of the specimen in grams. The specific gravity is taken as 7.5 for alloy or high-resistance steels, that is, steels having over 2 ohms resistance per metergram, and 7.7 for low-resistance steels having less than 2 ohms resistance per metergram. The wattmeter gives the total loss in the iron plus that in the secondary circuit. The latter is the sum of the I^2R losses in the voltmeter, the potential coil of the wattmeter and the secondary winding itself. It is, of course, deducted from the wattmeter reading.

375. Lloyd Epstein Method.[1]—The Lloyd form of Epstein apparatus as used at the Bureau of Standards is similar to the A. S. T. M. form. The principal difference is in the manner of closing the magnetic circuit at the corners. In the A. S. T. M. apparatus, the corners are simple butt joints with a piece of paper of specified thickness between the abutting surfaces, thus introducing an additional but definite reluctance. In the Lloyd apparatus the corners are completed with angle pieces of sheet iron which are interleaved with the ends of the two bundles, thus making two lapped joints at each corner. Each joint is clamped tightly with a special form of clamp. A correction is applied for the decreased reluctance which results at the corners.

376. Approximate Methods.—In factory testing of electrical sheet iron and steel, the principal object is to determine whether the material being purchased is at least equal to a certain steel which has been adopted as standard. Obviously, the precise Epstein method would ordinarily be much too costly in both the amount of material, and the amount of time required for the preparation of the sample and making the measurement. Consequently some modified form of the more precise methods is

[1] "The Testing of Transformer Steel," M. G. Lloyd and J. V. S. Fisher, *Bulletin*, Bureau of Standards, vol. 5, p. 453 (1908–09). *Circular* No. 17, "Magnetic Testing," Bureau of Standards (1916, 3d edition).

usually employed. For example, the Robinson method for hysteresis measurements (par. 371) is used, the apparatus being standardized with a specimen of the standard steel. The procedure is then simply one of substitution and material can be tested very rapidly.

377. Effect of Wave Form on Core Loss.—The eddy-current component of the core loss varies with the root-mean-square value of the applied voltage while the hysteresis component depends upon the maximum value of the induction which in turn varies with the average value of the applied voltage. Thus the core loss will vary with the wave shape of the applied voltage. A sine curve is taken as standard and core-loss values are determined with, or corrected to, a sine-wave voltage.

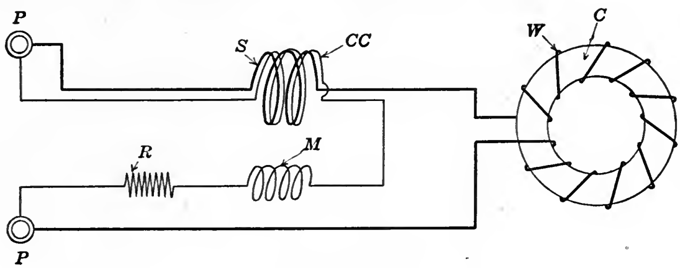


FIG. 217.

In commercial measurements of core losses a sine-wave voltage is not always available. A convenient method of obtaining the sine-wave loss under such conditions is to use a standardized test specimen in parallel with the one being tested. The loss in the standard specimen having been previously determined on a sine wave, the applied voltage is adjusted until this loss is indicated. Then the loss in the test specimen will be the equivalent sine-wave loss. The iron-loss voltmeter proposed by Chubb[1] is based on this principle. Fig. 217 shows the scheme of the instrument, C being a laminated core, W the magnetizing winding in series with the current coil S of a wattmeter and the terminals P_1P. The moving coil of the wattmeter M is connected in series with a non-inductive resistance R and an "adding" coil CC which is wound with, and has the same number of turns as, the

[1] "A Method of Testing Transformer Core Losses, Giving Sine-wave Results on Commercial Circuits," L. W. CHUBB, *Transactions*, A. I. E. E., vol. 28, p. 417 (1909). The instrument is manufactured by the Westinghouse Electric and Manufacturing Co.

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current coil. This "adding" coil causes the instrument indication to include its own shunt circuit copper loss in addition to the losses in C. Thus by changing R and the number of turns in W, a considerable variation in the ratio of equivalent eddy-current loss to total loss can be effected, so that the instrument can be adjusted to have the same ratio as the sample under test.

The instrument is used by connecting it in parallel with the test specimen or the transformer core to be measured, and the non-sinusoidal voltage adjusted until the indication corresponding to the specified voltage is obtained. The loss in the material being tested, as measured in the usual manner with a wattmeter, is then the same as it would have been with a sine wave. It has been found that the ratio of eddy-current loss in the instrument to the total loss, can be fixed at 20 per cent. at 0.6 full

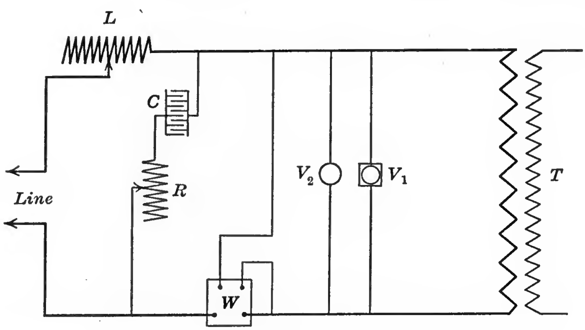

FIG. 218.

scale deflection once for all at the factory without introducing serious error with ordinary commercial steels and wave shapes. The instrument is calibrated on a sine wave in parallel with a root-mean-square value voltmeter.

When the greatest accuracy is required, the distorted wave may be altered to a sine wave by the scheme indicated in Fig. 218[1] and the true sine-wave loss obtained. An inductor, L, is connected in series with the line and the transformer whose core loss is being determined. An electrolytic cell, C, in series with an adjustable non-inductive resistor, R, is connected across the line as shown. A wattmeter, W, a root-mean-square voltmeter, V_2, and an iron-loss voltmeter, V_1, are connected as indicated.

[1] "A Method of Testing Transformer Core Losses, Giving Sine-wave Results on Commercial Circuits," L. W. CHUBB, *Transactions*, A. I. E. E., vol. 28, p. 426 (1909).

The effect of the inductance is to produce a peak in the wave but the cell short-circuits above its critical voltage and the wave is flattened out. The inductor L and the resistor R are adjusted until this critical voltage is such that the form factor is 1.11, which condition is indicated when the indications of the root-mean-square voltmeter and the iron-loss voltmeter are in agreement.

378. Separation of Losses.—The separation of the two components of the core loss, the hysteresis loss and eddy-current loss, is usually done by utilizing the fact that at a given value of maximum induction the hysteresis loss varies directly with the frequency and the eddy-current loss varies as the square of the frequency. If the core loss is measured at two different frequencies in the usual manner and with the same induction in both cases, two simultaneous equations will be obtained with two unknown quantities. From these two equations the two losses can be computed. Thus, suppose the core loss at 30 cycles, for example, is W and at double the frequency or 60 cycles, the loss is W'.

Then

$$W = W_e + W_h$$

and

$$W' = 4W_e + 2W_h$$

where W_e and W_h = eddy-current loss and hysteresis loss respectively at the first frequency. From these two equations

$$W_e = \frac{W' - 2W}{2} \text{ and } W_h = \frac{4\,W - W'}{2}.$$

If measurements are made at several frequencies, the mean value may be determined by graphical means as follows: The loss per cycle is plotted as ordinates against frequency as abscissæ. The points should fall in a straight line but if they do not, a mean straight line is drawn through the points. The intercept on the axis of ordinates gives the hysteresis loss per cycle and subtracting this quantity from the total loss at any frequency gives the eddy-current loss per cycle at that frequency.

Notes on Magnetic Measurements

379. Demagnetization.[1]—In induction measurements, the specimen should be carefully demagnetized before any measurements

[1] "The Best Method of Demagnetizing Iron in Magnetic Testing," C. W. Burrows, *Bulletin*, Bureau of Standards, vol. 4, pp. 205 (1907).

are made. This is best accomplished by first magnetizing to a value well above the highest that will be used. The current is then gradually reduced to zero, at the same time rapidly reversing it. Alternating current having a frequency of one cycle per second is very satisfactory for this purpose and is particularly convenient where much work is to be done. Obviously, continuous current, with a commutator which will reverse the current twice per second, will give the desired result.

380. Temperature.—The temperature has an effect in *B-H* measurements so that where an accuracy of 1 per cent. is desired at the steep part of the curve, the temperature should be kept at a standard value (*e.g.*, 20°C.).[1] In core-loss measurements, the temperature is obviously important because it affects the eddy-current loss component. Care should therefore be taken that the magnetizing winding has sufficient current-carrying capacity to avoid heating the specimen. In precise work, the apparatus should be immersed in an oil bath.

381. Test-coil Error.—In those methods of obtaining *B-H* data in which the induction is measured with a stationary test coil surrounding the specimen, some flux passes through the coil which does not pass through the specimen and the induction indicated is therefore too large. The error becomes zero when the area of the specimen and test coil are equal but as some insulation is always required there will always be an error. This error can be corrected for by subtracting from B the quantity

$$\frac{a - A}{A} \times H$$

where a = area of test coil and A = area of specimen. In general, however, it is usually possible to make the difference in area sufficiently small to reduce the error to a negligible quantity.

382. Computation of Area.—The area of a solid test specimen will be the same whether obtained by direct measurement of the diameter or from the mass, length, and the density, provided of course, that the specimen is homogeneous. Sheet material, however, is usually covered with a layer of oxide and as it is customary to leave the scale on the strips, the area computed from the thickness of the individual strips may differ materially from that obtained by weighing. The latter method is the standard practice

[1] "The Best Method of Demagnetizing Iron in Magnetic Testing," C. W. Burrows, *Bulletin*, Bureau of Standards, vol. 4, pp. 270, 273 (1907).

and, since the influence of the scale is less marked in this method, the induction may be as much as 10 per cent. higher than where the area is obtained by direct measurement.

383. Galvanometer Deflection.—Where a ballistic galvanometer is employed in inductive measurements, the deflection should be kept approximately constant by adjusting the series resistance, in order to keep the observational error constant.

384. Effect of Clamping.—Experimenters using magnetic apparatus in which the specimen is clamped in position may question whether or not the fit of the clamps and the pressure exerted on them has any effect on the results. Yensen[1] in his work on high-permeability steels, found that the tightness of the clamping in the Burrows apparatus did have a marked effect. This question has been investigated by others, however, including the author, and no appreciable difference could be found, with ordinary steels, between the condition of very loose clamps and that where a maximum pressure is exerted by the clamps. Of course, the clamps should be properly aligned so that no bending stresses are produced in the specimen.

385. Mechanical Vibration.—In induction measurements, the apparatus should be located where there is no appreciable vibration, because even the small vibrations produced by operating machinery, and so forth, affect the results appreciably.[2] A pad of felt ½ in. thick underneath the apparatus will ordinarily eliminate this source of error.

386. Preparation of Sheet Specimens.—Specimens of sheet materials should not be cut too narrow because of the hardening effect at the edges, due to the cutting. This effect is negligible with a width of 2 in. (5 cm.) In core-loss specimens, care should be taken to remove the burrs from the edges and thus avoid an excessive eddy-current loss which would result from short-circuiting between sheets. The standard practice is to measure core loss with the natural scale as the only insulation between sheets and with half the strips cut in the direction of rolling and half at right angles to the direction of rolling.

387. Knee of Magnetization Curve.—The "knee" of the magnetization curve is frequently used as a reference point in designs

[1] "The Magnetic Properties of Some Iron Alloys Melted *in Vacuo*," T. D. Yensen, *Transactions*, A. I. E. E., vol. 34, p. 2601 (1915).

[2] "The Best Method of Demagnetizing Iron in Magnetic Testing," C. W. Burrows, *Bulletin*, Bureau of Standards, vol. 4, p. 267 (1908).

and calculations and is therefore sometimes indicated in a report of a *B-H* measurement. Ball[1] has directed attention to the fact that the location of this point is largely a function of the scale selected for plotting the data, and therefore care should be taken to bear this in mind when interpreting curves of this character.

388. Plotting Magnetization Curves.—If logarithmic paper is used instead of standard rectangular paper for plotting *B-H* curves, the curve will be expanded at the bottom and condensed at the top, thus facilitating interpolations of *B* at low values and of *H* at high values.

389. Standard Data.—It is rarely necessary to determine the complete curve of the various magnetic properties of iron and steel. While there is no generally accepted practice in regard to the points at which measurements are made for commercial purposes, the specifications of the A. S. T. M. may be taken as sufficiently comprehensive for most purposes, particularly as they are in agreement with the practice of the Bureau of Standards for measurements of this character. These specifications prescribe tests as follows:

(*a*) *Normal Induction Data* (Standard test for solid or sheet iron and steel for electromagnets).—Determination of *H* at inductions of 2,000, 4,000, 6,000, 8,000, 10,000, 12,000, 14,000, 16,000, 18,000 and 20,000 gausses, or, up to *H* 300.

(*b*) *Hysteresis Data* (Standard test for steel for permanent magnets).—Determination of magnetizing force, residual magnetism and coercive force, all corresponding to a maximum induction of 14,000 gausses.

(*c*) *Core Loss.*—Measured at a maximum induction of 10,000 gausses and a frequency of 60 cycles per second.

[1] "Some Notes on Magnetization Curves," J. D. Ball, *General Electric Review*, January, 1915, p. 31.

CHAPTER XVI

CURVE-DRAWING INSTRUMENTS

390. General.—Curve-drawing or graphic instruments are essentially indicating instruments so arranged that a permanent, continuous record of successive, and approximately instantaneous values of the quantity being measured, is made on a chart. The electrical quantities for which standard types of graphic instruments have been developed are current, potential, power, power-factor and frequency. Some types of maximum demand instruments (Chapter 10) are curve drawing. Also, there are a number of graphic instruments which measure non-electrical quantities but which are essentially electrical instruments, such, for example, as temperature recorders and speed recorders. These instruments are, however, simply modifications of one of the standard types described in this chapter.

The fundamental principles in all graphic electrical instruments are those of corresponding indicating instruments of standard types with the addition of a suitable curve-drawing mechanism. However, this added feature introduces a number of problems, the principal ones of which are the friction of the pen on the record paper and the production of a continuous, legible record over long periods of time without the neccessity of continuous attention. The distinguishing features of the various types of instruments are, to a large extent, the various schemes which have been developed to solve these problems.

Graphic instruments may be divided in a general way into two classes. In one class, the record is made directly by the moving element of the instrument proper and in the other, the record is made by a separately operated mechanism. In all cases, the chart itself is driven by a clock mechanism which is entirely separate from the electrical instrument itself.

Graphic instruments may also be divided into two general classes on the basis of accuracy which in turn is fairly well indicated by the cost. In general the cheaper instruments (order of $50) are less accurate, but there are a great many classes of

measurements where a higher precision would be actually useless. Furthermore, the more expensive (order of $100 and over) and accurate instruments are also more delicate which prohibits their use under circumstances where the more rugged and simple instruments would operate satisfactorily.

Direct-acting Instruments

The best-known examples of graphic instruments where the record is made directly by the moving element are the instruments

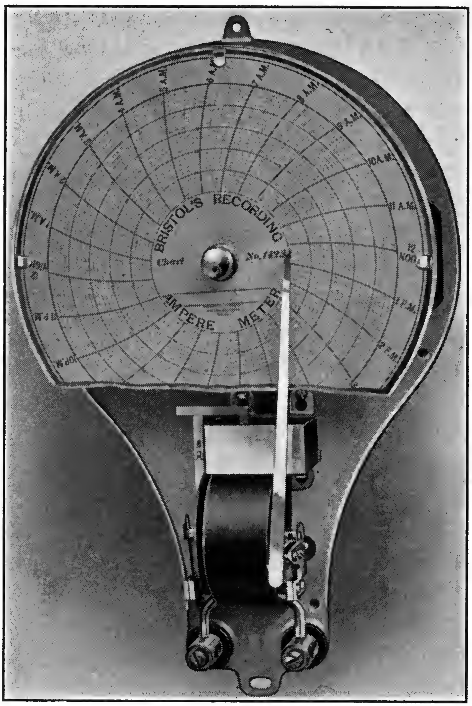

Fig. 219.

of that class made by the Bristol Co., the Westinghouse Co., the General Electric Co. and the Esterline Co.

391. Bristol Instruments.—The important features of the Bristol instruments are the use of a circular chart, simplicity, ruggedness and relatively low cost. They are made for the measurement of continuous current and potential, and for alternating-current, potential and power.

The construction of the ammeters (both continuous-current and alternating-current) is shown in Fig. 219. A stationary coil carries the current to be measured and attracts a disc armature rigidly attached to a non-magnetic shaft which passes through the center of the coil. This shaft and disc, which constitute the moving element, are mounted on knife edges and supported by vertical springs. The small movement of the armature is multiplied by suitable means and transmitted to the pen. The chart is rotated by a clock mechanism placed behind the chart.

The voltmeters employ the electrodynamometer principle and are used for either continuous or alternating current. Two coaxial coils wound with many turns of fine wire are connected in series with each other and through a non-inductive resistor to the line. One coil is stationary and the other is mounted on a non-magnetic shaft, the construction being similar to that of the ammeters. When so connected that the magnetic fields are cumulative, the movable coil is attracted and deflections are produced through a multiplying device, which are more than proportional to the voltage. This gives a higher sensitivity at the upper part of the scale.

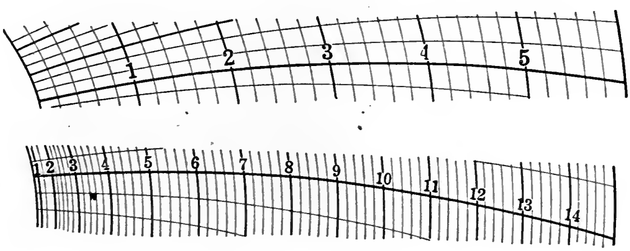

Fig. 220.

If a greater sensitivity at the lower part of the scale is desired, the instrument is furnished with the coils connected in opposition so that the movable coil is repelled instead of being attracted.

The wattmeters are similar to the voltmeters, the stationary coil being wound with heavy wire and connected in series with the circuit while the movable coil is wound with fine wire and connected across the circuit.

All Bristol instruments are made for three sizes of charts, 6, 8, and 12 in. in diameter and with various clock speeds. Fig.

220 shows two specimens of 12-in. charts, one with an equal-part scale and one with an increasing scale.

The record is ordinarily made with a special pen and ink, a small reservoir in the pen carrying sufficient ink for at least one chart. Where specially high sensitivity is required, a smoked chart is used, the record being made with a fine-wire stylus which produces much less friction than a pen. After the record is made it is protected from injury by dipping the chart in a weak solution of shellac which "fixes" the smoke.

Damping is obtained by a vane attached to the movable member and submerged in oil in the box shown just above the coils in Fig. 219.

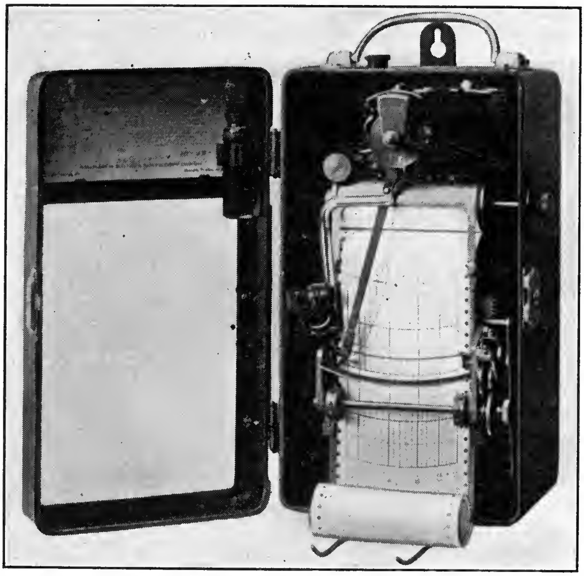

FIG. 221.

392. Westinghouse Type U Instruments.—These are low-price, rugged instruments suitable for either portable or switchboard use (Fig. 221). They are made for the measurement of current and potential only, either alternating or continuous current. The accuracy is, however, somewhat higher on alternating current.

The principle of operation is the solenoid and a spring-controlled core, the latter acting directly on an arm which carries the recording pen. The record is a continuous one made on a roll

of paper driven by a clock mechanism at a rate of 1 or 2 in. per hour. The record space is 2.5 in. wide. The only difference between the ammeters and the voltmeters is in the winding. The voltmeters are usually furnished with a suppressed zero in order to get an open scale at the working potential. Damping is obtained by means of an oil dashpot. The pen is a V-point similar to the Bristol pen but provision is made for keeping it supplied with ink by means of a thread wick feeding from the glass tube reservoir shown at the left of Fig. 221.

393. General Electric Instruments.—The General Electric Co. manufactures two forms of direct-acting instruments—type CR

Fig. 222.

which is an inexpensive instrument for use where high accuracy is not required, and type C_4, which is a more accurate and also a more expensive instrument.

The principle of operation of the type CR instruments (Fig. 222) is the same as that of the Westinghouse type U instruments, namely, a solenoid with a movable core which actuates the re-

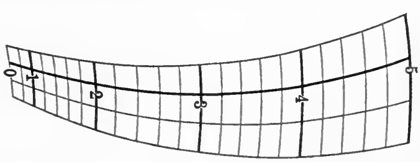

Fig. 223.

cording pen. The core, however, is controlled by gravity instead of by a spring. The chart is clock-driven and is circular in form, with an active width of about $2\frac{1}{4}$ in. (see Fig. 223). The

regulation V-point metal pen is used with a special fluid ink as in the other instruments of this class. Air damping is employed. Ammeters and voltmeters are furnished in both portable and switchboard forms. They may be used on either continuous or alternating current.

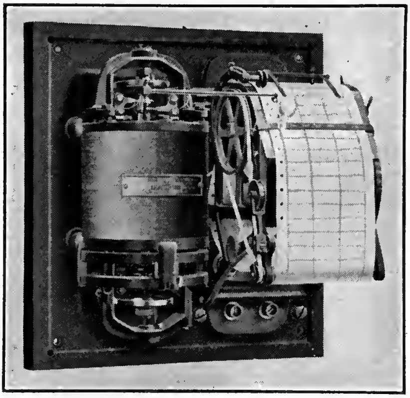

FIG. 224.

The type C instruments are made for measuring alternating current, potential, power, frequency, power-factor, and continuous current and potential. The general appearance of all is similar to that of the polyphase wattmeter shown in Fig. 224 (cover removed).

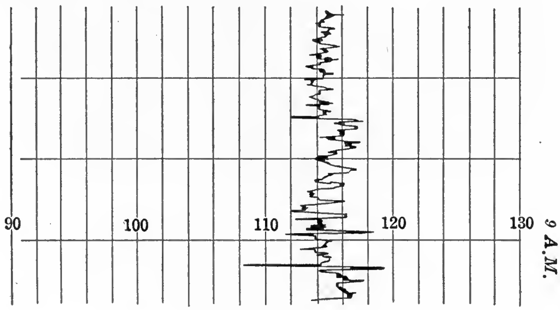

FIG. 225.

The alternating voltmeters, wattmeters and power-factor indicators are constructed on the direct-reading, dynamometer principle. The frequency indicator is of the tuned-circuit type and has an iron armature. Voltmeters and single-phase wattmeters have one fixed and one movable set of measuring coils; polyphase

wattmeters have two sets of fixed coils and two sets of movable coils, the latter rigidly attached to a common shaft. The alternating-current ammeters are constructed on the magnetic-cane principle. Power-factor instruments have two potential coils and one current coil, so arranged as to actuate a needle which shows on a single direct-reading scale the true power-factor of balanced two- or three-phase, three- or four-wire circuits. The continuous-current voltmeters are the same as the alternating-current voltmeters but the ammeters employ an electromagnetic field, the principle of operation being similar to that in the astatic type of indicating ammeters made by this company.

FIG. 226.

The distinctive features of the type C instruments are the laminated iron magnetic shield which surrounds the electrical elements, and the supporting of the movable element by a piano-wire suspension. The record is rectilinear (Fig. 225) and is made on a continuous roll with a standard rate of feed of 3 in. per hour. The active width of the chart is 3⅛ in. Effective damping is obtained by an aluminium vane between the poles of a permanent magnet. A special form of pen combined with a reservoir is used (Fig. 226). It is relatively heavy but friction is reduced to a minimum by counterweighting and by using jewel bearings at the joints connecting the pen arm and the movable element. These connecting links are so arranged that a straight line motion of the pen is obtained.

394. Esterline Instruments.—The continuous-current ammeters and voltmeters employ the standard D'Arsonval principle and the alternating-current ammeters, voltmeters and wattmeters utilize the electrodynamometer principle. The record is made by a very finely pointed pen on a continuous roll having a scale width of 4½ in. and driven by a clockwork mechanism which is provided with change gears so that various rates from ¾ in. to 12 in. per hour or per minute can be obtained with the same instrument. The pen is continuously supplied with ink from a stationary reservoir by siphon action.

The continuous-current ammeters are 50-millivolt voltmeters operated from shunts so that the range of the instrument is changed by simply changing the shunt.

These instruments (Fig. 227) are the most expensive of the standard commercial curve-drawing instruments of American manufacture, but they have a high accuracy and sensitivity. The high-chart-speed feature, together with the flexibility and high sensitivity, make these instruments particularly suitable for laboratory work.

Relay-type Instruments

395. Westinghouse Instruments.—The relay instruments made by the Westinghouse Electric and Manufacturing Co. are a conspicuous example of the class where the recording mechanism is separate from the instrument proper. The moving element operates, by means of contacts, a relay which in turn operates the recording mechanism. Thus the moving element does not have to develop a large torque, and ample power can be applied to the recording pen so that friction does not affect the sensitivity of the instrument.

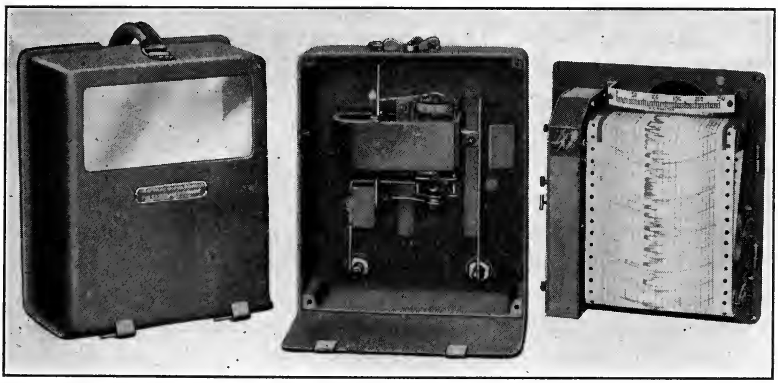

Fig. 227.

The record is made on a continuous roll having a scale width of 5¼ in. and ruled in rectangular coördinates—that is, the pointer moves in a straight line which is, at all times, perpendicular to the direction of travel of the paper. Adjustable damping is obtained with an oil dashpot. A distinctive feature is the clock which drives the paper. It is electrically wound at the end of each 2-in. period.

These instruments are made for the recording of all quantities which are ordinarily measured with indicating instruments—

voltage, current, power, frequency and power-factor. The meter elements of voltmeters, both continuous and alternating, of alternating-current ammeters and of wattmeters, are of the Kelvin

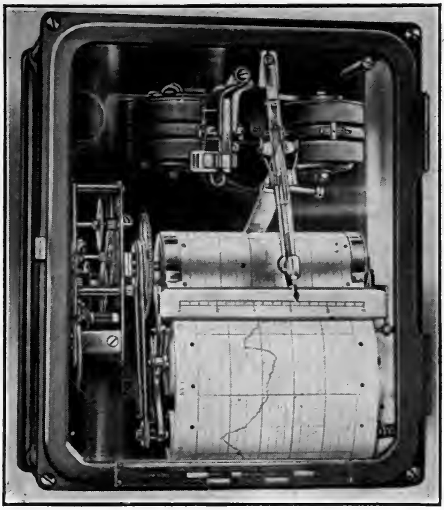

FIG. 228.

balance type. Continuous-current ammeter elements are of the permanent-magnet type with two sets of magnets and moving

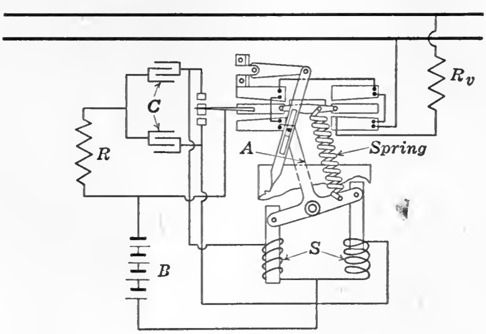

FIG. 229.

coils astatically arranged. The frequency and power-factor meters operate on the same principles that are used in the corresponding indicating instruments made by this company.

Fig. 228 is an illustration of a voltmeter and Fig. 229 is a schematic diagram of the connections. When the two movable coils are in the mid-position the forces of attraction and repulsion acting on them are balanced by the spring and the system is in equilibrium. If the voltage changes, the coils move up or down and one of the solenoids, *S*, is excited by the closing of either the upper or the lower contact shown at the left. This solenoid actuates a core attached to the lever *A* until the spring tension again balances the forces acting on the coils. The solenoids are

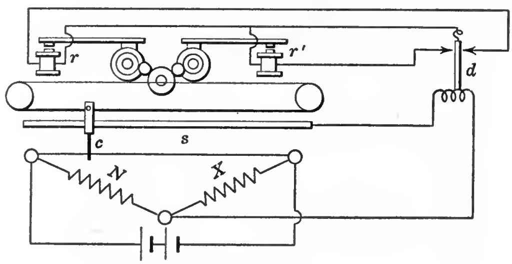

FIG. 230.

usually operated from a 110-volt source, represented by *B* which may be either continuous or alternating current. Provision is made for eliminating arcing at the contacts by means of a resistor *R* and condensers *C*.

396. Callender Recorder.—This instrument, made by the Cambridge Scientific Instrument Co., is a very elaborate and expensive instrument of high sensitivity capable of measuring very small quantities. It employs the principle of a slide-wire bridge (Fig. 230) in which the resistance of one arm, *X*, varies with the current, potential, or power to be measured. As soon as the bridge is unbalanced, a D'Arsonval galvanometer operates a relay, *r* or *r'*, which moves a contact, *c*, along the slide wire, *s*, until balance is restored, when the relay circuit opens. This contact also carries the recording pen, which leaves an ink record on a rectilinear roll chart.

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